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ESTIMATION OF COBB – DOUGLAS PRODUCTION FUNCTION FOR DEVELOPING COUNTRIES*

GELİŞMEKTE OLAN ÜLKELER İÇİN COBB – DOUGLAS ÜRETİM FONKSİYONUNUN KESTİRİMİ

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Abstract

Discussions on the production function have always taken care of the attention of economists. The production function is a mathematical expression that shows the relationship between inputs and outputs. The characteristics of this relationship can be expressed in three different concepts, scale flexibility, output flexibility, and substitution flexibility, respectively. Gross Domestic Product (GDP) is an indicator of economic growth. This study aims to estimate the Cobb – Douglas production function in developing countries by using capital, labor, and energy consumption input factors and investigate the effect of economic input factors on economic growth. For this purpose, the Cobb – Douglas production model was created using capital, labor, and energy consumption inputs. In this study, linear panel data analysis techniques were used for 22 developing countries with the data of the 1980-2016 period. Output elasticity of capital, labor, and energy consumption inputs in Cobb – Douglas production function is 0.602, 0.455, 0.147, respectively, which means that the economies of developing countries are capital intensive. The total share of all production factors is 1.204, and there is an increasing return to scale. Capital, labor, and energy consumption inputs of these economies have a positive impact on GDP. In addition, insufficient capital in these countries can be compensated by labor and/or energy.

Keywords: Production function, panel data analysis, economic growth, developing countries
JEL Classification: C01, C02, C13, C23

Öz

Üretim fonksiyonu hakkındaki tartışmalar her zaman iktisatçıların dikkatini çekmiştir. Üretim fonksiyonu, girdiler ve çıktılar arasındaki ilişkiyi gösteren matematiksel bir ifadedir. Bu ilişkinin özellikleri sırasıyla

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ölçek esnekliği, çıktı esnekliği ve ikame esnekliği olmak üzere üç farklı kavramla ifade edilebilir. Gayri Safi Yurtiçi Hasıla (GSYİH) ekonomik büyümenin bir göstergesidir. Bu çalışma, sermaye, işgücü ve enerji tüketimi girdi faktörlerini kullanarak gelişmekte olan ülkelerde Cobb – Douglas üretim fonksiyonunu tahmin etmeyi ve ekonomik girdi faktörlerinin ekonomik büyüme üzerindeki etkisini incelemeyi amaçlamaktadır. Bu amaçla, sermaye, emek ve enerji tüketimi girdileri kullanılarak Cobb – Douglas üretim modeli oluşturulmuştur. Bu çalışmada, gelişmekte olan 22 ülkeye ait 1980-2016 dönemi verilerinin analizi için doğrusal panel veri analizi tekniklerinden faydalanılmıştır. Cobb – Douglas üretim fonksiyonunda sermaye, emek ve enerji tüketimi girdilerinin çıktı esnekliği, sırasıyla, 0.602, 0.455, 0.147 olup, bu da gelişmekte olan ülke ekonomilerinin sermaye yoğun olduğu anlamına gelmektedir. Tüm üretim faktörlerinin toplam payı 1.204 olup, ölçeğe göre artan getiri söz konusudur. Bu ekonomilerin sermaye, işgücü ve enerji tüketim girdilerinin GSYİH üzerinde olumlu bir etkiye sahip olduğu söylenebilir. Ayrıca, bu ülkelerdeki yetersiz sermaye, emek ve / veya enerji ile telafi edilebilir.

Anahtar Kelimeler: Üretim fonksiyonu, panel veri analizi, ekonomik büyüme, gelişmekte olan ülkeler
JEL Sınıflandırması: C01, C02, C13, C23

1. Introduction

Production is the act of producing new goods and services using goods and services in its broadest definition (Eğilmez, 2016, p. 112). The resources used in the production process and enabling production are called inputs or factors of production (Besanko & Braeutigam, 2010, p. 228). The conversion of production factors into outputs is represented by mathematical expressions called the production function in the neoclassical tradition. In economic theory, the production function is simply described as the technical relationship between economic inputs and outputs (Cheng & Han, 2017). The most important condition for a country's economic growth is to increase production and to use production factors effectively to ensure this (M Songur & Saraç Elmas, 2017). It is generally believed that Philip Wicksteed (1894) first expressed the concept of production function algebraically. However, there is some evidence that Johann Von Thünen was the first economist to express his production functions as a formula in the 1840's (Mishra, 2010). It is difficult to establish the production functions to include all the inputs used in production. For this reason, the inputs in the production functions are generally grouped under two groups as "labor" and "capital" (Eğilmez, 2016, p. 115). While the production functions created using capital and labor inputs in the Neoclassical tradition do not take energy as an input factor due to its view of energy as an intermediate product, energy crises throughout history have emphasized the role of energy in economic growth. This has led some researchers to include energy in the production function (Brockway et al., 2017). In addition, one of the most significant issues in the economics literature is the relation between energy consumption and economic growth (Behera & Mishra, 2020).

Mathematically, there are many forms of production functions in the literature. In literature, very often used production functions are linear production function, Cobb – Douglas production function, Constant Elasticity of Substitution production function (CES), Variable Elasticity Substitution production function (VES), Leontief Production function, and Translog production function (Cheng & Han, 2017). In this study, Cobb – Douglas production function was used.

Cobb – Douglas production function has a lot of applications. One of the first studies in this field is Bronfenbrenner and Douglas's (1939) work. Later, Douglas analyzed the production function developed under different names in different years (Daly & Douglas, 1943; Daly, Olson, & Douglas, 1943; Gunn & Douglas, 1941, 1942). Regression analysis was used in all of these studies. Some of the studies using panel data analysis are as follows Wakelin (2001), Cantos, Gumbau-Albert, & Maudos (2005), Çermikli & Tokatlıoğlu (2015), Inglesi-Lotz (2016) and Chikabwi, Chidoko, & Mudzingiri (2017).

The popular production theory of Ferguson and Pfouts (1962) and Berndt and Christensen (1973) is advanced by Cameron and Schwartz (1980), Field and Grebenstein (1980).

There are many factors that have contributed to GDP, such that capital, labor, energy, optimal allocation of technology sources, innovations, etc. In this study, the Cobb – Douglas production function is used to scale the effect of capital, labor, and energy consumption at the macro level on economic growth in developing countries. The aim of this study is to estimate the Cobb – Douglas production function at the macro level for developing countries.

It is frequently claimed in the current press and financial markets that capital is needed in developing countries, and external borrowing is mandatory, and there will be no development without foreign capital. If this claim is correct or not can be seen by estimating the elasticity of substitution between production factors.

In this context, data from 22 developing countries were used in our study. In the study, the main reason for the restriction made in the context of country and period is to prefer working with a balanced data set. In this framework, the most comprehensive data set has been created as of the countries and period considered. The Human Development Index, published annually by the United Nations, was used for the separation of countries.

In the study, we used three different inputs in the production function. These are capital (K), labor (L), and energy consumption (E) inputs that must be involved in the production function. Gross Domestic Product (GDP) values, which are indicators of the economic growth of countries, were used to represent the output. The analysis in the study were carried out with the help of linear panel data analysis techniques.

The remaining part of this study is organized as follows. Section 2 reviews the Cobb – Douglas production function. Section 3 describes the data and methods used in the study. The study findings are expressed in detail in Section 4. Section 5 includes the results of the study.

2. Cobb – Douglas Production Function

Cobb and Douglas (1928) introduced the most famous and known production function in the form,

$$Q = f(K, L) = AK^{\alpha}L^{\beta} \tag{2.1}$$

where Q is a total production which means the value of all goods produced in a year, K is the capital input which represents by the total investment in fixed assets, L is the labor inputs which is the total number of person or hours worked in a year) and A is a positive constant which means total factor productivity (Cobb & Douglas, 1928). The parameter α and β show the output elasticities to capital and labor, respectively.

Output elasticity measures the sensitivity of output to a change in the levels of labor and capital used in production. Cobb – Douglas production function allows the change of the size of the inputs affected by factor price changes. One of the limitations of this production model is that it uses two input factors to explain production (Liao, Wu, & Xu, 2010).

The general form of production function is described by $f: D \rightarrow R_{+}, \;\; D \in R_{+}^{n}$ and,

$$Q = f(X_1, X_2, \ldots, X_n) \tag{2.2}$$

Where $X_1, X_2, \ldots, X_n$ are inputs and Q is production level. A production function with n input factors is called h –homogeneous, $h > 0$, if

$$f(kX_1, X_2, \ldots, k\,X_n) = k^h f(X_1, X_2, \ldots, X_n) \tag{2.3}$$

Where $k \in R$ (Onalan & Basegmez, 2018). In addition, in the production function,

If $h = 1$, function denotes the constant return to scale

If $h > 1$, it shows increasing returns to scale

If $h < 1$, function denotes the decreasing return to scale (Besanko & Braeutigam, 2010).

3. Data and Methodology

In this study, annual data are used from Penn World Tables (PWT 9.1) between 1980 – 2016. 22 out of a total of 24 developing countries classified in this way according to the Morgan Stanley Capital International (MSCI) Index were selected [1]. In practice, at the stage of creating a data set, a selection was made to the extent of the accessibility of the relevant data of the countries used in the study, and the most comprehensive data set was created for the period and countries considered. Russia and the Czech Republic were not included in the analysis since they had incomplete data since their establishment.

In the differentiation of developing countries, generally, two classification types come to the fore. One of them is the income classification according to the Atlas Method made by the World Bank, and the other is the Human Development Index (HDI) announced by the United Nations. The HDI has been included in the Human Development Reports prepared by the United Nations Development

1 https://www.msci.com/market-cap-weighted-indexes

Program since 1990 as a unit of measurement that evaluates the life quality and the standard of living of the countries. Unlike traditional income-weighted measures, HDI evaluates in terms of life span, education level, and having the resources to provide a good standard of living when evaluating a country for development.

In the study, three different inputs were used: capital (K), labor (L), and energy consumption (E). As output, the Gross Domestic Product (GDP) values of the developing countries were used. The data used in the analysis, namely, Gross Domestic Product (real GDP – million dollars), Labor (number of people engaged in any business in that country – million) and Capital (Realized in 2011-dollar prices – million dollars) variables is obtained from Penn World Table (PWT) 9.1. Quad Btu Energy consumption input is from U.S. Energy Information Administration.

The variables of GDP and K are realized with 2011-dollar prices (2011=1). The labor variable is the number of people engaged with any job in the present country. The quality of labor may not be the same for every country. In order to minimize this difference, the human capital index in PWT 9.1 data set has been multiplied with the labor variable, similar to the usage in Songur (2019). Barro – Lee (2013) and Cohen – Sato (2007) human capital indices were used in the Human Capital data included in the PWT 9.1 data set (Barro & Lee, 2013; Cohen & Soto, 2007). These indices include the return of education, private/social benefits of education and labor productivity. In this sense, it is an important index that measures human capital in terms of both qualitative and quantitative.

Table 1. Descriptive statistics (1980-2016)

Variable	Count	Mean	Std. D.	Min	Max
GDP	814	865170.3	1823405.32	8814.25	17546088
K	814	3265762	7330460.22	64209.74	87069408
L	814	137.76	338.07	0.20	2009.76
E	814	23.94	60.44	0.68	550.26

Energy data has been organized as Energy consumption data from International Information Statistics published on U.S. Energy Information Administration website. This variable is obtained by taking the sum of Petroleum, Primary Energy, Biofuel, Coal, Natural Gas, Hydroelectric and non-hydroelectric renewable energy consumption and arranged as Quad btu.

Descriptive statistics of the panel data set or developing countries are given in Table 1. This dataset is balanced.

3.1. Research Model

In physics, work is equal to force times distance. If this approach in physics is applied to the field of economic growth, we use the variables of capital K, labor L, and energy E to obtain the output as a growth rate. As a result, in the case where the energy variable is also included in the model, the production function is given as follows (Thompson, 2016),

$$GDP = A(KL)^{\beta_1}(KE)^{\beta_2} \tag{3.1}$$

This production function is expressed linearly by taking the logarithm of both sides in Eq. (3.1). In this study, linear panel data analysis techniques were used to estimate the Cobb – Douglas production function of developing countries. In this case, the model obtained using the static panel data method for the production function given in Equation (3.1), is taken in the linear form of,

$$ln\ GDP_{it} = \beta_0 + \beta_1(ln\ K_{it} + ln\ L_{it}) + \beta_2(ln\ K_{it} + ln\ E_{it}) + u_{it} \tag{3.2}$$

Where $\beta_0, \beta_1, \beta_2$ are parameters, u_{it} is the error term and $\beta_0 = ln\ A$ ..

3.2. Methods Used for Parameter Estimations the Cobb – Douglas Production Function

In the study, the factors affecting the growth of developing countries and the relationship between capital, labor, and energy consumption in these countries are analyzed with the fixed effects panel data estimation method.

Panel data models are created in different ways based on the assumptions of error terms and parameters used in the analysis. In this context, the existence of unit effect belonging to countries was investigated using F, LR, and LM test statistics which are panel regression models.

If it is understood that there are unit and/or time effects as a result of the tests, it is necessary to decide whether these effects are constant or random.

Hausman Specification test was conducted to determine whether the model is suitable for random or fixed effects model. The basic assumption of the random effects model is that the effects arising from the units are random. If this assumption is valid, both the fixed and random effects estimation methods will give consistent results. But the random effects method will be more efficient. If this assumption is not valid, then the generalized least squares estimation method, which is the random effects estimation method, is biased and not consistent. In this context, if the null hypothesis H_0 that "there is no correlation between the independent variables in the model and the unit effect" is rejected, the result is that the fixed effects model should be used, and if not, the random effects model should be used (Hausman, 1978). The basic hypothesis that the random effects estimator is valid by using statistics that fit the distribution χ^2 with k degrees of freedom is tested by Hausman test.

The variance covariance matrix of the error terms to be the unit matrix in the case of Heteroscedasticity, Autocorrelation, and Cross-Sectional Dependence, which are econometric assumptions. In such a case, a suitable correction method for deviations from the assumptions regarding the error term should be selected and applied (Yerdelen Tatoğlu, 2018).

After this stage, tests are carried out to see the compatibility of the model with the econometric assumptions, and the final regression analysis was obtained by performing the Driscoll-Kraay test, which eliminates the deviations from the resulting assumptions.

4. Findings

In the study, firstly, the existence of unit effect or time effect was tested. For this reason, F test, LR (Likelihood ratio) test, LM (Breusch – Pagan Lagrange Multiplier) test which are panel regression models were performed, respectively. We used STATA 14.0 (College Station, TX, USA) to analyze the data.

4.1. Testing the existence of unit and time effect

The F test is adapted from the Chow test and it is tested whether the data differs by unit or not (Chow, 1960). If the data does not differ according to units, the classical model is said to be suitable. LR test is used to test the classical model against the random effects model (Yerdelen Tatoğlu, 2018). LM test is based on the residuals of the least squares model and was developed by Breusch-Pagan to test the existence of individual heterogeneity, i.e. whether the pooled least – squares model is suitable or not against the random effects model (Breusch & Pagan, 1980).

The null hypothesis H_0 for these tests is that there is no unit effect, that is, the pooled (classical) model is valid. The alternative hypothesis H_1 is that there is unit effect; that is, the random (random) effects model is valid. Results for these tests can be seen in Table 2.

Table 2. Test results for F, LR, LM test

VARIABLES	(1) F test	(2) LM test	(3) LR test
lnK+lnL	0.455***	0.339***	0.420***
	(0.027)	(0.023)	(0.028)
lnK+lnE	0.147***	0.245***	0.177***
	(0.026)	(0.022)	(0.026)
Constant	2.398***	2.841***	2.521***
	(0.121)	(0.122)	(0.155)
Observations	814	814	814
Number of Country	22	22	22

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

When Table 2 is examined, as the H_0 hypothesis is rejected according to F test, LR test and LM test results. Therefore, it can be said that there is a unit effect in the model.

4.2. Hausman Test

LR test, LM test, and F test show the presence of unit effect. However, they do not provide information about the relationship between unit effect and at least one of the independent variables. Hausman test is used to decide whether the unit effect in our model is constant or random and to choose between estimators (Hausman, 1978). The null hypothesis H_0 for Hausman test is that there is no unit effect, that is, fixed effects are consistent, random effects are effective (consistent). The alternative hypothesis H_1 is that there is unit effect, that is, fixed effects are consistent, random effects are inconsistent. Hausman test results can be seen in Table 3. An estimator is effective means that it is consistent (Yerdelen Tatoğlu, 2018).

The Hausman test statistic is tested by comparing it with the χ^2 table value with 2 degrees of freedom. According to Table 3, the probability value of the analysis made for the current model is significant. As the H_0 hypothesis is rejected at the end of the test, the random effects estimator is inconsistent, and the fixed effects model is suitable for analyzing the model. To interpret the Cobb – Douglas production model, the fixed effects model is preferred because it is consistent. In this case, the results of the fixed effects estimator should be relied upon and interpreted.

Table 3. Hausman Test Results

VARIABLES	Hausman test
lnK+lnL	0.420***
	(0.028)
lnK+lnE	0.177***
	(0.026)
Constant	2.521***
	(0.155)
Hausman test statistics	67.39 ***
	(0.0000)
Observations	814
Number of Country	22

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

After this stage, constant variance test, autocorrelation test between error terms and cross-sectional dependence test which are tests for deviations from econometric assumptions will be performed.

4.3. Testing for Constant Variance

To groupwise heteroskedacity in the residuals of a fixed effect regression model, modified Wald statistic is used (Greene, 2018). The null hypothesis H_0 specifies that $\sigma_i^2 = \sigma^2$ for $i = 1, \ldots, N$, that is, the variance is constant for all units; there is no heteroskedacity. The alternative hypothesis H_1 specifies that there is varying variance.

Table 4. Modified Wald Statistic Results

VARIABLES	Modified Wald test
lnK+lnL	0.455***
	(0.027)
lnK+lnE	0.147***
	(0.026)
Constant	2.398***
	(0.121)
Test statistics	4021.36***
	(0.000)
Observations	814
Number of Country	22
R-squared	0.916

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

Modified Wald test statistic results can be seen in Table 4. This statistic is tested by comparing it with the χ^2 table with 22 degrees of freedom. When Table 4 is examined, the probability value of the analysis made for the current model is significant. According to this result, there is a changing variance problem in the model.

4.4. Testing Autocorrelation in Fixed Effect Model

Durbin-Watson and Baltagi-Wu LBI test statistics proposed by Bhargava, Franzini and Narendranatham are used to test the presence of autocorrelation in the fixed effects model (Bhargava, Franzini, & Narendranathan, 1982).

The null hypothesis H_0 for these tests is that there is no autocorrelation between error terms. Thus, the alternative hypothesis H_1 is that there is autocorrelation between error terms.

Durbin-Watson and Baltagi-Wu LBI test statistics can be seen in Table 5. Durbin-Watson test statistics value is 0.114< 2, and Baltagi-Wu LBI test statistics value is equal to 0.268< 2. Since both values are less than 2, it can be interpreted that there is an autocorrelation problem for the fixed effects model.

Table 5. Autocorrelation test results

Variables	Autocorrelation
lnK+lnL	0.419***
	(0.044)
lnK+lnE	0.285***
	(0.037)
Constant	0.498***
	(0.032)
Modified Bhargava et al. Durbin-Watson	0.114
Baltagi-Wu LBI	0.268
Observations	814
Number of Country	22

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

4.5. Testing Cross-Sectional Dependence

In panel data models, one of the general assumptions is that error terms are independent of units. This situation prevents the correlation matrix from being an identity matrix. Therefore, the assumption of non-correlation between units should be tested. There is a variety of tests for Cross-Sectional dependence in the literature. Pesaran's, Freedman's and Frees' tests were used in this study to test the existence of correlation between units (Friedman 1937; Frees 2004; Pesaran 2004).

Table 6. Cross-Sectional dependence statistics results

Pesaran's test of cross-sectional independence	13.658***
	(0.000)
Friedman's test of cross-sectional independence	129.775 ***
	(0.000)
Frees' test of cross-sectional independence	4.614***
	(0.000)

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

Table 6 shows the test statistics and probability value for the test of Cross-Sectional dependence with Pesaran's, Freedman's, and Frees' tests. The null hypothesis H_0 of Cross-Sectional dependence specifies that there is no cross-sectional dependence among the panels. It can be seen that H_0 can reject according to three tests. Thus, the results indicate that there is Cross-Sectional dependence among the variables.

4.6. Driscoll – Kraay Test

Autocorrelation, heteroskedacity and Cross-Sectional dependence problems in the Cobb – Douglas production model are present. If panel regression models have these problems, Driscoll–Kraay standard errors are adjusted. Driscoll-Kraay standard errors test eliminate bias and give robust estimators with autocorrelation, heteroskedacity, and Cross-Sectional dependence (Driscoll & Kraay, 1998; Hoechle, 2007).

When the results of the estimations in Table 7 are compared with the results of the fixed effects estimator in Table 2, it is seen that the coefficient estimations are the same. It was determined that the variables of Energy consumption, Capital and Labor included in the model created by using the data of the developing countries between 1980-2016 are significant at 95% confidence level. According to the results of the Driscoll – Kraay standard errors test, all variables in the model are significant either individually or as a whole. In addition, since $R^2 = 0.9165$, the independent variables in the model explain approximately 92% of variability in GDP.

Considering Driscoll – Kraay standard error test results, our model can finally be written as,

$$ln\ GDP = e^{2.398} + 0.455\ (ln\, K\ + ln\, L\,) + 0.147(ln\, K\ + ln\, E\,) + u_{it}. \tag{4.1}$$

After making the necessary arrangements, Eq. (4.1) becomes as follows,

$$ln\ GDP\ = 11.001 + 0.602\ ln\ K\ + 0.455\ ln\ L\ + 0.147\ ln\ E\ + u_{it}. \tag{4.2}$$

Table 7. Driscoll – Kraay standard errors test results

Variables	Driscoll – Kraay
lnK+lnL	0.455***
	(0.067)
lnK+lnE	0.147***
	(0.044)
Constant	2.398***
	(0.541)
Within R – squared	0.917
Observations	814
Number of groups	22

*Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1*

According to the model in Eq. (4.2), it can be say that the 1% increase in Capital, Labor and Energy consumption, which are the independent variables in the model, will increase the Real GDP by approximately 0.602%, 0.455%, 0.147%, respectively. There is increasing returns to scale, since the sum of the shares of all factors of production is 1.204.

The output elasticity of Capital, Labor and Energy consumption inputs were obtained as 0.602, 0.455, and 0.147, respectively. This situation indicates that the economies of developing countries are capital intensive. In addition, it can be said that the Capital, Labor, and Energy consumption inputs of these countries' economies positively affect GDP.

5. Results

The production function has been used in many studies until today. Gross Domestic Product (GDP) is an indicator of economic growth. The aim of this study is to estimate the Cobb – Douglas production function in developing countries by using capital, labor, and energy consumption input factors and to investigate the effect of economic input factors on economic growth.

Energy is an important tool for the growth of countries. For this reason, in addition to the classical production function inputs, energy consumption input is also included in our study. Studies on the relationship between energy consumption and economic growth or the share of energy consumption between growth dynamics differ according to the period, country group and method examined.

In addition, based on the expression of "Work is defined as force times distance" in physics, it was ensured that more reliable estimates were obtained for the economic growth estimation of developing countries between 1980 and 2016 by evaluating the individual interactions of capital with labor and energy consumption. In this study, the interaction of the inputs in the model with each other can be considered as one of the biggest contributions of the study.

The main reason for limiting the number of countries and periods in the study is the desire to work with a balanced data set. In this sense, the most comprehensive data set has been created as of the countries and period to be used in the analysis.

According to the empirical results of this study, it can be said that capital, labor, and energy consumption inputs of developing country economies affect GDP positively. According to the results of the regression analysis using the Cobb – Douglas production function, all variables in the model are meaningful either individually or as a whole. In addition, since $R^2 = 0.9165$, and the independent variables in the model explain about 92% of the variability in GDP. The output elasticity of capital, labor and energy consumption inputs were obtained as 0.602, 0.455, 0.147, respectively. This situation indicates that the economies of developing countries are capital intensive. The sum of the shares of all factors of production is 1.204, and there is an increasing return according to the scale. In addition, capital, labor, and energy consumption inputs of these countries' economies positively affect GDP.

Due to the nature of Cobb – Douglas production model, the marginal contribution of capital is higher. In line with Henry Thompson's view of the Cobb – Douglas production model, the marginal contribution of capital to the production function is the sum of the marginal contributions of labor and energy consumption. According to the estimated model, the decrease in one of the production factors can be compensated with the increase in the other factors. For instance, in 2016, estimated GDP value for $(C, L, E) = (6\,202\,699, 62.313, 22.537)$ input vector in Turkey is $1\,394\,141$.

When we reduce the capital by 10% in this input vector, to get the same GDP by keeping the energy consumption input constant, the value of the labor input must be 71.774. In other words, when we get 90% of the capital, the workforce has to increase 1.152 times to get the same GDP. Conversely, when we increase the labor input by 10% while keeping the energy consumption input constant, the capital must increase by 0.931 times to get the same GDP. It should be noted here that the parameters are taken as estimates.

Each model output calculated is a statistic, and it would be less misleading to interpret these statistics together with their standard errors and therefore with confidence intervals. Here, one of the reasons for the asymmetry in the capital requirement is 0.931 times when the labor force is increased by 10%, while the labor requirement is 1.152 times when the capital is reduced by 10%, the models used in the calculations are not deterministic.

According to the result, lack of capital in developing countries such as Turkey may not be mandatory to take external debt. Whether it is possible to compensate for insufficient capital with energy and/or labor can be investigated.

Author Contribution

The author conducted all stages of the study. The author read and approved the final manuscript.

Conflict of Interest

No conflict of interest was reported by the author.

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Resume

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