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Developing students' problem posing skills with dynamic geometry software and active learning framework

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ABSTRACT In the present study, the effects of using dynamic geometry software in active learning framework on students' problem posing skills and their views about problem posing were examined. The participants consisted of 16 eighth-grade students. Data were collected by problem posing tests, open-ended questions, student diaries, and dynamic geometry software supported tasks. The study, designed with the embedded mixed method, lasted 13 weeks. The dependent t-test was used in the analysis of quantitative data, and descriptive analysis was performed in qualitative data. Problem posing skills of students examined according to use of mathematical language, grammar and expression, suitability to acquisitions, quantity and quality of data, solvability, originality, solution of the problem criteria. In the present study, it was determined that the use of dynamic geometry software in active learning framework developed students' problem posing skills. The problems that students posed in the dynamic geometry software during the implementation process indicated improvement in terms of problem posing skills as the weeks passed. It was found that students had positive views about the problem posing process, while they experienced some difficulties in this process.

Keywords: *Active learning framework, Dynamic geometry software, Eighth-grade students, Problem posing skill, Triangles.*

Dinamik geometri yazılımı ve aktif öğrenme çerçevesi ile öğrencilerin problem kurma becerilerinin geliştirilmesi

ÖZ Araştırmada, aktif öğrenme çerçevesinde dinamik geometri yazılımı kullanımının öğrencilerin problem kurma becerilerine ve problem kurmaya yönelik görüşlerine etkisi incelenmiştir. Katılımcılar, 16 sekizinci sınıf öğrencisinden oluşmuştur. Veriler problem kurma testi, açık uçlu sorular, öğrenci günlükleri ve dinamik geometri yazılımı destekli etkinlikler ile toplanmıştır. Gömülü karma yöntem ile tasarlanan araştırma 13 hafta sürmüştür. Nicel verilerin analizinde ilişkili örneklemler t-testi, nitel verilerde betimsel analiz yapılmıştır. Öğrencilerin problem kurma becerileri; matematik dilini kullanabilme, dil bilgisi ve anlatım, kazanımlara uygunluk, veri miktarı ve niteliği, çözülebilirlik, özgünlük, problemin çözümü kriterlerine göre incelenmiştir. Araştırmada, aktif öğrenme çerçevesinde dinamik geometri yazılımı kullanımının öğrencilerin problem kurma becerilerini geliştirdiği belirlenmiştir. Öğrencilerin uygulama sürecinde dinamik geometri yazılımında kurdukları problemler ise haftalar ilerledikçe problem kurma becerileri açısından gelişim göstermiştir. Ayrıca öğrencilerin problem kurma sürecine yönelik görüşlerinin olumlu olduğu ancak bu süreçte bazı zorluklar yaşadıkları görülmüştür.

Anahtar Sözcükler: *Aktif öğrenme çerçevesi, Dinamik geometri yazılımı, Problem kurma becerisi, Sekizinci sınıf öğrencileri, Üçgenler.*

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INTRODUCTION

Problem posing is an intellectual activity defined as making changes in the given problem or creating problems in accordance with the presented mathematical situations (Cai & Hwang, 2020; Silver, 1994). Problem posing activities require many skills including the skill of students to pose complex mathematical problems as well as simple problems, formulate problems from daily life situations, take advantage of different mathematics topics and choose a suitable approach to a mathematical situation (Abu Elwan, 1999). However, problem posing activities that have an important place in the development of mathematical ideas of students are not sufficiently utilized (Ellerton, 2013). It is also stated that there is not enough information about mathematics teaching designed by using problem posing activities (Cai & Hwang, 2020; Cai et al., 2015; Xu et al., 2020). Chen and Cai (2020) pointed out that teaching mathematics with problem posing approach encourages students to think actively and creatively and has potential benefits such as supporting students' understanding mathematical concepts. In addition, the problem posing approach has been found to improve students' problem solving skills (Abu Elwan, 2002; Chen et al., 2015) and mathematical thinking (English, 1997). In addition, it is indicated that problem posing approaches improve students' problem posing skills (Abu Elwan, 1999, 2002; Cankoy, 2014; English, 1997). At this point, studies are needed on the design of the process and revealing the benefits of adopting a problem posing approach in mathematics lessons (Chen & Cai, 2020).

There is an increasing interest in the use of technology in mathematics education, and it has been emphasized that the exploration opportunities offered by technology are related to problem posing (Cai et al., 2015). Problem posing process supported with technology contributes positively to problem posing performance of students. For example, Beal and Cohen (2012) examined the problems posed by middle school students by using a web based content authoring and sharing system. This system also provided students with social behaviors such as solving problems written by their peers and interpreting each other's problems. Students were found to be successful in problem posing with this system. However, it was found that students preferred problem solving activities more than problem posing. Abu Elwan (2014) examined the algebraic problems prospective teachers posed with "what if" and "what if not" strategies in dynamic geometry software (DGS) environment. Prospective teachers posed more problems by using this strategy since they could make changes to the original problem more easily with "what if" strategy in DGS. The DGS helped prospective teachers to pose new problems and confirm their assumptions. Besides, prospective teachers stated their views that DGS provided an effective environment for problem posing and this environment was interesting.

Drawing, dragging and measurement tools of DGS contribute to students' problem posing and the process of validating the problems they pose (Christou et al., 2005; Öçal et al., 2020). In addition, DGS allows creating, reshaping, manipulating, moving geometric objects and examining their interactions in an interactive way (Christou et al., 2005). In this respect, using DGS in problem posing can be useful in developing problem posing skills of students. However we have limited knowledge about designing classroom environments to develop problem posing skills and attitudes of students and about how technology can be integrated into these environments (Ellerton et al., 2015). For this reason in the present study, DGS supported problem posing based learning process was designed and on the effect of this learning of environment students' problem posing skills was focused. In this respect, it is thought that this study will contribute to the literature in terms of integrating technology to problem posing environments and examining its reflections in the implementation process.

Although computer based technologies are suitable for problem posing in different mathematics topics, it may be more convenient to use them in geometry in order to provide opportunities such as dynamic visualization and exploration of objects (Cai et al., 2015). In the eighth-grade mathematics curriculum, the topic of triangles is handled in depth and covers an important part of the program (Ministry of National Education [MoNe], 2018). In the curriculum, the topic of triangles consists of auxiliary elements of the triangle, triangle inequality, angle-side relations, construct of the triangle, the Pythagorean Theorem, congruence, and similarity (MoNE, 2018) subtitles, respectively. Studies have shown that students have difficulties in explaining the auxiliary elements of a triangle (Şengün & Yılmaz, 2021) and they have misconceptions in triangles (Cutugno & Spagnolo, 2002; Kaya, 2018). For example, Şengün and Yılmaz (2021) found that eighth-graders had difficulties in explaining median and angle biosector in triangle and these concepts were confused with altitude. In addition, Kaya (2018) reported that eighth-graders had many misconceptions such as confusing bisector with median, thinking that a triangle can be drawn given the length of two sides or three interior angles and confusing the terms of drawing triangle with Pythagorean Theorem. Cutugno and Spagnolo (2002) showed that students had misconceptions only about drawing altitude inside the triangle. The topic of triangles was preferred in the present study because of the difficulties experienced by students about triangles and the contributions of DGS, which has the potential to overcome these difficulties, to the geometry learning environment.

Research Objectives

The present study aims to examine the effects of using dynamic geometry software in active learning framework on problem posing skills of eighth-grader students about triangles and their views about problem posing. Specifically, answers to the following problems have been sought:

1. Is there a significant difference between students’ pre and post-implementation problem posing skills?
2. How are the students’ pre and post- implementation problem posing skills?
3. How are the student groups’ during the implementation problem posing skills?
4. How are students’ views on pre and post-implementation problem posing based learning?

Theoretical Framework

Active learning framework

School mathematics programs have focused on developing students’ problem solving and problem posing skills does not receive enough attention in classroom activities (Ellerton, 2013; Ellerton et al., 2015). However, studies have shown that there is a significant relationship between problem solving and problem posing (e.g., Cai, 1998; Cai & Hwang, 2002; Chen et al., 2015; Silver & Cai, 1996) and these two activities affect each other mutually (Xie & Masingila, 2017). For example, in an international comparison study with sixth-grades, Cai and Hwang (2002) determined that there is a strong relationship between Chinese students’ problem solving and posing skills, and that this relationship is weak in US students. On the other hand, Silver and Cai (1996) concluded that students with high problem solving skills pose more complex problems and there was a strong positive relationship between problem solving and posing. In this context, problem posing based learning process was designed on the relationship between problem solving and problem posing in this study.

Different designs have been preferred in literature on how problem posing activities can be approached in classroom environments (e.g., Abu Elwan, 1999, 2002; Cankoy, 2014; Chen et al., 2015; English, 1997; Ellerton, 2013; Örnek & Soylu, 2021; Xie & Masingila, 2017). For example, Chen et al. (2015) adopted a problem posing process consisting of understanding the problem posing task presented, identifying its category, posing new problems by applying problem posing strategies and evaluating these problems. Örnek and Soylu (2021) designed a Problem Posing Learning Model in which the stages of understanding the desired situation, designing the story, forming the problem statement, solving the problem, assessing the problem and finalizing to pose a problem were followed. However, it can be said that these designs do not detail the actions of teachers and students in the process of problem posing and do not contain sufficient explanations about which stages to follow in teaching concepts. In this respect, Ellerton (2013) developed an active learning framework (ALF) based on the stages of modelling of examples, drawing attention, locating of examples, problem solving, problem posing and discussion. Ellerton (2013) aimed to integrate problem solving and problem posing activities to mathematics curriculum in parallel to each other with this framework. ALF stages were followed in the problem posing based classroom environment designed in this study. This framework was preferred because it supports active learning in problem posing process, it is based on the relationship between problem solving and problem posing and dominant classroom and student actions are defined at each stage (Ellerton, 2013).

In the present study, the following stages of ALF were followed in problem posing based learning process. In the modelling of examples stage, the teacher models examples, the students listen and imitate the samples presented by the teacher. In the process of drawing attention to examples, the students observe the examples presented by the teacher and engage in the examples. During the locating of examples stage, the presented examples are located in the student, and students are aimed to recognize the related concepts by doing research. In the problem solving stage, students try to solve the problems based on the model problem by recalling what they have learned and helped those who need support. In the problem posing stage, the students share their ideas with each other and pose problems with the same structure as the model problem by doing various experiments. In the discussion stage, the solution of the problems posed by the students and class discussion are made (Ellerton, 2013). In a study they conducted on prospective teachers, Xie and Masingila (2017) tried to determine the interaction between problem solving and problem posing and find out whether the order in which these two activities are carried out matters. In this respect, ALF was extended (EALF) by keeping the logic of students' progression from passive receiver to active learner role. Unlike ALF, stages were added to EALF in which first problem posing and then problem solving stages were followed. In the first stage of EALF, the teacher models examples. Next, the framework is divided into two different paths as problem solving or problem posing activities. In the first stage of the tasks that start with problem posing activities, the teacher designs problem posing activities. Students pose different problems from these activities and new concepts are discussed. The problems posed are solved by students and these problems are discussed. In the last stage, students try to pose problems according to the problems solved or new concepts. In the first stage of the tasks that start with problem solving activities, the teacher designs problem solving activities. Students try to solve these problems and they discuss new concepts. In the next stage, students pose new problems with the same structure to these problems or different types of problems. Finally, a class discussion is made and students try to solve the problems they pose (Xie & Masingila, 2017).

Problem posing activities performed after problem solving can make it easier for students to create new problems from the existing problem (Siswantoro & Siswono, 2019). In addition, it may be more useful to switch to problem posing activities after gaining experience in problem solving (Lavy, 2015). In this respect, the problem posing-problem solving ranking in Xie and Masingila's (2017) EALF may not be suitable for students without problem posing experience. Besides, result of the previous study has revealed the contribution of problem posing process by using ALF in developing students' problem solving and posing skills (Özgen et al., 2019). For this reason, in the present study, ALF of Ellerton (2013) has been adopted as a guide framework where there is a problem solving stage before problem posing. In this study, a learning environment was designed to enable students to learn about these concepts as well as develop their problem posing skills on triangles topic. Therefore, it is thought that the stages of ALF will contribute to teaching of targeted topic. It can be said that the problem posing stage of ALF, which has the same structure as the model problem (Ellerton, 2013), restricts students in terms of problem posing on different topics. However, this limitation of ALF may serve the criteria of suitability to acquisitions, which is one of the criteria that reflect students' problem posing skills in the present study. Although student actions such as listening and imitating emphasized in the early stages of ALF (Ellerton, 2013) are in line with the traditional approach, it can be said that there are traces from the constructivist learning philosophy in the later stages of the framework. Therefore, since this framework is considered to be suitable for our learning environment, the ALF of Ellerton (2013) was adopted in the present study. In addition, the ALF has been chosen since integrating problem solving and posing, step by step student progress to the role of active learner (Ellerton, 2013), and being a guide in designing the problem posing process. In addition, the stages are supported with dynamic geometry software (DGS).

Technology supported problem posing environment

Integrating technology to mathematics teaching enables students to understand concepts better and provides suitable environment for students to be included in deep cognitive activities (Ranasinghe & Leisher, 2009). Supporting problem posing activities with technology can enrich the problem posing process (Shriki & Lavy, 2012). For example, Öçal et al. (2020) examined the problems prospective teachers posed with paper-pencil test and in DGS environment in terms of creativity. It was found that prospective teachers were insufficient about posing creative problems in these two environments and the problems were mostly related to length. DGS has enabled prospective teachers to see the mistakes they make in the problems they pose and the errors in numerical values. In this respect, it was found that prospective teachers were more successful in posing conceptually valid problems in DGS environment. The measurement and drag properties of the DGS provide an understanding of the problem and help to examine the correctness of the possible solutions and assumptions of the problem (Christou et al., 2005). DGS provides learning environments suitable for creating complex shapes that cannot be made with pencil and paper and for making various transformations of these shapes. In addition, the dragging tool, which is a unique feature of DGS, provides students to test their assumptions and to make empirical justification (Marrades & Gutiérrez, 2000).

It was emphasized that DGS plays an effective role in the problem posing process (Abu Elwan, 2014; Christou et al., 2005; Leikin, 2015). For example, Lavy and Shriki (2010) found that structured problem posing activities carried out with "What if?" strategy in DGS supported environment were effective in deepening prospective teachers' knowledge of geometric concepts and shapes and improving their mathematical knowledge. In addition, the features of the software such as calculating and drawing made it easier for prospective teachers to examine without wasting time during the problem posing process. Shriki and Lavy (2012) examined the problem posing situations of mathematics teachers by using the "What If Not?" (WIN) strategy. It was defined that teachers are motivated with the help of WIN strategy in DGS, and their insight and understanding are improved. Leikin (2015) found that prospective teachers improved their problem posing skills, created new problems from a simple problem, and able to design problem posing activities for their peers through the investigations in DGS. On the other hand, Christou et al. (2005) reached the conclusion that DGS supports prospective teachers in terms of conjecture, experimentation, modelling and generalization in the problem solving-posing process. DGS provides the establishment of connections between the concepts during the problem posing process and facilitates the investigation of validity and solution control of the posed problems (Lavy & Shriki, 2010).

GeoGebra, which combines the features of DGS and computer algebra system in the same source, provides dynamic links between multiple representations of mathematical concepts (Hohenwarter & Jones, 2007). In addition, user-friendly GeoGebra dynamically helps students explore mathematical relationships and solve problems (Dikovic, 2009). GeoGebra can support creativity in the problem posing process and deepen the problem solving and posing process of students (Petkova & Velikova, 2015). Afrilianto et al. (2019) compared Project-Activity-Cooperative Learning (PACE) model, GeoGebra supported PACE (PACE-G) and direct learning environments in developing students' problem posing skills. It is concluded that PACE-G and PACE model is more effective than direct learning environments in developing students' problem posing skills. In another study, it was stated that problem posing learning based on GeoGebra applications positively affected students' conceptual understanding skills (Siswantoro & Siswono, 2019). It has also been found that classroom discussions of students while creating dynamic materials in a GeoGebra supported sociocultural learning environment contribute to mathematical communication (Zengin, 2018). In this line, in the problem posing based learning process GeoGebra was used because of the benefits that can be effective in improving students' problem posing skills.

Problem posing situations

Stoyanova and Ellerton (1996) classified problem posing situations in three categories as free, semi-structured and structured problem posing. In free problem posing situation, students are encouraged to pose problems from a natural or artificial situation provided without any limitations. In semi-structured problem posing, students are expected to explore a given open situation such as incomplete problems, equations, and picture by using their mathematical experience, knowledge and skills and to pose problems suitable for this situation. In structured problem posing, students are asked to produce new problems by making changes in the conditions of the existing problem (Stoyanova & Ellerton, 1996).

The problem posing tasks prepared in this study are based on Stoyanova and Ellerton's (1996) problem posing situations. Stoyanova (1998) stated that problems involving the use of a specific concept can be posed in situation of free problem posing. She gave the example of *"Make up some problems which relate to the right angled triangle."* (p. 173) to this situation. This study also included problem posing activities that included the use of concepts related to triangle topics discussed in the problem solving phase of ALF in free problem posing situation. Structured problem posing situations were discussed in the context of problem posing with a similar structure to this problem, after the problem based on the model problem was solved in problem solving phase of ALF. One of the situations presented to students in semi-structured problem posing is incomplete problem structures based on diagram or picture (Stoyanova, 1998; Stoyanova & Ellerton, 1996). In this sense, when the students are presented a triangle in DGS environment, they may tend to use different triangle concepts from the sub-topics of the triangle discussed in the implementation process. In the present study, sub-topics related to triangles stated in "Process" section were included every week during the implementation process. For this reason, problem posing activities prepared during the implementation process included free and structured problem posing situations and the students built all the structures themselves in the DGS environment for the problems they posed.

METHODOLOGY

Embedded mixed methods were used in this study. Qualitative and quantitative data were collected from the same participants, and qualitative data played a supportive role to promote dominant quantitative data (Creswell, 2014). The quantitative data in the present study were obtained by scoring the problems students posed according to a rubric and determining their levels. Quantitative data were collected using a one group pre-test and post-test design. In this design, the effect of the experimental process observed on a single group is measured using pretest-posttest pre and post-implementation (Creswell, 2014). The qualitative data consist of pre and post-implementation views of students about problem posing activities, and student diaries.

Participants

The participants of this study were 16 eighth-grade students who were studying in a public school in Turkey. Participants were selected from the volunteer students, according to the convenience sampling method. Students individually responded to the data collection tools implemented pre and post-implementation. In the tasks carried out during the implementation process, the binary groups were made through gathering the students in the upper and lower groups in the same group according to the mean score of the seventh-grade mathematics lesson report card. In seventh-grade mathematics lesson report card score averages, the lowest score is 0, while the highest score is 100. Seventh-grade mathematics lesson report card score averages of the students were obtained from the school administration.

When the students were classified according to their seventh-grade scores, the students within the range of (21-54) were grouped as low mathematics success. This group includes students named S_5, S_8, S_{13}, S_{14}, S_{15} and S_{16}. Students named S_1, S_2, S_4, S_6, S_7, S_9, S_{10} and S_{12} whose mathematics scores were between (55-84) had moderate mathematics success. Finally, students named S_3 ve S_{11} who had a score between (85-100) were grouped as high mathematics success level. Eight groups were formed in the present study and they were named as S_1-S_9, S_2-S_{16}, S_3-S_{14}, S_4-S_{13}, S_5-S_6, S_7-S_{10}, S_8-S_{12}, S_{11}-S_{15}. Before the implementation, it was determined that the students did not have experience in posing their own problems and they did not use GeoGebra in their lessons. However, students can use the computer sufficiently as they took information technology and software lessons in the fifth and sixth-grades.

Data Collection Tool and Process

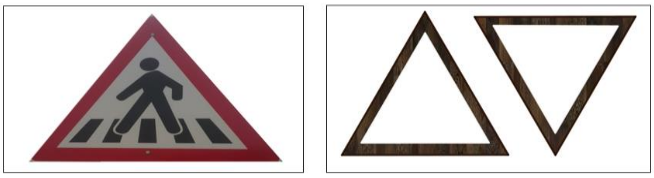
Tasks

In the present study, seven tasks were developed on the topic of triangles. The tasks consist of auxiliary elements of the triangle, triangle inequality, angle-side relations, construct of the triangle, the Pythagorean Theorem, congruence, and similarity (MoNE, 2018) subtitles, respectively. MoNE (2018) and ALF (Ellerton, 2013) were referred to while developing tasks. While preparing the tasks, dynamic materials on the GeoGebra website (www.geogebra.org) were considered relating to how the topic of triangles could be discussed in dynamic environment. The tasks were prepared by considering the six stages of ALF and these stages were supported with GeoGebra software. Students were encouraged to pose problems in a dynamic environment. In the first three weeks of the implementation process, the activities applied in the problem posing stage of ALF were structured and in the last four weeks, they were prepared according to free problem posing situations. Students were also provided with instructions (e.g., the problems you will pose must be solvable) about what they should pay attention to during the problem posing stage. These instructions have guided the students in terms of what they should pay attention to in the problems they pose. In addition, in the problem solving and posing stages of the tasks, activities within the context of daily life were developed to enable students to associate their mathematical knowledge with daily life.

The prepared tasks were examined by two mathematics education experts and two mathematics teachers. Necessary corrections were made in line with the feedback received from the experts. For example, the necessity to provide clues about the concepts related to the difficulties that students will face during the problem solving stage was added to the tasks. At this stage, possible scenarios were discussed in line with the feedback received from experts that concept-oriented specific information should be written in the discussion stage of tasks. It was also stated that students had to be presented with instructions during problem solving and problem posing stages and problem solving-posing instructions were added. Then, the tasks were piloted to 19 eighth-grade students at the school where the main implementation carried out. With the pilot study, necessary corrections were made in areas that are not understood on subjects such as the duration of the tasks, the suitability and adequacy of the used materials, encountered difficulties in implementation, and language-expression of the problems. Below is an example task used in the second week of the implementation process supported with GeoGebra software and designed according to ALF.

Stage 1- Modelling of Examples: The teacher tries to determine students' previous knowledge in the introduction. For this purpose, the teacher draws a triangle on the board and asks students the question of how to find out the side lengths of the triangle drawn. The triangular traffic sign and the triangular wall shelf, which we encounter in daily life, are shown to students on interactive board, they are explained how they need to learn the topic of triangle inequality and they are informed about the goals of the lesson. Students passively listen to the teacher and they imitate the visuals presented below (Figure 1).

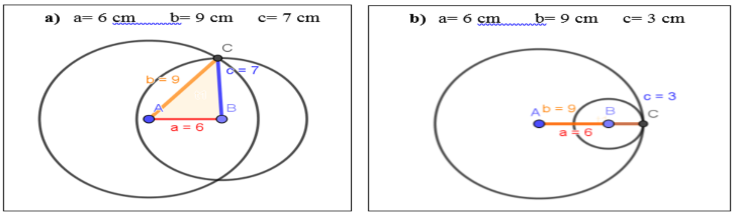
Figure 1.
Modelling of Examples Stage



Stage 2–Drawing Attention: The teacher forms the examples given below (Figure 2) by using GeoGebra software on interactive board. With the examples the teacher creates on GeoGebra software, students observe how a triangle is drawn, three side lengths of which are given and pay attention to the side lengths with which triangles can and cannot be drawn. At this stage, the teacher has an active role and draws triangles with GeoGebra software. The students engage with the information given in a passive position.

Figure 2.
Drawing Attention Stage

Example: Draw the triangles whose side lengths are given below in GeoGebra software.



Stage 3-Locating of Examples: This stage is the stage in which students will create the targeted acquisition and the worksheet below (Figure 3) is distributed to student groups. Students try to draw a triangle by following the instructions below in GeoGebra software. Students move the slider in the task they created with their group friend; they observe the changes in the triangle and research the relationships. Student groups fill in the table given on the worksheet, they begin to recognize the relationship between the sides of the triangle and to research whether it is possible to draw a triangle or not, while the teacher is in the guiding position.

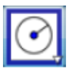
Figure 3.
Locating of Examples Stage

Worksheet:

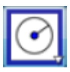
a) Open the GeoGebra software.

b) Select the **slider** tool and form three a, b, c sliders with a minimum value of 1, a maximum value of 10 and an increment value of 1.

c) Select the **segment with given length** tool and form a line segment with a length of a.



ç) Select the **circle: center & radius** tool, select point A of the line segment you created, type “b” in the new page that opens and create a circle with centre A and radius b.



d) Select the **circle: center & radius** tool, select the point B of the line segment you created, type “c” in the new page that opens and create a circle with centre B and radius c.

e) Select the **intersect** tool, find the intersection point of the circles you created by selecting the circles separately.

Figure 3. (Continued)
Locating of Examples Stage

f)  Select the **polygon** tool, create a triangle passing through points A, B and C by selecting points A, B and C respectively.

g)  Select the **distance or length** tool, find the side length by selecting separately the sides of the ABC triangle you created.

ğ) Write the side lengths of the triangle ABC that forms when you move the slides a, b, c the sum and the difference of the side lengths and whether a triangle is drawn or not in the table below.

a	b	c	a+b	a+c	b+c	|a-b|	|a-c|	|b-c|	Triangle	
									can be drawn	can't be drawn

h) Is there a relationship between the sides of the triangle according to the table above?

ı) Based on the relationship between the side lengths of the triangle, what kind of a conclusion can we reach about whether a triangle can be drawn or not?

Stage 4- Problem Solving: At this stage, students by recalling what they have learned and with their group friends try to solve the following problem related to daily life given in the worksheet in accordance with the problem solving instructions given below (Figure 4). The teacher supports groups having problems in problem solving with clues. The teacher tells the groups who have difficulties which have problems that the a, b and c sides of the triangle are the side lengths of the geometry strip in the problem, according to the table they filled in during the locating examples stage. Thus, students are given clues that they can solve the problem by using triangle inequality. The teacher tells the students experiencing difficulty in the evaluation phase that they can evaluate the problem solved by making use of GeoGebra software. The teacher also tells the students that they have to choose the maximum level as 20 instead of 10 in the sliders they create in the task they carry out in locating examples stage. The problem posing stage is started after all groups solve the problem.

Figure 4.
Problem Solving Stage

Ali wants to make similar of the above traffic sign using three geometry strips. The length of the given two strips are 13 cm and 7 cm, respectively. Find the sum of integer values that the length of the third geometry strip can take.

Stage 5-Problem Posing: At this stage, student groups try to pose a problem in GeoGebra software in the situation of structured problem posing same as the triangle inequality problem in the problem solving stage (see Figure 5). The teacher observes students passively, while student groups try to pose a problem about triangle inequality in daily life by reflecting their ideas to each other. While creating the triangle inequality in GeoGebra software for the problems posed by students, the teacher guides the students to give the maximum and minimum values of the sliders according to the side lengths of the triangles they have created.

Figure 5.
Problem Posing Stage

Pose a new problem in GeoGebra software by changing the state, conditions and assumptions of the triangle inequality problem you have solved above and solve the problem you have posed.

Stage 6-Discussion: In Figure 6, possible mistakes that students may make in the problems they have posed in triangle inequality are discussed during the discussion stage of the second week of the implementation process.

Figure 6.
Discussion Stage

At this stage, the problems posed are presented to the classroom. The problems posed are discussed by the class, the incomplete or wrong parts are evaluated by students and mistakes are noticed. In the problems posed, the problems in which the integer is not specified while asking the third side while drawing a triangle are discussed and it is questioned why integer should be specified. In the problems posed, the suitability of the values given to side lengths and the units used to real life is discussed and the errors made are fixed. The problems posed are solved and the task is ended after drawing attention to different strategies that can be applied in the solutions of students.

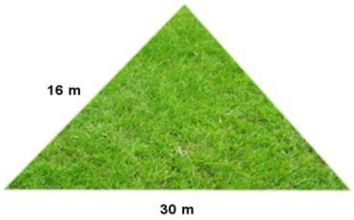
The problem posing test

In the present study, the problem posing test (PPT) was developed to determine students' problem posing skills. Because triangles covered through the seven acquisitions in Turkish eighth-grade mathematics curriculum, PPT formed by seven questions with respect to each acquisition. PPT was prepared according to the framework of Stoyanova and Ellerton (1996). First and fifth questions are free; second, sixth questions are semi-structured; third, fourth and seventh questions are structured problem posing situation.

For the validity and reliability of the developed test, two researchers who are experts in mathematics education and two mathematics teachers examined the test in terms of convenience, semantic and scope. Necessary corrections were made in the test according to the feedback. For example; "Pose and solve a problem by using auxiliary elements (median, angle bisectors, altitude) in the triangle." question that was developed in free posing situation was stated in the situation of general and difficult to evaluate in accordance with the expert view. For this reason, this question was rearranged according to the structured posing situation. In this context, the students were given a sample problem situation related to median. The students were asked to pose a new problem by using auxiliary elements in triangle by changing the conditions and assumptions of the given problem. PPT was applied to 19 eighth-grades in the pilot study and the test was finalized by correcting some expressions.

Daily life situations were used while developing PPT. Figure 7 shows an example of the daily life related problem posing activity prepared according to the triangle inequality acquisition in the seventh question of PPT. Students were given two lesson hours (80 minutes) to complete PPT.

Figure 7.
PPT Seventh Question



Ali will put up a fence around his triangular garden. Find the sum of the highest and lowest integer value the length of the fence can have.

Pose a new problem by changing the conditions and assumptions of the problem situation given above and solve the problem you have posed.

Open-ended questionnaire

In the present study, open-ended questions were prepared to determine the students' thoughts about problem posing activities before and after the implementation. Open-ended questions were used to find out students' thoughts about problem posing activities, the benefits of the implementation process and the difficulties experienced in the process. While preparing the questions, criteria such as being open-ended and easy to understand and avoiding channelling and multi-dimensional questions (Yıldırım & Şimşek, 2016) were taken into consideration. The prepared questions were applied in the pilot study and necessary corrections were made in terms of clarification and explanation of the questions. Open ended questions were as follows:

- How do you think the use of problem posing activities in mathematics learning process affects your learning?
- Explain in detail your views on problem posing activities carried out during GeoGebra supported problem posing based learning process.
- What kind of difficulties did you experience in GeoGebra supported problem posing based learning process?

In addition to these questions, students wrote their views about the tasks in the implementation process using diaries. Here, the aim was to find out students' views on tasks applied in the stages of ALF. In this way, the views of students about the problem posing environment based on GeoGebra supported ALF, which is carried out every week, were determined.

Process

The study was completed in 13 weeks. The implementation process was carried out by one of the researchers in the computer laboratory of the school, which has an interactive board. During the implementation process, the students worked in pairs (Figure 8).

Figure 8.
Class Environment

In the first four weeks of the implementation process, the GeoGebra software teaching plan was applied to the students for three lesson hours (3×40=120 minutes) per week. In this way, students were supposed to use the tools of GeoGebra software effectively. In addition, additional studies were carried out with students to practice with the tools they learned in the software outside the class hours. Students who have difficulty in using GeoGebra software are provided with one-to-one support. In this context, additional applications were made with students who had difficulties in constructing triangles for which the measurements of sufficient numbers of elements were given and using sliders in GeoGebra software. At the same time, students who have computers and tablets at their homes were informed about the installation of GeoGebra. Within the scope of the teaching plan, students were introduced to the interfaces, menus and tools of GeoGebra, and applications for the use of the tools. While determining the content within the GeoGebra teaching plan, the menus and tools to be used in the tasks were taken into consideration. In addition, possible tools that students can use in GeoGebra software are considered while posing problems in triangles. After observing that the students gained the desired skills with the applications supporting the activities in GeoGebra, pre-tests were applied and the learning process started. During the learning process, tasks prepared according to the ALF supported by GeoGebra were implemented on triangles for four lessons (4×40=160 minutes) per week. In the seven-week learning process, auxiliary elements of the triangle, triangle inequality, angle-side relations, construct of the triangle, the Pythagorean theorem, congruence and similarity acquisitions (MoNE, 2018) were considered. At the end of the implementation, the post-tests were applied and the study was completed.

Ethical Procedures

The ethical committee approval for this study was obtained from the Educational Sciences Ethics Committee at Dicle University (Approval Number is 2018/10-3). In addition, the study obtained the approval of the Batman National Education Directorate to collect data in school (Dated 2018, No. 40456018-480.99-E.20792091). Permission was also obtained from the parents of the students to conduct the study, and volunteers participated in the present study.

Data Analysis

The student posed problems before and after the implementation process were scored according to the rubric developed by Özgen et al. (2017). This rubric was preferred because it allows the posed problems to be scored and can be classified by frequency and percentages. It can be said that ALF's problem posing stage, which has the same structure as the model problem (Ellerton, 2013) is suitable for the rubric's suitability to acquisitions criterion. In addition, this rubric was used in the present study because the discussion stage of ALF provides opportunity to questioning the problems posed in terms of qualities such as grammar, mathematical language, and solution of the problem. In this study, students' problem posing skills were examined according to rubric's use of mathematical language, grammar and expression, suitability to acquisitions, quantity and quality of data, solvability, originality, solution of the problem (Özgen et al., 2017) criteria. In each criterion, level 1 (L1) is rated as 0, level 2 (L2) is 1 point, level 3 (L3) is 2 points and level 4 (L4) is 3 points (Özgen et al., 2017).

In the use of mathematical language criterion, if the mathematical language of the posed problem is correct and complete, it is evaluated as 3 points (L4), as 2 points (L3) if it is incomplete, as 1 point (L2) if the concepts are used incorrectly and as 0 points (L1) if left empty. In the grammar and expression criterion, if there are no incoherency and spelling mistakes in the problem statement, it is evaluated as 3 points (L4), as 2 points (L3) if there are spelling mistakes, as 1 point (L2) if there is incoherency, and as 0 points (L1) if left empty or if it includes incoherency and spelling mistake. In the suitability to acquisitions criterion, if the problem is suitable to acquisitions and complete and error free, it is evaluated as 3 points (L4), as 2 points (L3) if it is not suitable to acquisitions, but error free, as 1 point (L2) if it is suitable to acquisitions, but incomplete/erroneous and as 0 points (L1) if left empty or if it is not clear how it will be solved. In the quantity and quality of data criterion, if the data in the problem are sufficient and suitable, it is evaluated as 3 points (L4), as 2 points (L3) if they are not suitable or if there are missing/more data-expression, as 1 point (L2) if there are both inappropriate data and missing/more data-expression, and as 0 points (L1) if it is not understood how to solve or if there are no usable data. In solvability criterion, if the problem is solvable, it is evaluated as 3 points (L4), as 2 points (L3) if the data is sufficient but there are spelling mistakes or incoherency, as 1 point (L2) if the data is insufficient or if there is lack of expression and as 0 points (L1) if it is empty or if there is no text. In the originality criterion, if the problem is original (not found in textbooks and sources), it is evaluated as 3 points (L4), as 2 points (L3) if it is partly original (not the classic question type), as 1 point (L2) if it is ordinary (type of always been to) and as 0 points (L1) if it is empty or undetectable. In the solution of the problem criterion, if the solution is correct, it is evaluated as 3 points (L4), as 2 points (L3) if there is calculation error, as 1 point (L2) if given/asked could not be applied to the solution and as 0 points (L1) if there is no solution (Özgen et al., 2017). Below are examples of evaluating the student posed problems in the PPT according to the rubric (Table 1).

Table 1.
Examples for the Evaluation of PPT

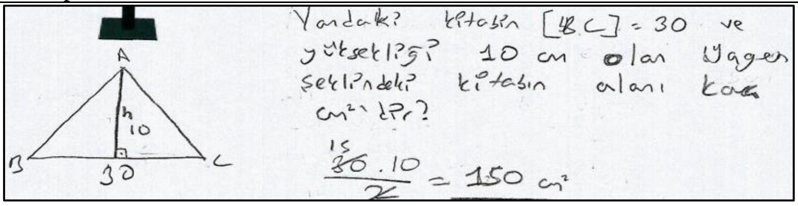
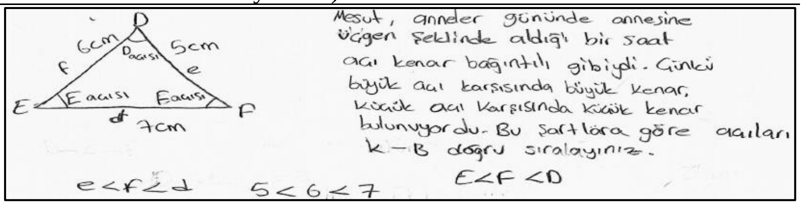
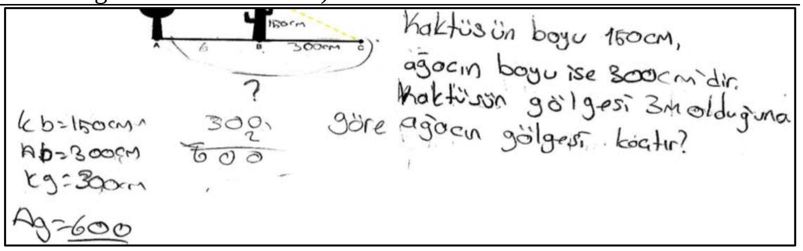
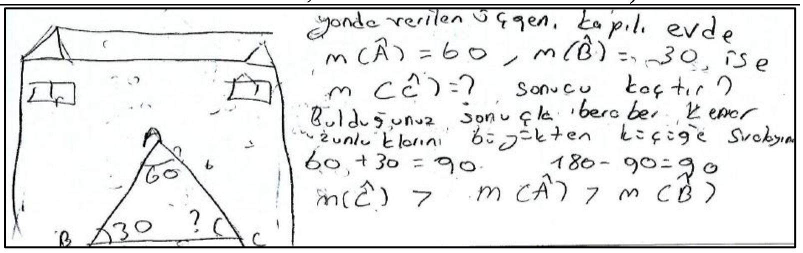
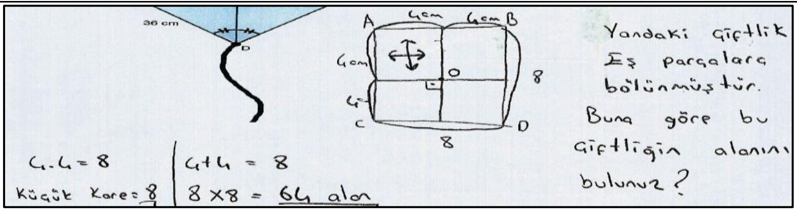
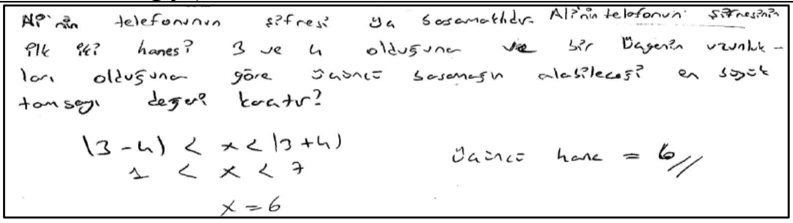
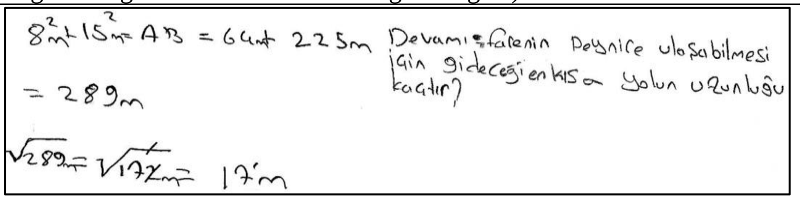
Criteria	Examples
Use of Mathematical Language: In post-test question 3, in the problem of S_{11}, the expression of the area of the book should be specified as the surface area. It was evaluated as L3 since mathematical language was used incompletely.	 ($\overline{BC}$=30 and 10 cm altitude of the book in the form of the triangular area of the book is how many cm^2?)
Grammar and Expression: In post-test question 5, there is an incoherency in the problem of S_{16} in the expression "clock angle seemed correlated". In addition, since unnecessary sentences are used in the problem, it is at L2.	 (Mesut seemed to relate the side of a clock angle he took to his mother in a triangle shape on Mother's Day. Because there was a big side opposite a large angle, a small side opposite a small angle. Order the angles s-b according to these conditions.)
Suitability to Acquisitions: S_2 gave the shadow of the cactus in the posed problem regarding the similarity in post-test question 6. The shadow of the tree wanted to be found with the similarity rate in the solution of the problem. The problem is at L4 since it is suitable to acquisitions and error free.	 (The length of the cactus is 150cm and the length of the tree is 300cm. Since the cactus shade is 3m, what is the shade of the tree?)
Quality and Quantity of Data: In the posed problem by S_{10} in the pre-test question 5, the values given to the inner angles of the triangle door and the triangle door are not logically appropriate. Therefore, the problem is at L3.	 (If m∠A= 60 and m∠B= 30 in the house with a triangular door given at the side then m∠C=? what is the result? Order the side lengths from large to small with your result.)

Table 1. (Continued)

Examples for the Evaluation of PPT

Solvability: The problem sentence of S_4 is understood in pre-test question 4. However, giving 8 cm to the side of the farm is not logically appropriate. Since the data in the problem are not suitable and sufficient, it is at L2.	 (The farm on the side is divided into identical parts. Find the area of this farm accordingly?)
Originality: S_{11} has created a different fiction than the given problem in the posed problem in post-test question 7. Therefore, since the problem is original, it has been evaluated as L4.	 (Ali's phone password has three digits. Since the first two digits of Ali's phone's password are 3 and 4, and the lengths of a triangle, what is the largest integer value that the third digit can get?)
Solution of the Problem: In the post-test question 2, the problem posed by S_{13} has been evaluated as L4 since it has been solved correctly and completely.	 (Continued: What is the shortest path for the mouse to reach the cheese?)

In the quantitative data of the study, Shapiro-Wilk test, Q–Q plot, skewness and kurtosis coefficients were examined in the normality analysis of the PPT pretest-posttest scores of the students. The data were found to have a normal distribution at the 0.05 significance level, and the dependent t-test was used. The effect size was calculated with the formula $r = \sqrt{t^2/(t^2 + df)}$ (Field, 2009). The calculated r value is considered as 0.10 small, 0.30 medium, and 0.50 large effect (Cohen, 1992). In calculating the reliability for scoring of the problems posed, the formula proposed by Miles and Huberman (1994) was used. In the scoring of the problems posed by students, the same data were evaluated twice periodically by the first researcher. The percentage of agreement regarding the scoring of the problems posed by the students was calculated as 90% for pre-test, as 84% for post-test and as 84% during the implementation process. Different scorings were reviewed and agreement was reached.

Descriptive analysis was used to analyze the qualitative data obtained from open-ended questionnaire and diaries of the students. In descriptive analysis, the data are described in a systematic and clear way, presented in an interpreted way with cause and effect relationships and direct quotations are included in the presentation of the data obtained (Yıldırım & Şimşek, 2016). In the findings section, direct quotations were given in the students' views and the students were named as $S_1 \ldots S_{16}$.

RESULTS

Students' Problem Posing Skills

This section presents results regarding the first sub-problem "Is there a significant difference between students' pre and post-implementation problem posing skills?" In the PPT, the t-test results according to the criteria in the rubric and total scores of the posed problems by the students are presented below (Table 2).

Table 2.
T-test Results of Students' Problem Posing Skill Scores

Criteria	Test	*n*	$\bar{X}$	*ss*	*sd*	*t*	*p*
Use of mathematical language	Pre-test	16	7.37	1.7	15	-5.36	.00
	Post-test	16	11.06	3.8			
Grammar and expression	Pre-test	16	4.87	2.21	15	-4.54	.00
	Post-test	16	9.56	5.45			
Suitability to acquisitions	Pre-test	16	2.81	1.79	15	-5.19	.00
	Post-test	16	9.56	6.17			
Quantity and quality of data	Pre-test	16	4.18	2.25	15	-5.91	.00
	Post-test	16	10.68	5.97			
Solvability	Pre-test	16	4.43	2.87	15	-6.1	.00
	Post-test	16	10.5	5.92			
Originality	Pre-test	16	2.43	1.63	15	-4.14	.00
	Post-test	16	5.43	3.53			
Solution of the problem	Pre-test	16	5.81	2.22	15	-5.09	.00
	Post-test	16	11.06	5.7			
Total score	Pre-test	16	31.93	13.09	15	-5.53	.00
	Post-test	16	67.87	35.97			

According to Table 2, there is a significant difference in student use of mathematical language [$t(15)=-5.36$, $p<.05$, $r=.81$], grammar and expression [$t(15)=-4.54$, $p<.05$, $r=.76$], suitability to acquisitions [$t(15)=-5.19$, $p<.05$, $r=.80$], quantity and quality of data [$t(15)=-5.91$, $p<.05$, $r=.83$], solvability [$t(15)=-6.1$, $p<.05$, $r=.84$], originality [$t(15)=-4.14$, $p<.05$, $r=.73$], solution of the problem [$t(15)=-5.09$, $p<.05$, $r=.79$] skills and PPT total scores [$t(15)=-5.53$, $p<.05$, $r=.81$] in favor of post-test. When the effect size values of the problem posing criteria were examined, it was found that the largest effect size was in solvability criterion ($r=.84$), while the smallest effect size was in originality criterion ($r=.73$). Moreover, since PPT total scores and effect size values calculated in all skills were higher than .50, it can be said that the effect size of the implementation process was large (Cohen, 1992). PPT average total scores increased from 31.93 to 67.87. Therefore, it can be said that DGS use in ALF developed students' problem posing skills.

Levels of the Problems Posed by Students

This section presents results regarding the second sub-problem "How are the students' pre and post-implementation problem posing skills?" The levels of the problems students posed in problem posing pretest-posttest according to the criteria in rubric were given below (Table 3) with frequency and percentage table.

Table 3.
Levels of Students' Problem Posing Skills

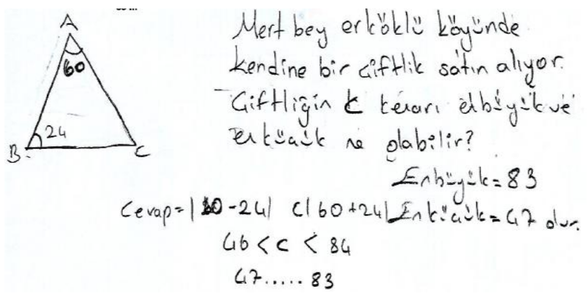
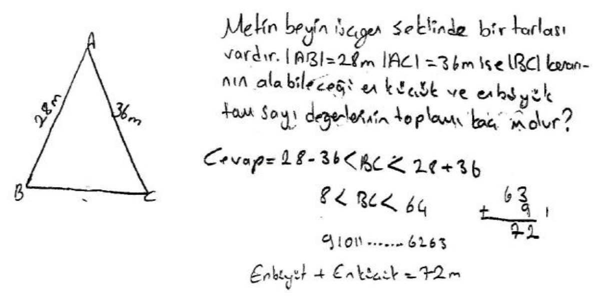
Criteria	PPT	L1	L2	L3	L4
Use of mathematical language	Pre-test	12 (10.7%)	84 (75%)	14 (12.5%)	2 (1.8%)
	Post-test	5 (4.5%)	52 (46.4%)	40 (35.7%)	15 (13.4%)
Grammar and expression	Pre-test	35 (31.3%)	76 (67.9%)	1 (0.9%)	0 (0%)
	Post-test	29 (25.9%)	41 (36.6%)	14 (12.5%)	28 (25%)
Suitability to acquisitions	Pre-test	75 (67%)	31 (27.7%)	4 (3.6%)	2 (1.8%)
	Post-test	40 (35.7%)	31 (27.7%)	1 (0.9%)	40 (35.7%)
Quantity and quality of data	Pre-test	67 (59.8%)	28 (25%)	12 (10.7%)	5 (4.5%)
	Post-test	34 (30.4%)	26 (23.2%)	11 (9.8%)	41 (36.6%)
Solvability	Pre-test	56 (50%)	46 (41.1%)	5 (4.5%)	5 (4.5%)
	Post-test	32 (28.6%)	33 (29.5%)	6 (5.4%)	41 (36.6%)
Originality	Pre-test	74 (66.1%)	37 (33%)	1 (0.9%)	0 (0%)
	Post-test	44 (39.3%)	53 (47.3%)	11 (9.8%)	4 (3.6%)
Solution of the problem	Pre-test	39 (34.8%)	56 (50%)	14 (12.5%)	3 (2.7%)
	Post-test	26 (23.2%)	32 (28.6%)	17 (15.2%)	37 (33%)

Note. Frequencies (percentages)

In Table 3, it is seen that most of the student posed problems in the pre-test are gathered in levels 1 and 2 in terms of all the criteria of rubrics. In this case, it can be said that the problems students posed in the pre-test were insufficient. It can be said that in the posed problems in the post-test, a positive development towards levels 3 and 4 has been shown in terms of all criteria. This finding shows that the learning process improves the students in terms of the criteria in posed problems by them. Below (Figure 9) is an evaluation of the problems posed by S_3 in PPT question 7.

Figure 9.

Problems Posed by S_3

	
Pre-test: Mr. Mert buys himself a farm in Erköklü Village. What can be the largest and smallest c side of the farm?	Post-test: Mr. Metin has a triangular field. What is the sum of the largest and smallest integer values that the |BC| side can take if |AB|=28 m, |AC|=36 m?

In Figure 9, in the posed problem in the pre-test by S_3, the values given as the inner angle of the triangle are used as side lengths in the solution. Therefore, it is at L2 in terms of the use of mathematical language. Since there is an incoherency in the problem sentence, it is at L2 in grammar and expression criteria. Although the problem is suitable for the triangle inequality acquisition, it is evaluated at L2 in the criterion of suitability to acquisitions as being incorrect. In the problem, since the side lengths of the triangle is not given and the "integer values" statement is not used when asking the side c, it is at L2 in terms of quantity and quality criterion. In the posed problem, the side lengths are missing and semantically missing, so it is at L2 in terms of solvability. Since the problem does not contain an original context, it is at L2 in the originality criterion. Since the internal angles of the triangle are applied to the solution as side lengths in the problem, it is evaluated at L2 in the criterion of the problem solution.

In the posed problem in the post-test by S_3, the length symbols, units and concepts are used correctly, hence it is at L4 in the criterion of the use of mathematical language. Since the problem statement is understandable, it is at L4 in grammar and expression criteria. Since the problem is suitable and accurate for the acquisition of triangle inequality, it is evaluated at L4 in terms of suitability to acquisitions. In the problem, the values given to the side lengths of the field is logical. In addition, since the statement "integer values" is used when asking for the BC side, it is at L4 in terms of data quantity and quality. Side lengths are given in the problem and the problem statement is sufficient in terms of expression, so it is at L4 in the solvability criterion. The context of the posed problem is similar to the given problem, and since it is not original, it is at L2 in terms of originality. Since the problem was solved by the student correctly, it was evaluated at L4 in the solution of the problem. In this case, it can be said that DGS use in ALF was effective in developing the problem posing skills of S_3

Levels of the Problems Posed by Students during the Implementation Process

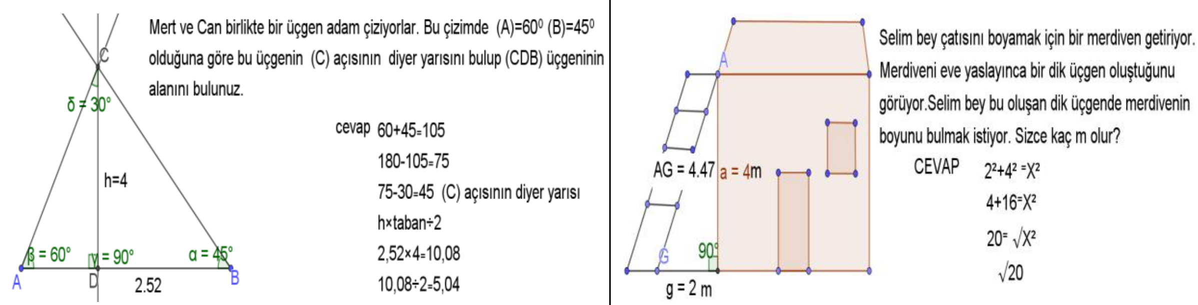
This section presents results regarding the third sub-problem "How are the student groups' during the implementation problem posing skills?" During the implementation period, student posed problems in pairs in GeoGebra during seven weeks are given according to their problem posing skills with frequency and percentage tables. Values given in the tables are presented as frequency (percent). The tables are supported by direct quotations from the student posed problems.

Table 4.
Levels of the Posed Problems in terms of Use the Language of Mathematical

Weeks	L1	L2	L3	L4	Total
Week-1	0 (0%)	6 (75%)	1 (12.5%)	1 (12.5%)	8 (100%)
Week-2	0 (0%)	5 (62.5%)	3 (37.5%)	0 (0%)	8 (100%)
Week-3	0 (0%)	4 (50%)	3 (37.5%)	1 (12.5%)	8 (100%)
Week-4	0 (0%)	0 (0%)	6 (75%)	2 (25%)	8 (100%)
Week-5	0 (0%)	1 (12.5%)	4 (50%)	3 (37.5%)	8 (100%)
Week-6	0 (0%)	2 (25%)	4 (50%)	2 (25%)	8 (100%)
Week-7	0 (0%)	2 (25%)	6 (75%)	0 (0%)	8 (100%)
Total	0 (0%)	20 (35.7%)	27 (48.2%)	9 (16.1%)	56 (100%)

According to Table 4, it can be seen that in terms of the use of mathematical language, the problems students posed in the early weeks concentrated at L2, and they progressed towards L3 and L4 in the problems they posed starting from week-4. Therefore, it can be said that the students made mistakes in using mathematical language in the problems they posed in early weeks, while they progressed in terms of using mathematical language well as the weeks progressed. It can be said that this situation is due to the fact that students gained experience in problem posing and also due to GeoGebra's characteristics supporting correct use of mathematical language.

Figure 10.
Problems Posed by S_3-S_{14} during Implementation



Week-1: Mert and Can draw a triangle man together. In this drawing, as (A)=60° (B)=45°, find the other half of the angle (C) of this triangle and find the area of the triangle (CDB).

Week-5: Mr. Selim brings a ladder to paint his roof. When he leans a ladder against the home, he sees a right triangle formed. Mr. Selim wants to find the length of the ladder in this right triangle. Do you think, how many meters it could be?

In Figure 10, in the posed problem by S_3-S_{14} at week-1, it is seen that the angle symbols are used poorly and the lengths given in the figure are not given in the problem sentence. In addition, the expression "the other half of the angle C" is used incorrectly and the angle is not divided into two identical parts. Since there are errors in terms of mathematical language in the problem, it is at L2 in the criterion of using mathematical language. In the student posed problem at week-5, it is seen that they use the unit "*m*" and define the formed right triangle correctly. However, the length of the right triangle is not specified in the problem sentence. For this reason, since mathematical language is used incompletely, the problem is evaluated as L3.

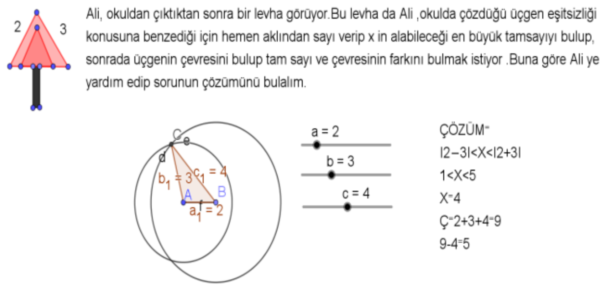
From the view of S_4 as *"...we were drawing a triangle and we were finding half of it by clicking the median feature in the software to be sure that we divide it by half of its base, and do the angles through the angle feature, and we were seeing all of the features of the shape that we made with the algebra window in GeoGebra…"*, it is understood that GeoGebra tools support using the concepts correctly. Similar situation reflected the view of S_7 as *"... I learned new mathematical shapes while posing problems: ...polygon, center angle, hypotenuse, perpendicular bisector, etc. We saw the mathematical tools... when we pressed the animate the triangle we made, the angles were changing and I could see different aspects, and other shapes of triangles emerged…"* Therefore, it can be said that GeoGebra is effective in developing students' skills to use the language of mathematical.

Table 5.
Levels of Posed Problems in terms of Grammar and Expression

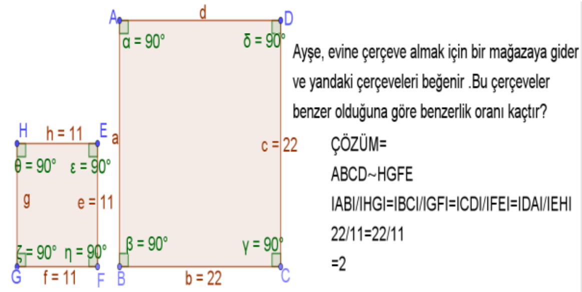
Weeks	L1	L2	L3	L4	Total
Week-1	2 (25%)	5 (62.5%)	0 (0%)	1 (12.5%)	8 (100%)
Week-2	1 (12.5%)	5 (62.5%)	1 (12.5%)	1 (12.5%)	8 (100%)
Week-3	0 (0%)	7 (87.5%)	0 (0%)	1 (12.5%)	8 (100%)
Week-4	0 (0%)	4 (50%)	2 (25%)	2 (25%)	8 (100%)
Week-5	0 (0%)	2 (25%)	1 (12.5%)	5 (62.5%)	8 (100%)
Week-6	1 (12.5%)	2 (25%)	2 (25%)	3 (37.5%)	8 (100%)
Week-7	0 (0%)	4 (50%)	2 (25%)	2 (25%)	8 (100%)
Total	4 (7.1%)	29 (51.8%)	8 (14.3%)	15 (26.8%)	56 (100%)

According to Table 5, it was determined that the students had difficulties in terms of grammar and expression and approximately half of the posed problems (51.8%) were semantically incorrect. It was found that the problem statements students made in the first weeks included incoherencies; therefore, there were more problems posed at L2. However, it can be said that there was a progress in problems posed after week-3 in terms of expression.

Figure 11.
Problems Posed by S_{11}-S_{15} during Implementation



Week-2: Ali sees a sign after leaving school. Since this sign is similar to the triangle inequality that he solved at school, Ali wants to find the largest integer that x can get from the mind and then find the perimeter of the triangle and find the difference between the integer and its perimeter. Accordingly, let's help Ali and find a solution to the problem.



Week-7: Ayşe goes to a store to buy a frame for her house and likes the frames on the side. Since these frameworks are similar, what is the similarity ratio?

In Figure 11, in the posed problem at week-2 by S_{11}-S_{15}, the side lengths of the sign and the unknown side are not clearly expressed in the problem sentence. There is also an incoherency because unnecessary words are used in the problem (e.g., because it is similar to the triangle inequality that solved at school). Therefore, it was evaluated at L2 in terms of grammar and expression. The problem sentence that the students posed at week-7 is understandable and there is no spelling mistake, so the problem is at L4. In this case, it can be said that as the implementation process progresses, the students develop in terms of grammar and expression, but the development is in limited level.

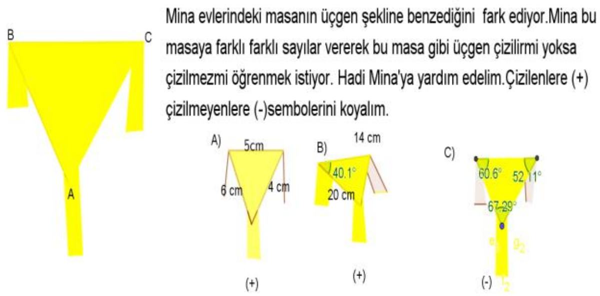
It is understood from the view of S_3 that the discussion stage contributed to seeing the mistakes: *"…when we share and discuss our problems with our friends, we find spelling and mathematical mistakes and do not make the same mistakes in posing the next problem. For example; I prepared a problem with a door. When we examined this posed problem with my friends, we found spelling and mathematical mistakes and I took care not to make these mistakes in posing the next problem."* This situation also stated in the views of S_{10} as *"…We did not use symbols such as | | length m ( ) angle, our problem was not clear, we had such mistakes. Then I and my group friend got up on the board, explained our problems, we had our mistakes, our problem was not fully explanatory, but we will pay close attention to them."* In this case, it can be said that the limited development shown in terms of grammar and expression resulted from the discussion stage.

Table 6.
Levels of Posed Problems in terms of Suitability to Acquisitions

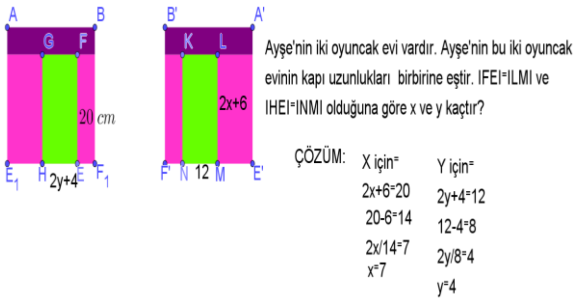
Weeks	L1	L2	L3	L4	Total
Week-1	2 (25%)	4 (50%)	0 (0%)	2 (25%)	8 (100%)
Week-2	0 (0%)	7 (87.5%)	0 (0%)	1 (12.5%)	8 (100%)
Week-3	0 (0%)	4 (50%)	0 (0%)	4 (50%)	8 (100%)
Week-4	0 (0%)	5 (62.5%)	0 (0%)	3 (37.5%)	8 (100%)
Week-5	0 (0%)	2 (25%)	0 (0%)	6 (75%)	8 (100%)
Week-6	1 (12.5%)	1 (12.5%)	0 (0%)	6 (75%)	8 (100%)
Week-7	0 (0%)	4 (50%)	0 (0%)	4 (50%)	8 (100%)
Total	3 (5.4%)	27 (48.2%)	0 (0%)	26 (46.4%)	56 (100%)

According to Table 6, it is determined that 27 of the posed problems are suitable for the acquisitions, but are incomplete or incorrect, and 26 problems are complete and error free. Therefore, it can be said that most of the posed problems are suitable to acquisitions. It was also found that students did not pose problems at L3; in other words, problems which were not suitable to acquisitions. It can be said that this situation results from ALF's instructions of posing problems same structure to the model problem. The posed problems in the first weeks are low level (L1-L2) in terms of suitability to acquisitions. However, it can be said that the implementation process positively affects students in terms of posing problems suitable for the acquisitions.

Figure 12.
Problems Posed by S_7-S_{10} during Implementation



Week-4: Mina realizes that the table in their home looks like a triangular shape. Mina wants to learn whether or not to draw a triangle like this table by giving different numbers to this table. Let us help Mina. Let us put the symbols (+) on the ones drawn and (-) on the ones not drawn.



Week-6: Ayşe has two toy houses. The door lengths of these two toy houses of Ayşe are congruent. What are x and y if |FE|=|LM| and |HE|=|NM|?

In Figure 12, the posed problem by S_7-S_{10} in week-4 is suitable to the acquisition of construct of the triangles. However, the elements of the triangle given in the choices are not expressed mathematically. In addition, although the triangles are constructed, it is asked which triangle cannot be constructed. Therefore, since the posed problem is incorrect, it is at L2 in the suitable to acquisitions criterion. In week-6, in the posed problem by S_7-S_{10} about congruence, two congruent gates are given and x, y were asked. The problem is at L4 since it is suitable to acquisition and error free. Therefore, it can be said that the implementation process had developed the students in terms of problem posing that is suitable to acquisitions.

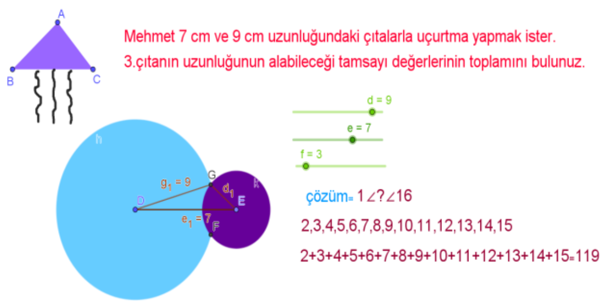
It is understood from the view of S_3 that the student tries to find a problem in the same acquisition after the problem solving activity: *"Today we solved similarity activities in the GeoGebra lesson and checked it through GeoGebra. Later, I and my group friend prepared a problem about the similarity…"* A similar situation was also stated in the views of S_{10} as *"…The problem that we solved was related to the triangle drawings, in fact, it was very simple because it would be drawn if the numbers are given in the problem corresponded to the triangle drawing formula, it would not be drawn if it did not fit … Then we pose a problem. There was a triangular table in our problem. We gave them angle measurements and side lengths. We put the (+) sign on the drawn and (-) sign on the non-drawn…"* Therefore, it can be said that the problem solving stage of ALF contributes to students' problem posing in accordance with the relevant acquisitions.

Table 7.
Levels of Posed Problems in terms of Data Quantity and Quality

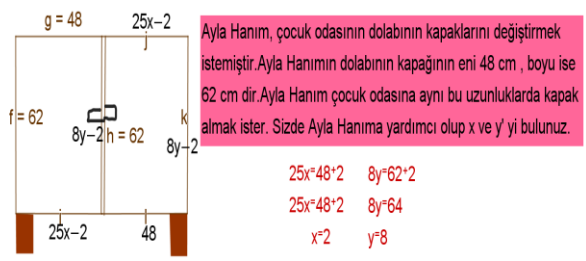
Weeks	L1	L2	L3	L4	Total
Week-1	2 (25%)	3 (37.5%)	1 (12.5%)	2 (25%)	8 (100%)
Week-2	0 (0%)	0 (0%)	7 (87.5%)	1 (12.5%)	8 (100%)
Week-3	0 (0%)	3 (37.5%)	1 (12.5%)	4 (50%)	8 (100%)
Week-4	0 (0%)	2 (25%)	3 (37.5%)	3 (37.5%)	8 (100%)
Week-5	0 (0%)	1 (12.5%)	1 (12.5%)	6 (75%)	8 (100%)
Week-6	0 (0%)	2 (25%)	0 (0%)	6 (75%)	8 (100%)
Week-7	0 (0%)	2 (25%)	2 (25%)	4 (50%)	8 (100%)
Total	2 (3.6%)	13 (23.2%)	15 (26.8%)	26 (46.4%)	56 (100%)

According to Table 7, it is determined that the data used in approximately half (46.4%) of the posed problems are sufficient and appropriate. Since the data in the posed problems by the students in the first weeks were not logically/operational appropriate, the posed problems were evaluated at a low level (L1-L2). However, as the weeks progress, it can be said that the students pay attention to the data and expressions that they use and that the posed problems developed.

Figure 13.
Problems Posed by S_5-S_6 during Implementation



Week-2: Mehmet wants to make a kite with 7 cm and 9 cm long bars. Find the sum of the integer values that the length of the third bar can take.



Week-6: Mrs. Ayla wanted to change the doors of the cabinet of the children's room. The width of the door of Mrs. Ayla's cabinet is 48 cm wide and 62 cm long. Mrs. Ayla wants to buy a door for the child's room at the same length. Find x and y to help Mrs. Ayla.

In Figure 13, in the posed problem at week-2 by S_5-S_6, it is not logically appropriate to give "7 cm and 9 cm" to the value of the two sides of the kite. For this reason, the posed problem is at L3 according to the quantity and quality of data criterion. In the posed problem at week-6, the given values to the door of the cabinet are suitable for real life. In addition, the width and length expressions of the cabinet were used correctly. Therefore, the posed problem was evaluated at L4. In this case, it can be said that over time, students have improved in terms of using logical data.

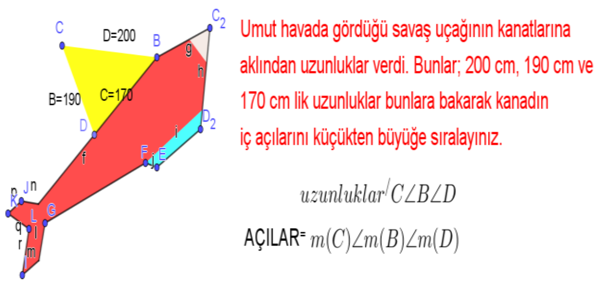
It was stated in S_6's view that the used data was illogical when posing a problem and this situation was criticized during the discussion: *"We were making mistakes mostly in giving numbers to the shapes in the problem posing process. For example, when we made a triangle hat, we gave 4 cm or something. But at that time, it did not seem like a mistake to us, and when we argued, our mistakes were noticeable in our view…"* Similarly, S_7 emphasized that the discussion process allows paying attention to the data in the posed problems in the view as *"…I was able to do problem posing, but I was not paying attention to what was given and desired… When I pose problems, I pay attention to "do the sides that I gave fit the triangle drawing", what is given and desired. When I pose the problem and solved it before, sometimes there was no answer, but what shape fit to the sides that I gave, I pay attention to it now. Narrow angle, right triangle or wide? I pay attention to the measure of that angle. The discussion has* *greatly contributed to me."* Therefore, it can be said that the development shown in terms of data quantity and quality resulted from questioning errors at the discussion stage.

Table 8.
Levels of Posed Problems in terms of Solvability

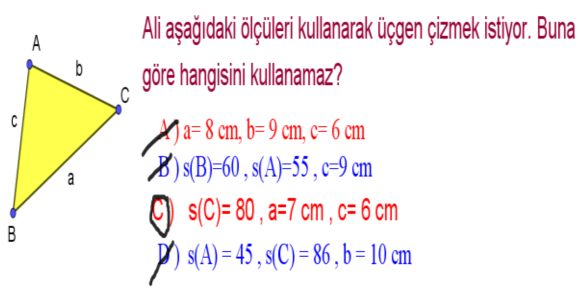
Weeks	L1	L2	L3	L4	Total
Week-1	2 (25%)	3 (37.5%)	1 (12.5%)	2 (25%)	8 (100%)
Week-2	0 (0%)	7 (87.5%)	0 (0%)	1 (12.5%)	8 (100%)
Week-3	0 (0%)	4 (50%)	0 (0%)	4 (50%)	8 (100%)
Week-4	0 (0%)	5 (62.5%)	0 (0%)	3 (37.5%)	8 (100%)
Week-5	0 (0%)	2 (25%)	0 (0%)	6 (75%)	8 (100%)
Week-6	0 (0%)	2 (25%)	0 (0%)	6 (75%)	8 (100%)
Week-7	0 (0%)	3 (37.5%)	1 (12.5%)	4 (50%)	8 (100%)
Total	2 (3.6%)	26 (46.4%)	2 (3.6%)	26 (46.4%)	56 (100%)

According to Table 8, there is a lack of inappropriate data and expression in 26 of the posed problems. 26 of them were determined to be solvable problems. It was determined that the students had difficulty in posing problem solvable especially in the first weeks and that the posed problems were at low level. However, it can be said that the implementation process affects students positively in terms of solvability.

Figure 14.
Problems Posed by S_4-S_{13} during Implementation



Week-3: Umut gave the value of lengths of the wings of the warplane he saw in the air. These are lengths of 200 cm, 190 cm, and 170 cm, and sort the internal angles of the wing from small to large through looking at these.



Week-4: Ali wants to draw a triangle using the following dimensions. Accordingly, which of the following cannot be used?

In Figure 14, in the problem posed at week-3 by S_4-S_{13} the given values to the side lengths of the plane (200 cm, 190 cm, and 170 cm) are not logical and suitable for real life. There is also a lack of expression in the problem sentence. Therefore, the posed problem is at L2 in terms of solvability. The posed problem at week-4 is understandable and the used data is sufficient. For this reason, it was evaluated at L4. In this case, it can be said that the students showed improvement in posing solvable problem.

It is understood that the mistakes made in problem posing affect the solvability of the problem from the view of S_{13}: *"When we discussed the problems that we made at GeoGebra with our friends, we saw verbal and numerical errors in the problem. We tried to be more careful when we posed or solved a problem, we did more control and checkup of it…"* This situation also reflected the view of S_{16} as *"I think it is more useful for us to see our mistakes verbally and mathematically in the problem that we posed when we discuss the problems that we pose with our group friends in our GeoGebra lesson. And after the mistakes that we made, we try to explain and write more carefully and concisely in the next problem posing stage…"* Therefore, it can be said that the mistakes made by the students in terms of semantic, mathematical, and logical points affect the solvability of the posed problems.

Table 9.

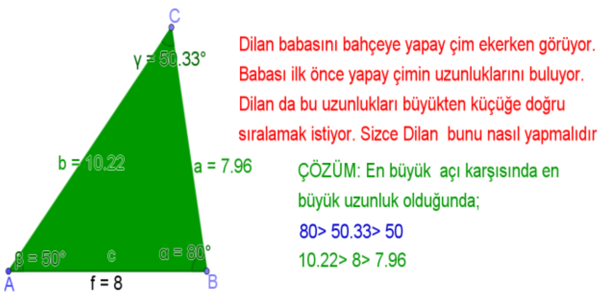
Levels of Posed Problems in terms of Originality

Weeks	L1	L2	L3	L4	Total
Week-1	2 (25%)	5 (62.5%)	1 (12.5%)	0 (0%)	8 (100%)
Week-2	0 (0%)	5 (62.5%)	3 (37.5%)	0 (0%)	8 (100%)
Week-3	0 (0%)	3 (37.5%)	5 (62.5%)	0 (0%)	8 (100%)
Week-4	1 (12.5%)	6 (75%)	1 (12.5%)	0 (0%)	8 (100%)
Week-5	1 (12.5%)	1 (12.5%)	4 (50%)	2 (25%)	8 (100%)
Week-6	2 (25%)	1 (12.5%)	4 (50%)	1 (12.5%)	8 (100%)
Week-7	2 (25%)	3 (37.5%)	3 (37.5%)	0 (0%)	8 (100%)
Total	8 (14.3%)	24 (42.9%)	21 (37.5%)	3 (5.4%)	56 (100%)

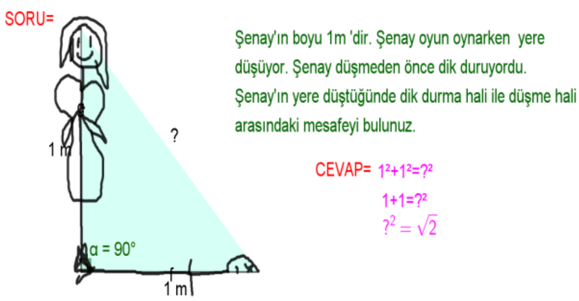
According to Table 9, it is determined that the majority of the posed problems are ordinary problems (type of always been to). It was found that only 3 of the 56 problems posed were original. This shows that students had difficulties in posing original problems and they tended to pose ordinary/classical problems. Therefore, it can be said that the implementation process affects students at a low level in terms of originality.

Figure 15.

Problems Posed by S_2-S_{16} during Implementation



Week-3: Dilan sees his father planting artificial grass in the garden. Her father first finds the length of the artificial grass. Dilan wants to rank these lengths from large to small. In your opinion how should Dilan do this



Week-5: Şenay's height is 1m. Şenay falls to the ground while playing games. Şenay stood upright before falling. When Şenay falls to the ground, find the distance between standing upright and falling down.

Figure 15 is given the posed problem at week-3 by S_2-S_{16}. In this posed problem in the acquisition of angle-side relationships, there are many data in the problem since both angle and side lengths are given. This posed problem includes an ordinary context for ordering the lengths of the sides by giving the angle values of the triangle. It is at L2 in terms of originality. On the other hand, the problem posed at week-5 is a type of question that is not included in textbooks or resources. Therefore, since the problem is original, it is evaluated as L4. In this case, students' problem posing each week positively affected their original problem posing skills during the implementation process. However, it can be said that the development is limited.

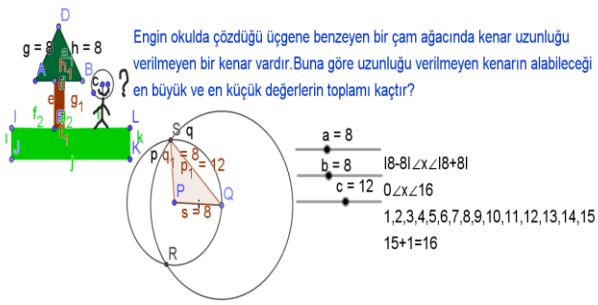
It is understood that the student does not strive to create an original problem from the view of S_{11}: *"…As always, I and my group friend finished at first. Because, as always, we were taking the easy way out, so we were posed easy problems and completed…"* It was also stated that the students tend to pose ordinary problems in the view of S_{13} as *"…We did not cross our borders by always making easy problems when we were posing problems with triangles …"* Therefore, it can be said that the limited development shown in terms of originality is caused by problem posing of the students in easy and considering their levels.

Table 10.
Levels in terms of Solving the Posed Problem

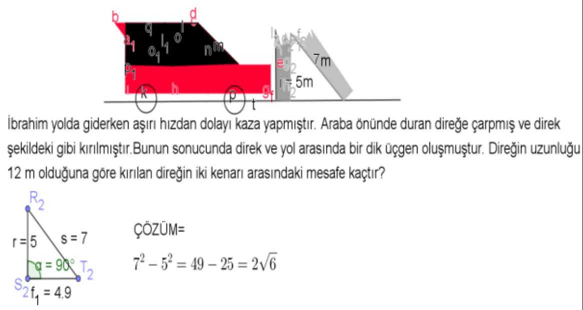
Weeks	L1	L2	L3	L4	Total
Week-1	1 (12.5%)	2 (25%)	4 (50%)	1 (12.5%)	8 (100%)
Week-2	0 (0%)	2 (25%)	6 (75%)	0 (0%)	8 (100%)
Week-3	0 (0%)	3 (37.5%)	1 (12.5%)	4 (50%)	8 (100%)
Week-4	0 (0%)	4 (50%)	2 (25%)	2 (25%)	8 (100%)
Week-5	0 (0%)	1 (12.5%)	1 (12.5%)	6 (75%)	8 (100%)
Week-6	0 (0%)	2 (25%)	0 (0%)	6 (75%)	8 (100%)
Week-7	0 (0%)	2 (25%)	2 (25%)	4 (50%)	8 (100%)
Total	1 (1.8%)	16 (28.6%)	16 (28.6%)	23 (41.1%)	56 (100%)

According to Table 10, 23 of the posed problems were solved by the students correctly. Especially in the first weeks, it was observed that there were errors and deficiencies in the solutions of the problems due to the mistaken student posed problems. However, it can be said that as the weeks progressed, they developed solvable problem posing and this development contributed to solving the problems correctly.

Figure 16.
Problems Posed by S_8-S_{12} during Implementation



Week-2: A pine tree, similar to the triangle that Engin solves at school, has a side that is not given a side length. According to this, what is the sum of the largest and smallest values that the side of which length is not given can take?



Week-5: While going on the road, Ibrahim had an accident due to excessive speed. The car hit the pole standing in front of it and the pole was broken directly as in the figure. As a result, a right triangle formed between the pole and the road. Since the length of the pole is 12 m, what is the distance between the two sides of the broken pole?

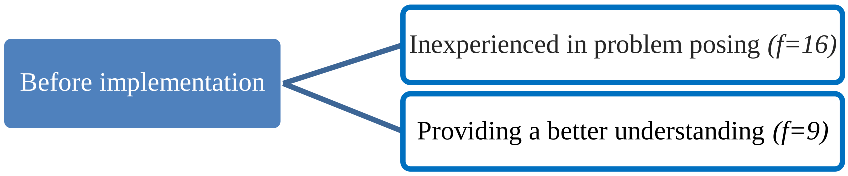
In Figure 16, S_8-S_{12} solved the problem they had posed in week-2, but the solution is erroneous because the word "integer values" is not given in the problem sentence. Therefore, it is at L3 in terms of solving the problem. The posed problem at week-5 was evaluated as L4 since it was solved by students correctly. In this case, it can be said that the students have improved in terms of solving the problems they have posed over time.

It is understood that GeoGebra's calculation feature was used in the view of S_4 as *"…Through the algebra window feature, we could see everything we did… The GeoGebra program was both practical and contributing to finding the right result."* It is also stated that GeoGebra tools are used in the solution of the problem in the view of S_5 as *"… It was very easy and simple when posing a problem in GeoGebra, and we could find the right answer through GeoGebra when our answer was wrong."* Therefore, it can be said that the features of GeoGebra contribute to the correct solution of the problems posed by the students.

Students' Views about Problem Posing Based Learning

This section presents results regarding the fourth sub-problem "How are students' views on pre and post-implementation problem posing based learning?" Codes obtained as a result of the analysis of the answers given by students to open-ended questions are presented below.

Figure 17.
Students' Views Before the Implementation



In Figure 17, it can be seen that before the implementation, all of the students stated that problem posing was not used sufficiently in mathematic lessons and they were inexperienced about posing problems. In addition to these views, the students stated that making use of problem posing activities would contribute to understanding the topics better. In this regard, S_6 stated, *"We do not benefit from problem posing in mathematics lessons. Okay, we are solving problems, but we are not posing problems ourselves. But if we could pose problems, we would have better understood topics and the topics would be simpler for us…"* S_7 said, *"We do not pose problems in the lesson. But if I pose problems, it shows that I understand this topic…"* S_{15} said, *"We do not pose problems in lessons. But if we pose problems, we could understand better and write better problems and solve them…"* These views of the students show that they are inexperienced about problem posing but they have a positive perspective towards problem posing activities.

Figure 18.
Students Views After the Implementation (Problem Posing)

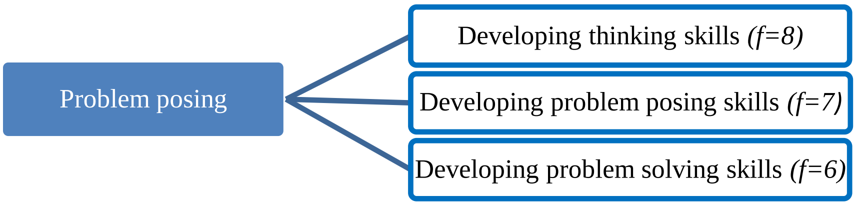
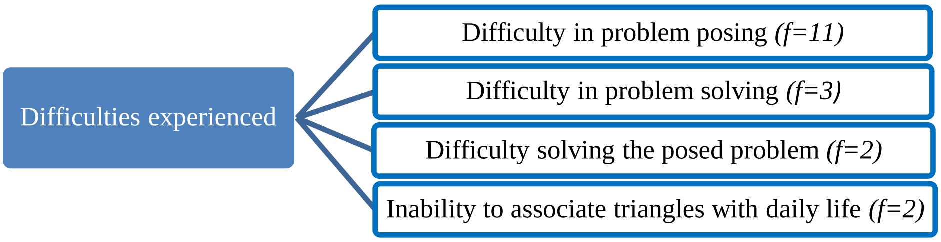


Figure 18 shows that after the implementation, the students stated that problem posing developed their thinking skills and problem solving-posing skills. For example, S_3 said *"…Problem posing makes us think logically. While posing a problem, posing a solvable problem in accordance with the logical content for the person in front of us to understand the problem improves our thoughts and improves us both in posing and solving problems…"* S_4 stated *"…Through the problem posing in GeoGebra, I can pose a problem as much as I want, so I can plan how to solve the problem and of the lesson, I have improved my problem solving skill while solving it. I can better do than before."* S_{11} said *"…In the GeoGebra lesson, our problem posing skill improved and helped us to understand the topics better. Through the problem posing, I can easily understand the topics and solve them easily. Through the problem posing, our logic skill improves…"* In line with these views, it can be said that all of the planning students made during the process of shaping the problem they posed contributed to their problem solving-posing skills in addition to their high level thinking skills.

Figure 19.
Students Views After the Implementation (Difficulties Experienced)



When Figure 19 is examined, it can be seen that in terms of their views after the implementation, the students stated that they had difficulties in solving and posing problems. In this regard, S_2 stated, *"Problem posing; because I had difficulties while thinking, designing and finding problems related with the topic…"* S_7 stated, *"I had difficulties in posing a problem. Because I could not solve the problem when I could not pose the problem meaningfully. But we posed a lot of problems, we discussed about these problems, we saw visuals and I began to develop as I made researches…"* According to the views obtained, it can be understood that a great majority of the students had difficulties in problem posing process. However, it can be said that the stages of ALF and the studies conducted for posing problems during the implementation process supported students' problem posing skills. In addition, some of the students stated that they had difficulties in solving the problems they posed and associating triangles with daily life. S_4 said, *"The fact that the problem posing stage of GeoGebra supported problem posing based learning process was very useful for me. I can pose problems with the measurements I learned, but I experienced difficulties in this process. For example: After I pose the problem, sometimes I cannot solve that problem or I have difficulties in solving…"* S_{10} said, *"…In the past, I could not pose a problem alone, I could not associate the things in our daily life with triangles, but thanks to GeoGebra, I can do these now…"* Therefore, it can be said that in the modelling of examples stage of ALF, different triangle models presented each week affected students positively in terms of associating these with daily life.

DISCUSSION AND CONCLUSION

This study examined the effects of using DGS in ALF on students' problem posing skills. As a result of the study, it was found that the use of DGS in ALF developed students' problem posing skills. This result is in line with studies showing that problem posing approaches improve students' problem posing skills (e.g., Abu Elwan, 1999, 2002; Cankoy, 2014; English, 1997). Similarly, Lavy (2015) stated that the integration of the problem posing process with the WIN strategy and DGS supports the self-confidence and problem posing skills of prospective teachers. In addition, Beal and Cohen (2012) concluded that middle school students could successfully pose problems in a web based implementation. Students actively participated in the problem posing process through the stages of ALF (Ellerton, 2013) and the design of this lesson provided students with a rich learning environment that is different and they had not experienced before. Therefore, it can be said that the development of students is due to the stages of ALF adopted in the problem posing based learning process and the positive effect of GeoGebra.

Regarding the use of mathematical language, it was determined that most of the students posed problems in the pre-test were at a low level, and that they developed in the post-test. While the mistakes in the use of mathematical language were higher in the posed problems in the first weeks of the implementation process, the students improved as the weeks progressed. In line with the views of the students, it can be said that the development is due to the features of GeoGebra which support the use of mathematical language correctly. The conclusion of the Zengin (2018) that GeoGebra supports the mathematical communication skills of prospective teachers in the sociocultural learning environment is in line with this view. Similarly, Lavy and Shriki (2010) found that problem posing supported by DGS and WIN strategy was effective in developing the mathematical knowledge of prospective teachers and deepening their knowledge about geometric concepts. The errors and deficiencies made in the use of mathematical language in the problems posed by students during the implementation process were questioned at the discussion stage. In this process, situations such as incorrect and incomplete use of symbols such as length and angle in triangles, the information on the figure being shown mathematically incomplete or incorrect in the problem statement were discussed as a classroom. In this context, it can be said that the stage of discussing ALF has an important effect on the development shown.

Most of the problems that students posed in the pre-test are low level in grammar and expression skills. Through the implementation process, it was determined that although the students showed improvement in this skill in the post-test, they had difficulty in expressing the problem. Similarly, Lin and Leng (2008) determined that one of the reasons for the unsolvable problems posed by secondary school students is incomprehensible statements in the problem sentence. In terms of grammar and expression criterion, it can be said that the posed problems in GeoGebra in the first weeks are at low level and students have difficulty in this criterion although they have improved over time. It can be said that this situation was caused by the students not paying attention to the spelling rules while posing problems and having difficulty in writing the problem sentence clearly. In addition, the fact that the students used unnecessary sentences in the problem sentence while trying to pose problems in daily life situations caused this result.

In the suitability to acquisitions, which is another problem posing skill, the problems posed by students in the pre-test were developed in the post-test in terms of this criterion. It was found that most of the problems posed with GeoGebra during the implementation were suitable to acquisitions. In line with the views of the students, it can be said that this development is caused by the problem solving stage of the ALF suitable for each acquisition. In addition, in this development, the ALF is thought to be effective at the problem posing stage, which has the same structure as the model problem (Ellerton, 2013). Students who can realize which topic is related to the presented problem posing activity, may have problems in the targeted acquisitions. It can be said that in problem solving activities related to the acquisitions before the problem posing in the learning process, students can realize which acquisition is related to the solved problem. In this way, it is thought that students do not tend to pose problems related to different topics.

In terms of data quantity and quality skills, it was determined that inappropriate data, expressions or incomprehensible expressions were used in the majority of the problems that students posed in the pre-test. In the post-test, it was observed that the students paid attention to the data they used and showed improvement. It was determined that the data in the problems posed in GeoGebra by the students in the first weeks are not suitable for real life, therefore there are logical errors in the problems. During the discussion stage of ALF, posed problems with inappropriate data were criticized by the class. Class discussions in the problem posing process allow students to hear ideas they could not think of from others (Lavy, 2015). In this study, through the discussion stage of ALF, the students noticed the mistakes they made in the data in the posed problems with their peers. Therefore, it can be said that the development stems from the discussion stage. In fact, S_6 and S_7's views that the discussion process reduces their mistakes in the problems they pose support this view.

When the students' problem posing skills were examined in terms of solvability that is another criterion, it was seen that there was improvement in favor of the post-test. This result is compatible with the study results of English (1997) and Cankoy (2014). For example, Cankoy (2014) examined the effects of interlocked problem posing and conventional problem posing teaching on fifth-graders' problem posing skills in the situation of free problem posing. As a result of five-week long implementation, it was found that interlocked problem posing developed students more in terms of posing solvable and reasonable problems of start-unknown when compared with the conventional teaching. It has been observed that the most important factors affecting the solvability in the posed problems are the situations where the data are not suitable and the lack of expression in the problem sentence (Özgen et al., 2017). Approximately half of the posed problems in GeoGebra are solvable problems. As the weeks progress, it can be said that the problem posing based learning process affects students positively and there is improvement in terms of solvability in posed problems.

Although the posed problems developed in favor of post-test in the originality criterion, which is another indicator of students' problem posing skills, low level original problems were posed. Considering that the students in this study did not have problem posing experience before the study, it can be said that this is an expected situation. Similarly, in studies performed with students at different grade levels, it was determined that students had difficulties in posing creative problems (e.g., Özgen et al., 2019; Yığ & Ay, 2021; Xie & Masingila, 2017). For example, Yığ and Ay (2021) examined the qualities of the problems posed by seventh-graders on linear equations with support from problem posing approach. It was found that students were unsuccessful about posing original problems and they were affected by the problems in textbooks in most of the problems they posed. In the implementation process, it is determined that a few of the posed problems in GeoGebra are original problems. In line with the views of the students, it can be said that this situation stems from the fact that students tend to pose easy problems while posing problems. Although students have reached the level to do basic activities by GeoGebra teaching, the fact that they did not use GeoGebra before the implementation may have affected this result. It is suggested that students should spend more time on problem posing activities to pose original problems, and especially the free posing situations without restrictions.

In the context of the relationship between problem solving and posing (Cai, 1998; Cai & Hwang, 2002; Chen et al., 2015; Silver & Cai, 1996; Xie & Masingila, 2017), the solution of problems posed by students can be an effective criterion in reflecting problem posing skills. In this respect, when the student posed problems are analyzed in terms of the solution of the problem, it is determined that there is improvement in favor of the post-test. Therefore, it can be said that problem posing based learning process is effective in developing students' problem solving skills. This result is in line with the results of the study, which determined that problem posing approaches improve students' problem solving skills (Abu Elwan, 2002; Chen et al., 2015). In the post-test, although approximately 37% of the problems posed by students are solvable, 33% of them are correct. Therefore, it can be said that some students have difficulties in solving their own posed problems correctly (Özgen et al., 2017) and students' problem solving skills affect the problem posing process. In the first weeks of the process of implementation, while the problems posed by the students were at a low level in terms of this criterion, improvement was shown as the weeks progressed. According to the S_4 and S_5's views that GeoGebra helps in reaching to the correct result in problem posing, it can be said that the calculation feature of GeoGebra contributes to the correct solution of the posed problems. The result of Bülbül et al. (2020) that GeoGebra was used for visualization, calculation and verification during problem solving process was in parallel with these results. As a result, it has been determined in the present study that the student posed problems in GeoGebra develop in terms of problem posing skills as the weeks progress. However, it was observed that the development was not linear, especially in the problems posed by students in similarity acquisition at week-7, they had difficulty in terms of all skills compared to previous weeks. This situation may be due to learning differences among acquisitions in triangles topic. In the future studies, it can be examined in depth whether the student posed problems in triangles differ according to acquisitions and the reasons of learning differences between them.

It was stated before the implementation in views of students regarding problem posing activities that they did not have problem posing experiences in mathematics lessons. However, students expressed their views that problem posing activities would enable them to understand topics better. This finding shows that students have a positive perspective on problem posing activities. Similarly, Van Harpen and Presmeg (2015) stated that high school students have little or no problem posing experience, but students have a positive attitude towards problem posing. Students' post-implementation views included having difficulties in problem solving-posing and associating triangles with daily life. It can be said that this situation results from the fact that students had not done any problem posing activities before the implementation process. However, during the implementation process, this situation was tried to be overcome with students' doing problem solving-posing activities with stages based on ALF and examining modelling of examples related with triangles. Indeed, in addition to their views above, the students stated that problem posing had a positive effect on their thinking skills and problem solving-posing skills. Abramovich and Cho's (2015) view that technology supported problem posing had the potential to support reasoning and thinking skills and also developing students' problem solving skills support these findings. By following the instructions in the activities in ALF's locating of examples stage, the students tried to find out the characteristics of triangles such as the relationship between the sides of triangle with their group friends using GeoGebra software. Therefore, it is thought that the inquiries made in the process of determining the relationships between concepts contributed to the development of students' high level thinking skills. Moreover, it is thought that students' consideration of many situations such as the appropriateness of data and expressions in the problem statement, language and expression makes great contributions in terms of developing their thinking skills.

In line with the results obtained from the study, some suggestions were made. First of all, it can be said that DGS use in ALF supports students' problem posing skills and the classroom environment designed meets the problem posing learning environment, the lack of which was emphasized. In this case, similar problem posing environments can be designed especially for students who do not have problem posing experience. In addition, the stages of ALF can be a guide to researchers in designing problem posing process. In addition, due to the positive effects on student problem posing skills, it is recommended that problem posing activities be carried out in DGS supported environments. Thanks to the discussion stage of ALF, students questioned the errors they made in the problems they posed and they had the chance to see these errors. Therefore, supporting problem posing activities with classroom discussions can make significant contributions to students in terms of improving the quality of problems posed. Based on the result that students have positive perspectives towards problem posing, it is understood that such activities should be included more in mathematics lessons. In this context, it may be useful to follow these problem posing stages followed in this study in problem posing activities to be carried out with students in different learning areas and different grades. In addition, future studies can work with fewer students and examine students' thoughts in problem posing process and examine in more detail the interaction between students through observation.

Limitations

The main limitation of this study is that there was no control group to compare the problem posing skills of the experimental group students and increase the generalizability of the results. Therefore, a similar experimental design can be realized in the future by using a control group equivalent to the experimental group. Another limitation is that given time to students for the GeoGebra teaching was limited. GeoGebra teaching was carried out according to the tools in tasks and the possible tools students can use in the process of posing problem in triangles. This situation limited the students in terms of the tools they could use in GeoGebra software. Therefore, the GeoGebra teaching provided to students may cover a longer period of time and more comprehensive teaching in a similar study in the future. In addition, due to the inadequacy of the devices in the computer laboratory, students at GeoGebra posed problems in pairs. Although this situation contributed to cooperative learning, it caused certain students to be dominant in some groups. Therefore, in a future study, students may be able to pose problems individually during the implementation process.

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TÜRKÇE GENİŞLETİLMİŞ ÖZET

Problem kurma, verilen problemde değişiklikler yapma ya da sunulan matematiksel durumlara uygun problem oluşturma olarak tanımlanan zihinsel bir aktivitedir (Cai & Hwang, 2020; Silver, 1994). Problem kurma etkinliklerinin teknoloji ile desteklenmesi problem kurma sürecini zenginleştirebilir (Shriki & Lavy, 2012). Bu nedenle çalışmada GeoGebra destekli problem kurma temelli bir öğrenme süreci tasarlanmıştır ve bu öğrenme ortamının öğrencilerin problem kurma becerileri üzerindeki etkisine odaklanılmıştır. Öğrencilerin problem kurma becerilerini geliştirmek amacıyla tasarlanan sınıf ortamında problem çözme ve problem kurma etkinliklerinden yararlanılmıştır. Bu nedenle çalışmada, Ellerton (2013) tarafından geliştirilen aktif öğrenme çerçevesi (AÖÇ) kullanılmıştır. Bununla birlikte AÖÇ'nin aşamaları dinamik geometri yazılımı (DGY) ile desteklenmiştir.

Araştırmada, AÖÇ'de DGY kullanımının sekizinci sınıf öğrencilerinin üçgenler konusundaki problem kurma becerilerine ve problem kurmaya yönelik görüşlerine etkisini incelemek amaçlanmıştır. Araştırma, gömülü karma yöntem ile tasarlanmıştır ve 13 hafta sürmüştür. Katılımcılar kolay ulaşılabilir durum örneklemesine göre belirlenen 16 sekizinci sınıf öğrencisinden oluşmuştur. Araştırmada yedi adet etkinlik planı, problem kurma testi (PKT) ve öğrencilerin problem kurma etkinliklerine yönelik düşüncelerini belirlemek için açık uçlu sorular hazırlanmıştır. Uygulama sürecinin ilk dört haftasında haftada üç ders saati (3×40=120 dakika) öğrencilere GeoGebra yazılımı öğretim planı uygulanmıştır. Daha sonra ön testler uygulanmıştır ve öğrenme sürecine geçilmiştir. Öğrenme sürecinde haftada dört ders saati (4×40=160 dakika) üçgenler konusunda GeoGebra destekli AÖÇ'ye göre hazırlanan etkinlikler uygulanmıştır. Yedi haftalık öğrenme sürecinde sırasıyla üçgenin yardımcı elemanları, üçgen eşitsizliği, açı-kenar ilişkileri, üçgen çizimi, Pisagor teoremi, eşlik ve benzerlik kazanımları (MEB, 2018) ele alınmıştır. Uygulama sonunda ise son testler uygulanmıştır ve araştırma tamamlanmıştır.

Öğrencilerin kurdukları problemlerin değerlendirilmesinde Özgen vd. (2017) tarafından geliştirilen "Problem kurma becerilerinin değerlendirilmesine yönelik rubrik" kullanılmıştır. Öğrencilerin problem kurma becerileri matematik dilini kullanabilme, dil bilgisi ve anlatım, kazanımlara uygunluk, veri miktarı ve niteliği, çözülebilirlik, özgünlük, problemin çözümü kriterlerine (Özgen vd., 2017) göre değerlendirilmiştir. Araştırmanın nicel verilerinde, öğrencilerin PKT ön test-son test puanlarının normal dağılım gösterdiği belirlenmiştir. Bu nedenle ilişkili örneklemler t-testi kullanılmıştır ve etki büyükleri hesaplanmıştır. Öğrencilerin görüşleri ve günlüklerinden elde edilen nitel verilerin analizinde ise betimsel analiz kullanılmıştır.

Araştırmada öğrencilerin matematik dilini kullanabilme, dil bilgisi ve anlatım, kazanımlara uygunluk, veri miktarı ve niteliği, çözülebilirlik, özgünlük, problemin çözümü becerilerinde ve PKT toplam puanlarında son test lehine anlamlı fark olduğu belirlenmiştir. Dolayısıyla, AÖÇ'de DGY kullanımının öğrencilerin problem kurma becerilerini geliştirdiği söylenebilir. Öğrencilerin ön testte kurdukları problemlerin rubriğin tüm kriterleri açısından büyük bir kısmının 1. ve 2. düzeyde yoğunlaştığı belirlenmiştir. Son testte kurulan problemlerde ise tüm kriterler açısından 3. ve 4. düzeye doğru olumlu yönde bir gelişim gösterildiği söylenebilir. Bu bulgu öğrenme sürecinin öğrencileri kurdukları problemlerdeki kriterler açısından geliştirdiğini göstermektedir.

Yedi haftalık öğrenme sürecinde öğrencilerin ikili gruplar halinde GeoGebra'da kurdukları problemler rubrikteki kriterler açısından hafta hafta incelenmiştir. Öğrencilerin ilk haftalarda kurdukları problemlerde matematiksel dili kullanmada hatalar yaptıkları, hafta 4'den itibaren matematiksel dilin kullanımı açısından 3. ve 4. düzeye doğru gelişim gösterdikleri söylenebilir. Öğrencilerin dil bilgisi ve anlatım açısından zorlandığı ve kurulan problemlerin yaklaşık yarısının (%51,8) anlamsal açıdan hatalı olduğu belirlenmiştir. Kurulan problemlerden 27'sinin kazanımlara uygun ancak eksik ya da hatalı olduğu, 26 problemin ise kazanımlara uygun eksiksiz ve hatasız olduğu görülmüştür. Ayrıca kurulan problemlerin yaklaşık yarısında (%46,4) kullanılan verilerin yeterli ve uygun olduğu belirlenmiştir. Kurulan problemlerden 26'sında uygun olmayan veri ve ifade eksikliği olduğu, 26'sının ise çözülebilir problemler olduğu belirlenmiştir. Kurulan 56 problemden sadece 3'ünün özgün problem olduğu ve 23'ünün öğrenciler tarafından doğru çözüldüğü tespit edilmiştir.

Çalışmanın sonunda, AÖÇ'de DGY kullanımının öğrencilerin problem kurma becerilerini geliştirdiği belirlenmiştir. Matematik dilini kullanabilme becerileri açısından öğrencilerin ön testte kurdukları problemlerin büyük bir kısmının düşük düzeyde olduğu, son testte ise geliştiği belirlenmiştir. Uygulama sürecinin ilk haftalarında kurulan problemlerde matematiksel dil kullanımında yanlışlıklar daha fazla iken, öğrenciler haftalar ilerledikçe ilerleme göstermiştir. Öğrencilerin ön testte kurdukları problemlerin büyük bir kısmı dil bilgisi ve anlatım becerisinde düşük düzeydedir. Uygulama süreci sayesinde öğrenciler son testte bu beceri açısından gelişim göstermesine rağmen problemi ifade etmede zorlandıkları belirlenmiştir. Dil bilgisi ve anlatım kriteri açısından GeoGebra'da ilk haftalarda kurulan problemlerin düşük düzeyde olduğu, öğrencilerin zamanla gelişim göstermesine rağmen bu kriterde zorlandıkları söylenebilir. Bir diğer problem kurma becerisi olan kazanımlara uygunluk kriterinde, ön testte öğrenciler tarafından kurulan problemler son testte bu kriter açısından gelişmiştir. Uygulama sürecinde GeoGebra'da kurulan problemlerin ise çoğunluğunun kazanımlara uygun olduğu belirlenmiştir. Veri miktarı ve niteliği becerisi açısından, öğrencilerin ön testte kurdukları problemlerin çoğunluğunda uygun olmayan veri, ifadelerin olduğu ya da anlaşılmayan ifadelerin kullanıldığı belirlenmiştir. Son testte ise öğrencilerin kullandıkları verilere dikkat ettikleri ve gelişim gösterdikleri görülmüştür. Öğrenciler tarafından ilk haftalarda GeoGebra'da kurulan problemlerdeki verilerin gerçek yaşama uygun olmadığı bu nedenle problemlerde mantıksal açısından hatalar olduğu belirlenmiştir. Öğrencilerin problem kurma becerileri bir diğer kriter olan çözülebilirlik açısından incelendiğinde son test lehine gelişim gösterildiği görülmüştür. GeoGebra'da kurulan problemlerin ise yaklaşık yarısı çözülebilir problemlerdir. Öğrencilerin problem kurma becerilerinin bir diğer göstergesi olan özgünlük kriterinde kurulan problemler son test lehine gelişim göstermesine rağmen düşük düzeyde özgün problemler kurulmuştur. Uygulama sürecinde de GeoGebra'da kurulan problemlerin çok az bir kısmının özgün problemler olduğu görülmüştür. Öğrencilerin kurdukları problemler problemin çözümü açısından incelendiğinde son test lehine gelişim gösterildiği belirlenmiştir. Dolayısıyla problem kurma temelli öğrenme sürecinin öğrencilerin problem çözme becerilerini geliştirmede etkili olduğu söylenebilir. Son testte, öğrenciler tarafından kurulan problemlerin yaklaşık %37'si çözülebilir olmasına rağmen bunlardan %33'ünün çözümü doğrudur. Dolayısıyla bazı öğrencilerin kendi kurdukları problemleri doğru çözmede zorlandıkları (Özgen vd., 2017) ve öğrencilerin problem çözme becerilerinin problem kurma sürecini etkilediği söylenebilir. Uygulama sürecinin ilk haftalarında ise öğrencilerin kurdukları problemler bu kriter açısından düşük düzeyde iken haftalar ilerledikçe gelişim gösterilmiştir.

Problem kurma etkinliklerine yönelik öğrencilerin uygulama öncesi görüşlerinde, matematik derslerinde problem kurma deneyimlerinin olmadığı belirtilmiştir. Ancak öğrencilerin problem kurma etkinliklerinin konuları daha iyi anlamayı sağlayacağına yönelik düşünceleri bulunmaktadır. Öğrenciler uygulama sonrası görüşlerinde ise yaşadıkları zorlukları problem çözme ve kurmada zorlanma, üçgenleri günlük yaşamla ilişkilendirememe olarak belirtmişlerdir. Ancak uygulama sürecinde öğrencilerin AÖÇ'ye dayalı aşamalarla problem çözme-kurma etkinlikleri yapmaları, üçgenlerle ilgili model örneklemeleri incelemeleri sayesinde bu durum aşılmaya çalışılmıştır. Nitekim öğrenciler yukarıdaki görüşlerine ek olarak problem kurmanın, düşünme becerisine ve problem çözme-kurma becerilerine olumlu yönde yansıdığını ifade etmişlerdir. Araştırmadan elde edilen sonuçlar doğrultusunda, AÖÇ'de DGY kullanımının öğrencilerin problem kurma becerilerini desteklediği ve tasarlanan sınıf ortamının eksikliği vurgulanan problem kurma öğrenme ortamını karşıladığı söylenebilir. Bu durumda özellikle problem kurma deneyimi olmayan öğrenciler için benzer problem kurma ortamları tasarlanabilir.