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Research Article

Applications of the Sub Equation Method for the High Dimensional Nonlinear Evolution Equation

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Abstract

In this article, the Generalized (3+1)-dimensional Shallow Water-Like (SWL) equation is taken into consideration and exact solutions have been constructed of the SWL equation using sub equation method. This method is an easier and efficient method for finding analytic solutions of the nonlinear evolution differential equation. The method appears to be easier and faster for symbolic computation. Moreover, 2D, 3D and contour graphical representation of the obtained results of the specified equation is made using a ready-made package program for certain values and thus the conformity of the founded results has been demonstrated.

Keywords: Sub equation method, the Generalized (3+1)-dimensional Shallow Water-Like equation, Exact solution, Traveling wave solution.

Yüksek Boyutlu Doğrusal Olmayan Evrim Denklemi için Alt Denklem Metodunun Uygulanması

Öz

Bu makalede Genelleştirilmiş (3+1) boyutlu sığ su benzeri (SWL) denklemi dikkate alınmış ve SWL denkleminin alt denklem yöntemi kullanılarak kesin çözümleri oluşturulmuştur. Bu metot, doğrusal olmayan evrim denklemlerinin analitik çözümlerini bulmak için daha kolay ve verimli bir yöntemdir. Metot, sembolik hesaplama için daha kolay ve daha hızlı görünmektedir. Ayrıca, belirtilen denklemin elde edilen sonuçlarının 2 boyutlu, 3 boyutlu ve kontur grafiksel gösterimi belirli değerler için hazır paket programı kullanılarak yapılmış ve böylece bulunan sonuçların uygunluğu gösterilmiştir.

Anahtar Kelimeler: Alt denklem metodu, Genelleştirilmiş (3+1) boyutlu sığ su benzeri denklemi, Tam çözüm, Yürüyen dalga çözümü.

1. Introduction

The models of many physical phenomena that occur in the world appear in applied mathematics as nonlinear evolution differential equations (NLEEs). Therefore, these NLEEs analytical and numerical solutions are very important in fields such as fluid dynamics (Benetazzo et al., 2021), virus spread (Gao et al., 2020), biology (Yavuz and Yokus, 2020), and so on (Duran, 2020; Yokus and Yavuz, 2020; Eckart, 1948; Sulaiman, et al., 2018; Fellman, 2007; Akturk, 2020; Duran et al., 2017; Yokus et al. 2019; Kaya et al. 2020; Yokus et al., 2021). In particularly, the soliton obtained as a result of these NLEEs and their applications were first defined by Scott

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Russell (1845). Later, it was developed with advanced studies on the concept of soliton by Scott et al., (1973). To reach these solutions, many analytical methods have been developed for the solutions of NLEEs. Some of these methods: exponential function method (Bulut et al., 2016), Bernoulli sub equation method (Durur et al., 2020; Baskonus and Bulut, 2015), modified exponential function method (Silambarasan and Kilicman, 2021), the sine-Gordon expansion method (Ali et al., 2020), $(1/G')$-expansion method (Yokus et al., 2020), $(G'/G, 1/G)$-expansion method (Duran, 2020; Duran 2021) and so on (Durur, 2020; Durur et al., 2020; Duran, 2021; Tabassum, 2021).

$(3+1)$-dimensional Shallow Water-Like (SWL) equation (Dusunceli, 2019) is a nonlinear evolution equation that has become quite popular recently

$$u_{xxxy} + 3u_{xx}u_y + 3u_x u_{xy} - u_{yt} - u_{xz} = 0. \quad (1)$$

When we examine the literature, the SWL equation has been examined by many researchers with the help of different methods. We can define the hierarchical order of some of the important ones as follows: Traveling wave solutions were created by Tran and Gao (1996) with the help of the generalized tanh algorithm method. Traveling wave solutions have been reached by Zayed (2010) with the help of (G'/G)-expansion method. These solutions were classified as hyperbolic, trigonometric and rational functions. By giving special values to the parameters obtained here, solitary waves were created from traveling waves. Grammian and Pfaffian solutions were obtained for the SWL equation by Tang et al. (2012). Rational and lump solutions for Eq. (1) with the help of a generalized bilinear operator were presented by Zhang et al. (2017). The exact solutions of Eq. (1) were obtained with the help of the Bernoulli sub-equation method by Dusunceli (2019).

The aim of this study is to obtain traveling wave solutions with the help of sub equation method (Durur et al., 2020) for the SWL equation, which is very rich in literature. In these traveling wave solutions, we have obtained, graphs are presented with the help of special values given to parameters.

2. Sub-Equation Method

Let us explain the methodology of the sub equation method to solve NLEEs. Consider the NLEEs as

$$T\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0. \quad (2)$$

Applying the wave transformation

$$\boldsymbol{u(x, y, z, t) = U(\xi) = U},$$

$$\xi = x + ky + mz - wt,$$

where w, k are constants and w is the velocity of the wave. We may be converted into the following nonlinear ordinary differential equation for $U(\xi)$:

$$\mathrm{L}(U, U', U'', U''', \dots) = 0. \tag{3}$$

The solution of Eq. (3) is assumed to have the form

$$\boldsymbol{U}(\boldsymbol{\xi}) = \sum_{i=0}^{n} \boldsymbol{a}_i \left(\boldsymbol{G}(\boldsymbol{\xi})\right)^{i}, \boldsymbol{a}_{\mathbf{n}} \neq \mathbf{0}, \tag{4}$$

where $a_i, (0 \le i \le n)$ are constants. These a_i values will then be found with the help of the balancing term. The positive integer *n* defined in Eq. (4) is the balance term and is found with the help of two factors. These factors are taken into consideration the property of balancing and the solution of the Riccati equation.

$$G'(\xi) = (G(\xi))^2 + \mu, \tag{5}$$

where μ is an arbitrary constant. **The Riccati equation in Eq. (5) gives certain exclusive solutions as follows.**

$$G(\xi) = \begin{cases} -\sqrt{-\mu}\tanh(\sqrt{-\mu}\,\xi)\,, \mu < 0 \\ -\sqrt{-\mu}\coth(\sqrt{-\mu}\,\xi)\,, \mu < 0 \\ \sqrt{\mu}\tan(\sqrt{\mu}\,\xi)\,, \mu > 0 \\ -\sqrt{\mu}\cot(\sqrt{\mu}\,\xi)\,, \mu > 0 \\ -\frac{1}{\xi + R},\ \mu = 0(R\ is\ const.) \end{cases} \tag{6}$$

In Eq. (3) if we apply the Eq. (5) and Eq. (4), with respect to $G(\xi)$, a nonlinear algebraic equation system, we reached the new polynomial. All the coefficients $G(\xi)^i, (i = 0,1, \dots, n)$ of the obtained polynomial to find constants a_i are set to zero. In the nonlinear algebraic equation system, we have obtained as described above, constants are determined as $\mu, \tau, R,$ $a_i,\ (i = 0,1, \dots, n)$. Solving this system with the help of a ready-made package program and substituting the constants in the relevant equations with the help of Eq. (6), we obtain an exact solution for Eq. (2).

3. Solutions of the Generalized

(3+1)-Dimensional SWL Equation

We consider Eq. (1) and using transformation

$$\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}) = \boldsymbol{U}(\boldsymbol{\xi}) = \boldsymbol{U},$$

$$\xi = x + ky + mz - wt,$$

$$\mathrm{k}U^4 + 6kU'U'' + (kw - m)U'' = 0. \tag{7}$$

If the Eq. (7) is integrated according to ξ, we can write the following equation

$$kU''' + 3k(U')^2 + (kw - m)U' = 0. \tag{8}$$

According to the homogeneous balancing principle, the equilibrium term of Eq. (8) is $n = 2$. By using this balancing term in Eq. (4), the following situation is obtained:

$$U(\xi) = a_0 + a_1(G(\xi)) + a_2(G(\xi))^2, \tag{9}$$

where $a_1 \neq 0$ or $a_2 \neq 0$. Eq. (9) is substituted in Eq. (8). Then, by using the polynomial equation, the coefficients are equalled to zero. After doing the necessary operations, the following algebraic equation system is obtained:

$$(G(\xi))^0: -m\mu a_1 + kw\mu a_1 + 2k\mu^2 a_1 + 3k\mu^2 a_1^2 = 0,$$

$$(G(\xi))^1: -2m\mu a_2 + 2kw\mu a_2 + 16k\mu^2 a_2 + 12k\mu^2 a_1 a_2 = 0,$$

$$(G(\xi))^2: -ma_1 + kwa_1 + 8k\mu a_1 + 6k\mu a_1^2 + 12k\mu^2 a_2^2 = 0,$$

$$(G(\xi))^3: -2ma_2 + 2kwa_2 + 40k\mu a_2 + 24k\mu a_1 a_2 = 0,$$

$$(G(\xi))^4: 6ka_1 + 3ka_1^2 + 24k\mu a_2^2 = 0,$$

$$(G(\xi))^5: 24ka_2 + 12ka_1 a_2 = 0,$$

$$(G(\xi))^6: 12ka_2^2 = 0. \tag{10}$$

Constants a_1, a_2, k, μ, w, m are attained from Eq. (10) system with the aid of package program.

Case1.

$$a_1 = -2,\ a_2 = 0,\ w = \frac{m+4k\mu}{k}. \tag{11}$$

Substituting these obtained values into Eq. (9), the hyperbolic wave solution for Eq. (1) is obtained

$$u_1(x,y,z,t) = a_0 + 2\sqrt{-\mu}\text{Tanh}[\sqrt{-\mu}(x + ky + mz - \frac{t(m+4k\mu)}{k})]. \tag{12}$$

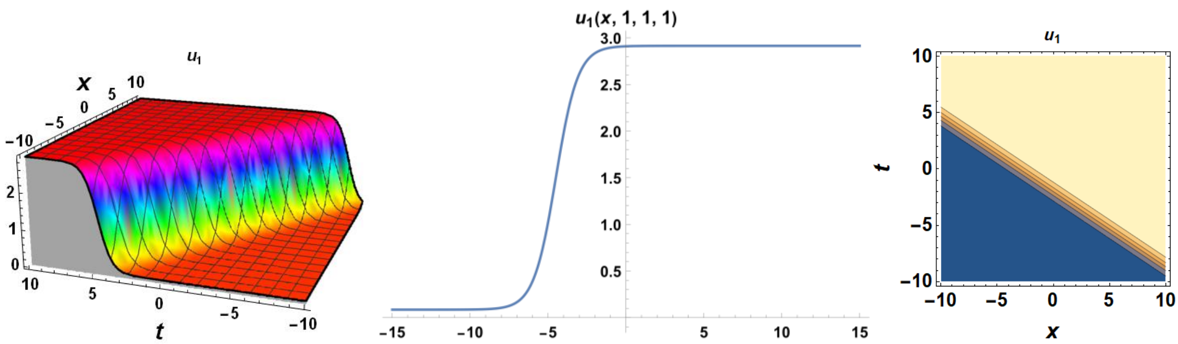

Figure 1. 3D, 2D and contour graphs of $u_1(x,y,z,t)$ respectively for $\mu = -0.5, a_0 = 1.5, m = 1, k = 2, y = 1, z = 1$.

Case2.

$$a_1 = -2,\ a_2 = 0,\ w = \frac{m+4k\mu}{k}. \tag{13}$$

Substituting these obtained values into Eq. (9), the hyperbolic wave solution for Eq. (1) is obtained

$$u_2(x,y,z,t) = 2\sqrt{-\mu}\mathrm{Coth}[\sqrt{-\mu}(x+ky+mz-\frac{t(m+4k\mu)}{k})] + a_0. \tag{14}$$

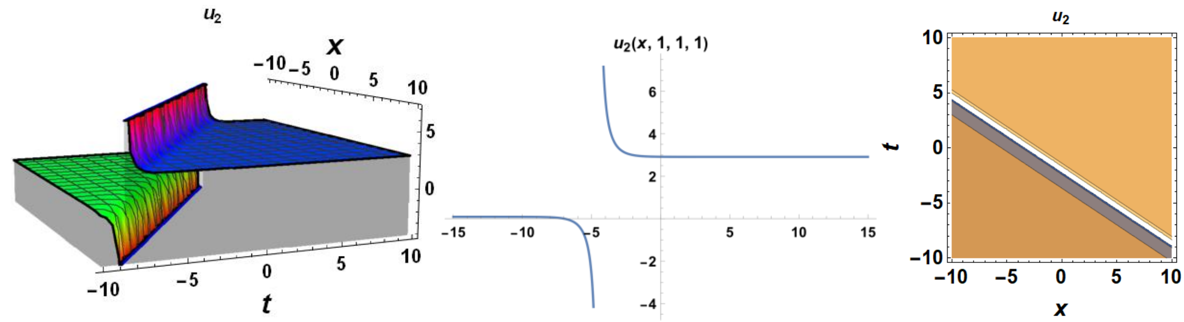

Figure 2. 3D, 2D and contour graphs of $u_2(x,y,z,t)$ respectively for $\mu = -0.5, a_0 = 1.5,$ $m = 1, k = 2, y = 1, z = 1.$

Case3.

$$a_1 = -2,\ a_2 = 0,\ w = \frac{m+4k\mu}{k}. \tag{15}$$

Substituting these obtained values into Eq. (9), the trigonometric wave solution for Eq. (1) is obtained

$$u_3(x,y,z,t) = a_0 - 2\sqrt{\mu}\mathrm{Tan}[\sqrt{\mu}(x+ky+mz-\frac{t(m+4k\mu)}{k})]. \tag{16}$$

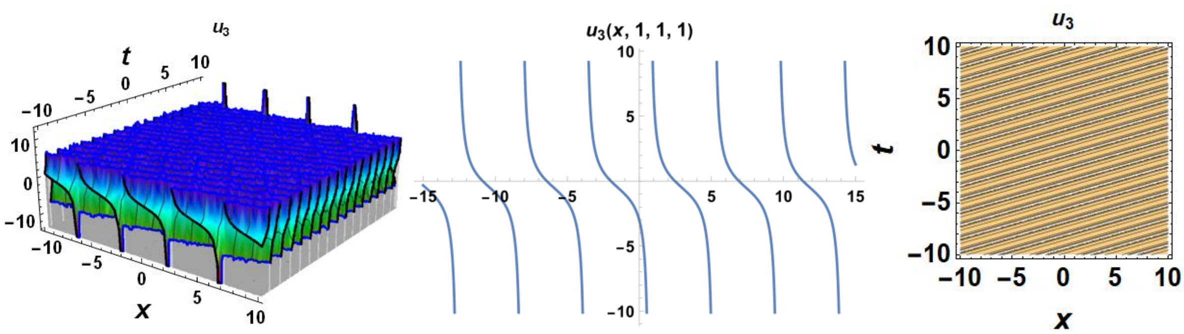

Figure 3. 3D, 2D and contour graphs of $u_3(x,y,z,t)$ respectively for $\mu = 0.5, a_0 = -0.5,$ $m = 3, k = 2, y = 1, z = 1.$

Case4.

$$a_1 = -2,\ a_2 = 0,\ w = \frac{m+4k\mu}{k}. \tag{17}$$

Substituting these obtained values into Eq. (9), the trigonometric wave solution for Eq. (1) is obtained

$$u_4(x,y,z,t) = 2\sqrt{\mu}\mathrm{Cot}[\sqrt{\mu}(x + ky + mz - \frac{t(m+4k\mu)}{k})] + a_0. \tag{18}$$

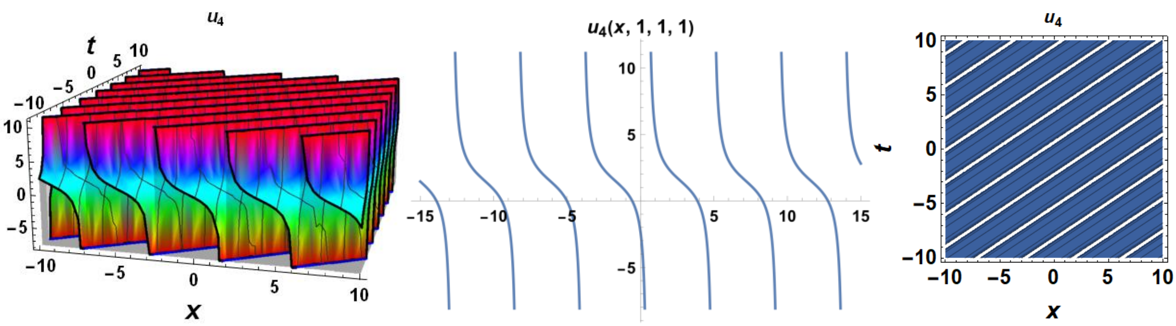

Figure 4. 3D, 2D and contour graphs of $u_4(x,y,z,t)$ respectively for $\mu = 0.5, a_0 = 1.5, m = -1, k = 2, y = 1, z = 1$.

Case5.

$$a_1 = -2,\ a_2 = 0,\ w = \frac{m+4k\mu}{k}. \tag{19}$$

Substituting these obtained values into Eq. (9), the rational wave solution for Eq. (1) is obtained

$$u_5(x,y,z,t) = \frac{2}{R - \frac{mt}{k} + x + ky + mz} + a_0. \tag{20}$$

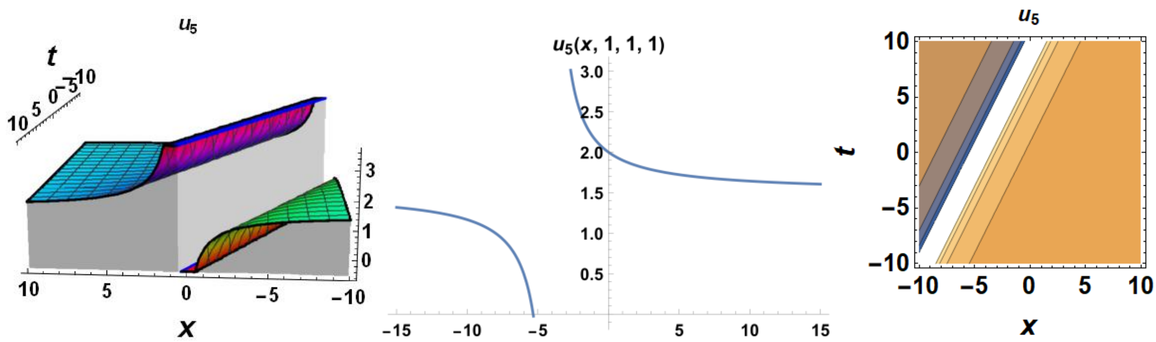

Figure 5. 3D, 2D and contour graphs of $u_5(x,y,z,t)$ respectively for $\mu = 0, R = 1.5, a_0 = 1.5, m = 1, k = 2, y = 1, z = 1$.

4. Conclusion

In this study, we have obtained the traveling wave solutions of the (3+1) dimensional SWL equation. When these traveling wave solutions were examined, they were classified in hyperbolic, trigonometric and rational forms. The sub equation method applied here is an effective method because it produces solutions from all different classes. Therefore, this method, which is easier to apply than other methods, is a very efficient and reliable method to find solutions to NLEEs. Besides, the solutions for the five different situations obtained are graphically shown as 3D, 2D and contour with the help of a ready-made package program.

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