Adıyaman University Journal of Science
https://dergipark.org.tr/en/pub/adyujsci
DergiPark AKADEMİK
ISSN 2147-1630
e-ISSN 2146-586X

Solitary Wave Solutions of the Generalized (3+1)-Dimensional Shallow Water-Like Equation by Using Modified Kudryashov Method

Asıf YOKUŞ[1,*]

[1]*Fırat University, Faculty of Science, Department of Mathematics, Elazığ, 23100, Turkey*
asfyokus@yahoo.com, ORCID: 0000-0002-1460-8573

Received: 22.02.2021 **Accepted:** 24.05.2021 **Published:** 30.06.2021

Abstract

In this study, the generalized (3+1)-dimensional Shallow Water-Like (SWL) equation, which is one of the evolution equations, is taken into consideration. With the help of this evolution equation discussed, the modified Kudryashov method, traveling wave solutions are successfully obtained. In these solutions, graphs of solitary waves to be obtained by giving special values to arbitrary parameters are presented. At the same time, the effect of change of velocity parameter on the behavior on the solitary wave is examined in the solution obtained. The breaking of the wave is discussed. In this study, complex operations and graphic presentations are presented with the use of a ready-made package program.

Keywords: Generalized (3+1)-dimensional Shallow Water-Like equation; Modified Kudryashov method; Traveling wave solution.

Modifiye Kudryashov Metodu Kullanılarak Genelleştirilmiş (3 + 1)-Boyutlu Sığ Su Benzeri Denkleminin Solitary Dalga Çözümleri

* Corresponding Author DOI: 10.37094/adyujsci.883428

CC BY SA

Öz

Bu çalışmada evrim denklemlerinden biri olan genelleştirilmiş (3+1) boyutlu sığ su benzeri denklemi dikkate alınır. Ele alınan bu evrim denklemi modifiye Kudryashov metodu yardımıyla yürüyen dalga çözümleri başarılı bir şekilde elde edilir. Bu çözümlerde keyfi parametrelere özel değerler verilerek elde edilecek solitary dalgaların grafikleri sunulur. Aynı zamanda elde edilen çözümde hız parametresinin değişiminin solitary dalga üzerindeki davranışlara olan etkisi incelenir. Dalganın kırılma olayı tartışılır. Bu çalışmada karmaşık işlemler ve grafik sunumları hazır paket programının kullanımıyla sunulur.

Anahtar Kelimeler: Genelleştirilmiş (3+1) boyutlu Sığ Su Benzeri denklem; Modifiye Kudryashov metodu, Yürüyen dalga çözümü.

1. Introduction

Nonlinear partial differential equations (NLPDEs) have many application areas such as fluid dynamics, hydromagnetic, optics, physics, chemistry, biology and others [1-5]. With the solutions of these NLPDEs and the values given to the special parameters in these solutions, many physical phenomena we encounter in daily life are modelled [6-10]. Therefore, there has been an increasing interest in the solution methods of NLPDEs by many scientists. Especially the methods existing in the last twenty years are updated and applied to these differential equations. With the help of these methods, many traveling wave solutions that satisfy the equation have been obtained [11-14]. Some of the methods are very efficient in NLPDEs and generate solutions from many different types. These properties of the methods are very important for the application area. Some of these methods are (G'/G)-expansion method [15], (G'/G^2)-expansion method [16], $(1/G')$-expansion method [17-19], $(m+G'/G)$-expansion method [20], $(m+1/G')$-expansion method [21], $(G'/G,1/G)$-expansion method [22] and so on [23].

$(3+1)$-dimensional Shallow Water-Like (SWL) equation is a nonlinear evolution equation that has become quite popular recently [24].

$$u_{xxxy}+3u_{xx}u_y+3u_xu_{xy}-u_{yt}-u_{xz}=0. \tag{1}$$

There have been many studies on this equation recently. The (G'/G)-expansion method was obtained by Zayed in 2010 [25], and the traveling wave solutions were obtained with the help of the generalized binary operator by Zhang in 2017 [26]. In 2019, the traveling wave solutions of Eqn. (1) were obtained by Dusunceli with the help of the Bernoulli sub equation method [24].

Then, by applying the sine-Gordon method, traveling wave solutions in complex form were reached by Baskonus and Eskitascıoglu in 2020 [27].

In this study, we aimed to reach traveling wave solutions for Eqn. (1) with the help of the the modified Kudryashov method [28]. At the same time, in the solutions obtained, special values are given to the parameters and presented with the help of graphics.

2. Materials and Methods

2.1. Methodology of the modified Kudryashov method

Assume you have a NLPDE in the form below

$$T\left(u,u_t,u_x,u_y,u_z,u_{xx},...\right)=0, \tag{2}$$

where T is a function in $u\left(x,y,z,t\right)$and its partial derivatives in which nonlinear terms and highest-order derivatives. We give the basic steps of this method in the following.

Step 1. Using the wave transmutation

$$u\left(x,y,z,t\right)=U\left(\xi\right), \quad \xi=x+ky+mz-wt, \tag{3}$$

we can transform it the following nODE for $U\left(\xi\right)$:

$$S\left(U,U',U'',...\right)=0, \tag{4}$$

where Eqn. (4) is the ODE, where k,w,m are constants. Here w is a physical quantity and is the speed parameter of the wave.

Step 2. We assume that Eqn. (4) has the formal solution

$$U\left(\xi\right)=a_0+\sum_{i=1}^{n}(a_iQ\left(\xi\right)^i+a_{-i}Q\left(\xi\right)^{-i}), \tag{5}$$

where $a_i,a_{-i}, \quad i=\left\{1,...,n\right\}$are constants to be determined, such that $a_n\neq 0\ or\ a_{-n}\neq 0$, and $Q\left(\xi\right)$is the solution of the equation

$$Q'\left(\xi\right)=\left[Q^2\left(\xi\right)-Q\left(\xi\right)\right]\ln a, \tag{6}$$

Eqn. (6) has solutions

$$Q(\xi)=\frac{1}{1\pm a^{\xi}}, \tag{7}$$

where $a>0, a\neq 1$ is a real number.

Step 3. In Eqn. (4), a positive integer n is calculated according to the balance principle.

Step 4. Substitute Eqn. (5) with Eqn. (6) into Eqn. (4), we compute all the required derivatives $U', U'', \ldots$ of the function $U(\xi)$. Thus, we get a polynomial of $Q^j(\xi), (j=0,1,2,\ldots)$. Computed polynomial, we add all the terms of the same powers of $Q^j(\xi)$ and equal them to zero, we get a system of algebraic equations that can be solved by a computer package program to attain the unknown parameters $a_i, a_{-i},\quad i=\{1,\ldots,n\}$, k and w. As a result, we get exact solutions of the Eqn. (2).

2.2. Application of modified Kudryashov method

We consider Eqn. (1). By using

$$u(x,y,z,t)=U(\xi),\quad \xi=x+ky+mz-wt, \tag{13}$$

Inserting Eqn. (13) into Eqn. (1), we obtain

$$kU^{(4)}+6kU'U''+(kw-m)U''=0, \tag{14}$$

Once the Eqn. (14) is integrated

$$kU'''+3k(U')^2+(kw-m)U'=0. \tag{15}$$

In the Eqn. (15), we get the balancing term $n=2$ and by considering in the Eqn. (5),

$$U(\xi)=a_0+a_1Q(\xi)+a_2Q(\xi)^{-1}+b_1Q(\xi)^2+b_2Q(\xi)^{-2}, \tag{16}$$

if Eqn. (16) is written in Eqn. (15) and if necessary adjustments are made, the following systems of equations can be written:

$$
\begin{aligned}
&Const: \quad m\log[a]a_2 - kw\log[a]a_2 - k\log[a]^3 a_2 - 6k\log[a]^2 a_1 a_2 + 3k\log[a]^2 a_2^2 \\
&\qquad + 6k\log[a]^3 b_2 + 24k\log[a]^2 a_1 b_2 - 24k\log[a]^2 b_1 b_2 = 0, \\
&\frac{1}{Q[\xi]}: \quad -m\log[a]a_2 + kw\log[a]a_2 + k\log[a]^3 a_2 - 6k\log[a]^2 a_2^2 + 2m\log[a]b_2 \\
&\qquad -2kw\log[a]b_2 - 14k\log[a]^3 b_2 - 12k\log[a]^2 a_1 b_2 + 12k\log[a]^2 a_2 b_2 = 0, \\
&\frac{1}{Q[\xi]^2}: \quad 3k\log[a]^2 a_2^2 - 2m\log[a]b_2 + 2kw\log[a]b_2 + 8k\log[a]^3 b_2 \\
&\qquad -24k\log[a]^2 a_2 b_2 + 12k\log[a]^2 b_2^2 = 0, \\
&\frac{1}{Q[\xi]^3}: \quad 12k\log[a]^2 a_2 b_2 - 24k\log[a]^2 b_2^2 = 0, \\
&\frac{1}{Q[\xi]^4}: \quad 12k\log[a]^2 b_2^2 = 0, \\
&Q[\xi]: \quad m\log[a]a_1 - kw\log[a]a_1 - k\log[a]^3 a_1 + 12k\log[a]^2 a_1 a_2 \\
&\qquad -12k\log[a]^2 a_2 b_1 - 12k\log[a]^2 a_1 b_2 + 48k\log[a]^2 b_1 b_2 = 0, \\
&Q[\xi]^2: \quad -m\log[a]a_1 + kw\log[a]a_1 + 7k\log[a]^3 a_1 + 3k\log[a]^2 a_1^2 \\
&\qquad -6k\log[a]^2 a_1 a_2 + 2m\log[a]b_1 - 2kw\log[a]b_1 - 8k\log[a]^3 b_1 \\
&\qquad +24k\log[a]^2 a_2 b_1 - 24k\log[a]^2 b_1 b_2 = 0, \\
&Q[\xi]^3: \quad -12k\log[a]^3 a_1 - 6k\log[a]^2 a_1^2 - 2m\log[a]b_1 + 2kw\log[a]b_1 \\
&\qquad +38k\log[a]^3 b_1 + 12k\log[a]^2 a_1 b_1 - 12k\log[a]^2 a_2 b_1 = 0, \\
&Q[\xi]^4: \quad 6k\log[a]^3 a_1 + 3k\log[a]^2 a_1^2 - 54k\log[a]^3 b_1 - 24k\log[a]^2 a_1 b_1 + 12k\log[a]^2 b_1^2 = 0, \\
&Q[\xi]^5: \quad 24k\log[a]^3 b_1 + 12k\log[a]^2 a_1 b_1 - 24k\log[a]^2 b_1^2 = 0, \\
&Q[\xi]^6: \quad 12k\log[a]^2 b_1^2 = 0.
\end{aligned} \tag{17}
$$

a_1, a_2, b_1, b_2 and m, k, w constants are obtained from Eqn. (17) the system utilizing a software program.

Case 1: If

$$a_1 = -2log[a], \quad a_2 = 0, \quad b_1 = 0, \quad b_2 = 0, \quad m = k\left(w + log[a]^2\right), \tag{18}$$

replacing values Eqn. (18) into Eqn. (16), we get traveling wave soliton for Eqn. (1)

$$u_1\left(x,y,z,t\right)=-\frac{2\,log\left[a\right]}{1+a^{-tw+x+ky+kz\left(w+log\left[a\right]^2\right)}}+a_0. \tag{19}$$

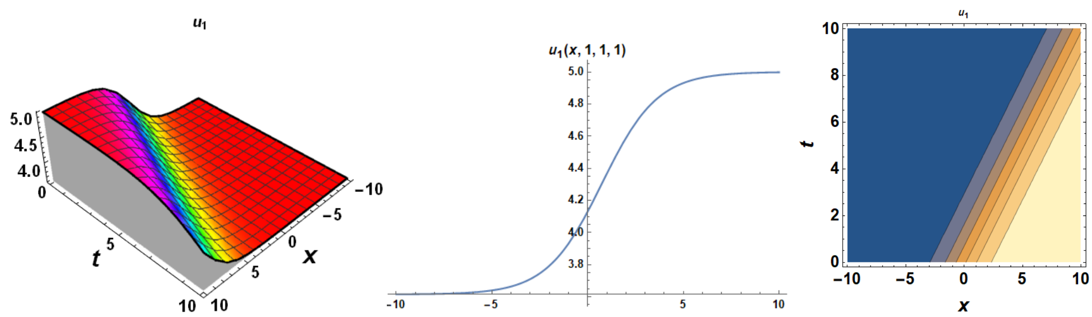

Figure 1: 3D, 2D and contour graphs for the Eqn. (19) for $a_0 = 5, w = 1, k = 0.1, y = 1, z = 1,\ a = 2$

3. Results and Discussions

In this study, we have obtained the traveling wave solution of the SWL equation with the modified Kudryashov method. It can be said that the solution obtained by this method is general from the solutions obtained in $\left(1/G'\right)$-expansion method. This is usually because the base of the exponential function contains an arbitrary parameter. It is the *"e"* expression defined as the base exponential function in the $\left(1/G'\right)$-expansion method. However, the term without exponential expression is constant in modified Kudryashov method, while it contains arbitrary parameters in $\left(1/G'\right)$-expansion method [29]. When the solutions obtained are examined physically, let's examine the effect of the change of velocity parameter on the traveling wave solution obtained. The *"w"* expression in the classical wave transformation is a parameter representing the frequency of the wave and therefore its speed. We can present the effect of velocity on the wave with the following 3D simulation provided that other parameters except *"w"* are taken as constant Fig. 2 as seen in the simulation, as the speed increases, the changes in the behavior of the wave and the refraction phenomenon occur. Here is $w = 2.05$. The distortions at the end point of the value of the wave and the breaking event at $w = 2.055$ are clearly seen. In the future, the effect of other parameters on the wave can be observed.

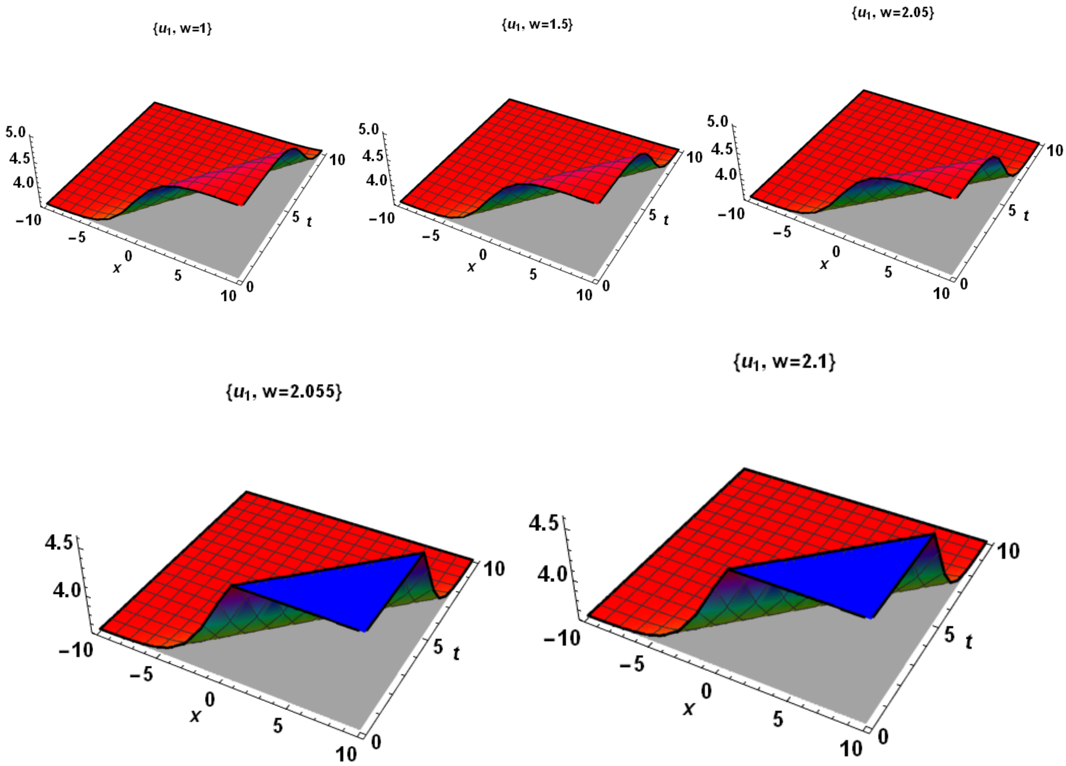

Figure 2: Simulation graphs of the Eqn. (19) for $a_0 = 5, k = 0.1, y = 1, z = 1,\ a = 2$

4. Conclusions

In this study, the traveling wave solution of the generalized SWL equation has been successfully obtained. Solitary wave solutions were created for specific values of arbitrary parameters in the traveling wave solution. 2D, 3D and contour graphics of these solitary waves are presented. At the same time, the effect of the change of velocity parameter on the behavior on the solitary wave in the solution obtained. Fig. 2 also presented and discussed. In addition, the value in the speed parameter at which the breakage of the wave occurred was determined. It was concluded that the modified Kudryashov method is valid, reliable and applicable. In future studies, many studies can be done on non-linear evolution equations with the help of this method.

References

[1] Yavuz, M., Sene, N., *Approximate solutions of the model describing fluid flow using generalized ρ-laplace transform method and heat balance integral method*, Axioms, 9(4), 123, 2020.

[2] Dungey, J.W.*, Hydromagnetic Waves. In Physics of the Magnetosphere,* Based upon the Proceedings of the Conference Held at Boston College, Springer Science & Business Media, 10, 218, 2012.

[3] Rezazadeh, H., Mirhosseini-Alizamini, S.M., Eslami, M., Rezazadeh, M., Mirzazadeh, M., Abbagari, S., *New optical solitons of nonlinear conformable fractional Schrödinger-Hirota equation*, Optik, 172, 545-553, 2018.

[4] Alam, M.N., Akbar, M.A., *Traveling wave solutions for the mKdV equation and the Gardner equations by new approach of the generalized (G'/G)-expansion method*, Journal of the Egyptian Mathematical Society, 22(3), 402-406, 2014.

[5] Duran, S., Askin, M., Sulaiman, T.A., *New soliton properties to the ill-posed Boussinesq equation arising in nonlinear physical science*, An International Journal of Optimization and Control: Theories & Applications (IJOCTA), *7*(3), 240-247, 2017.

[6] Duran, S., *Solitary Wave Solutions of the Coupled Konno-Oono Equation by using the Functional Variable Method and the Two Variables (G'/G, 1/G)-Expansion Method*, Adıyaman Üniversitesi Fen Bilimleri Dergisi, 10(2), 585-594, 2020.

[7] Durur, H., *Different types analytic solutions of the (1+ 1)-dimensional resonant nonlinear Schrödinger's equation using (G'/G)-expansion method,* Modern Physics Letters B, 34(03), 2020.

[8] Saleem, S., Hussain, M.Z., Aziz, I., *A reliable algorithm to compute the approximate solution of KdV-type partial differential equations of order seven*. *Plos one*, *16*(1), e0244027, 2021.

[9] Duran, S., *Exact Solutions for Time-Fractional Ramani and Jimbo—Miwa Equations by Direct Algebraic Method*, Advanced Science, Engineering and Medicine, 12(7), 982-988, 2020.

[10] Yokus, A., Durur, H., Ahmad, H., Yao, S.W., *Construction of different types analytic solutions for the Zhiber-Shabat equation*, Mathematics, 8(6), 908, 2020.

[11] Yokuş, A., Durur, H., Abro, K.A., Kaya, D., *Role of Gilson–Pickering equation for the different types of soliton solutions: a nonlinear analysis*, The European Physical Journal Plus, 135(8), 1-19, 2020.

[12] Duran, S., Kaya, D., *Applications of a new expansion method for finding wave solutions of nonlinear differential equations*, World Applied Sciences Journal, 18(11), 1582-1592, 2012.

[13] Sulaiman, T.A., Bulut, H., Yokus, A., Baskonus, H.M., *On the exact and numerical solutions to the coupled Boussinesq equation arising in ocean engineering*, Indian Journal of Physics, 93(5), 647-656, 2019.

[14] Durur, H., Yokuş, A., *Analytical solutions of Kolmogorov–Petrovskii–Piskunov equation*, Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 22(2), 628-636, 2020.

[15] Zhang, J., Wei, X., Lu, Y., *A generalized (G' G)-expansion method and its applications*, Physics Letters A, 372(20), 3653-3658, 2008.

[16] Rehman, S.U., Yusuf, A., Bilal, M., Younas, U., Younis, M., Sulaiman, T.A., *Application of (G'/G^ 2)-expansion method to microstructured solids, magneto-electro-elastic circular rod and (2+ 1)-dimensional nonlinear electrical lines*, Journal| MESA, 11(4), 789-803, 2020.

[17] Yokuş, A., Durur, H., Ahmad, H., *Hyperbolic type solutions for the couple Boiti-Leon-Pempinelli system*, Facta Universitatis, Series: Mathematics and Informatics, 35(2), 523-531, 2020.

[18] Durur, H., Yokuş, A., *Vakhnenko-Parkes denkleminin hiperbolik tipte yürüyen dalga çözümü*, Erzincan Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 13(2), 550-556, 2020.

[19] Yokuş, A., Durur, H., *Complex hyperbolic traveling wave solutions of Kuramoto-Sivashinsky equation using (1/G') expansion method for nonlinear dynamic theory,* Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 21(2), 590-599, 2019.

[20] Ismael, H.F., Bulut, H., Baskonus, H.M., *Optical soliton solutions to the Fokas–Lenells equation via sine-Gordon expansion method and (m+ ({G'}/{G}))-expansion method,* Pramana, 94(1), 35, 2020.

[21] Durur, H., Ilhan, E., Bulut, H., *Novel complex wave solutions of the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation*, Fractal and Fractional, 4(3), 41, 2020.

[22] Yokus, A., Durur, H., Ahmad, H., Thounthong, P., Zhang, Y.F., *Construction of exact traveling wave solutions of the Bogoyavlenskii equation by (G'/G, 1/G)-expansion and (1/G')-expansion techniques,* Results in Physics, 19, 103409, 2020.

[23] Duran, S., Doğan, K., *New wave solutions for nonlinear differential equations using an extended Bernoulli equation as a new expansion method*, In ITM Web of Conferences (Vol. 22, p. 01035). EDP Sciences (2018).

[24] Dusunceli, F., *Exact solutions for generalized (3+ 1)-dimensional Shallow Water-Like (SWL) equation*, In Conference Proceedings of Science and Technology, 2(1), 55-57, 2019.

[25] Zayed, E.M.E., *Traveling wave solutions for higher dimensional nonlinear evolution equations using the G'/G-expansion method*, Journal of Applied Mathematics & Informatics, 28(1_2), 383-395, 2010.

[26] Zhang, Y., Dong, H., Zhang, X., Yang, H., *Rational solutions and lump solutions to the generalized (3+1)-dimensional Shallow Water-Like equation*, Computers & Mathematics with Applications, 73(2), 246-252 2017.

[27] Baskonus, H.M., Eskitascioglu, E.I., *Complex wave surfaces to the extended shallow water wave model with (2+1)-dimensional*, Computational Methods for Differential Equations, 8(3), 585-596, 2020.

[28] Kumar, D., Seadawy, A.R., Joardar, A.K., *Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology.* Chinese journal of physics, 56(1), 75-85, 2018.

[29] Yokus, A., Tuz, M., Güngöz, U., *On the exact and numerical complex travelling wave solution to the nonlinear Schrödinger equation,* Journal of Difference Equations and Applications, 1-12, 2021.