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A Dynamic Programming Model to Determine the Optimal Harvest Decision for a Fir Forest that Provides Both Timber Harvest Volume and Carbon Sequestration Services

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Abstract

Carbon sequestration in forests is being considered as a mechanism to slow or reverse the trend of increasing concentrations of carbon dioxide in the atmosphere. We present results to determine the optimal harvest decision for a forest stand in the fir forest of Turkey that provides both timber harvest volume and carbon sequestration services by using a dynamic programming model. The state of the system at any point in time is described by stand age and the amount of carbon in the dead organic matter pool. Merchantable timber volume and biomass are predicted as a function of stand age. As a result of decay and litterfall, carbon stocks in the dead organic matter pool changes.

The results of the study indicate that initial carbon stock levels significantly affect economic returns to carbon management while optimal harvest age is relatively insensitive to carbon stocks in dead organic matter.

Keywords: Optimal rotation, fir forest, carbon market

Introduction

Carbon sequestration in forests is a considered way to mitigate the effects of greenhouse gas (GHG). Forests, as potential carbon sinks, absorb substantial amounts of CO_2 from the atmosphere through photosynthesis.

CBM-CFS3 is a detailed model that recognizes more than 20 different carbon pools within a forest stand and tracks the transfer of carbon between these pools and the atmosphere (Fig. 1).

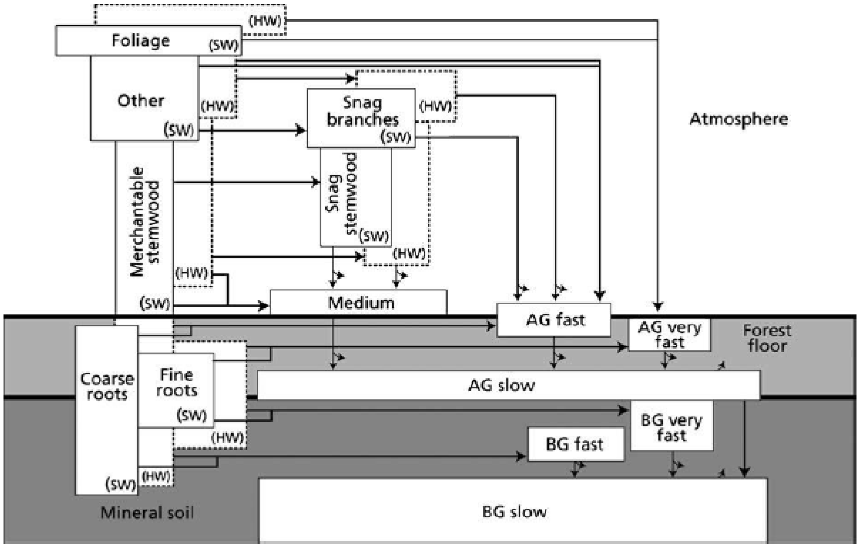


Fig. 1. The carbon pool structure of the CBM-CFS3

The Intergovernmental Panel on Climate Change (IPCC) is the leading international body for the assessment of climate change and provides the world with a clear scientific view on the current state of knowledge in climate change and its potential environmental and socio-economic impacts (IPCC, 2006). Carbon Budget Model of the Canadian Forest Sector (CBM-CFS3) is developed by Canadian Forest Service to track and report the changes in forest carbon stocks (Kult et al., 2007). Faustmann's formula (1849) gives the present value of the income stream for forest rotation. Hartmann (1976) extends the model and includes value associated with standing trees. The classic problem is the determination of the harvest age for even-aged forest stand which maximizes the net present value (Patrick, 2011).

A simulation model, which includes representation of carbon stored in live biomass, dead and downed wood, of the economics of timber and carbon management was developed by Gutrich and Howarth (2007).

In this study, a dynamic programming model was developed to find optimal stand management policy when both timber harvest and carbon sequestration values are considered. Forest stand is described in this analysis in terms of age and carbon stored in DOM pool.

We present the dynamic programming model used in this paper to examine the sensitivity of optimal harvest age to stocks of carbon in dead organic matter and carbon prices. The dynamic programming model was presented here to also examine the sensitivity of the net present value of the forested land stand age, stocks of carbon and carbon prices. We examine protected trajectories of carbon stocks in DOM given optimal harvest rules for a given carbon price and the impact of ignoring carbon stocks in DOM on the optimal harvest decision.

Data

The timber yield used in this study comes from the study about *Abies nordmanniana* subsp. *equi-trojani* (Asan, 1984). The cost information is provided by Trabzon Forestry Regional Directorate.

We resemble the table of the merchantable timber yield table using a Chapman-Richards growth function, $V(a) = v_1(1-e^{-v_2 a})^{v_3}$ represents the timber volume in m^3/ha at age a and v_1, v_2 and v_3 are parameters. All of the cost and prices used in this paper are Turkish Lira (TL).

The average lumber price of killed dried, standard, Turkish fir is approximately 255 TL/m^3 and also we used low and high lumber prices of 170 and 340 TL/m^3. The price of the wood chips is 70 TL/m^3. The parameter P^w represents the selling price of final products used in terms of wood input. The total revenue at any harvest age is calculated as $P^w V(a)$.

The cost of tree to end products is the sum of all costs for hauling, milling and overhead costs which are 66 TL/m^3, 72 TL/m^3 and 131 TL/m^3. We use F^a to represent the area based cost and F^v for volume based cost. The total harvesting and processing cost at any age are calculated as $F^a + F^v V(a)$.

Chapman-Richard function to represent the biomass carbon pool as a stand age function which is: $B(a)$ is the mass of carbon in tC/ha, stored in the living trees at age a, and b_1, b_2 and b_3. This function provides a reasonable representation of the tabulated biomass at different stand ages.

Decomposition, litterfall and harvest are the three processes to affect the development of the DOM pool. DOM is decomposed at a rate α, and litterfall rate β is expressed in this proportion. The DOM pool grows according to *Eq. (1)* with no timber harvest. The estimated parameters are $\alpha = 0.0184$ and $\beta = 0.028772$.

$$D_{t+1} = (1 - \alpha)D_t + \beta B(a) \qquad (1)$$

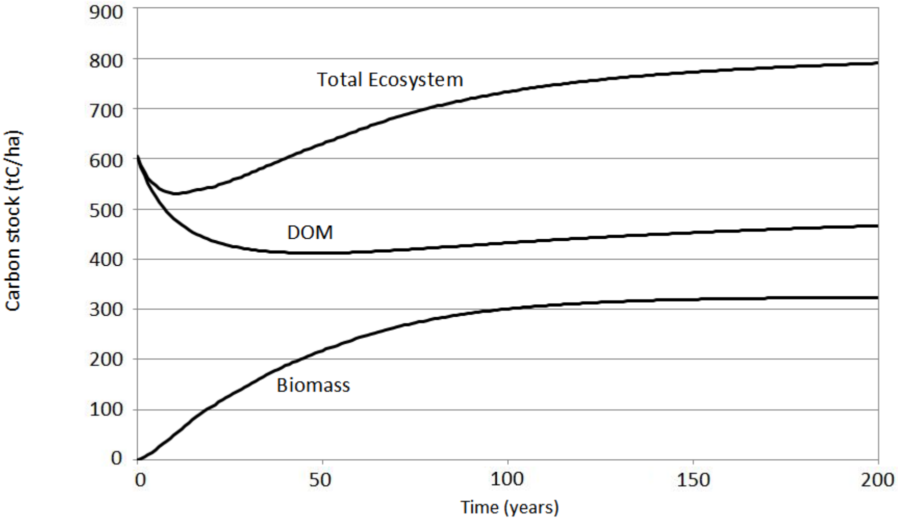
When timber harvest, the volume of merchantable timber is removed and the roots, stumps, tops branches and leaves are assumed to die and become DOM pool (Patrick, 2011). The mass of carbon removed from the site is calculated as $\gamma V(a)$. $\gamma = 0.2$ is used to convert wood volume to the mass of carbon stored wood (Asan, 1995). The DOM pool grows according to *Eq. (2)* with timber harvest.

$$D_t = (1-\alpha)D_t + B(a) - \gamma V(a) \qquad (2)$$

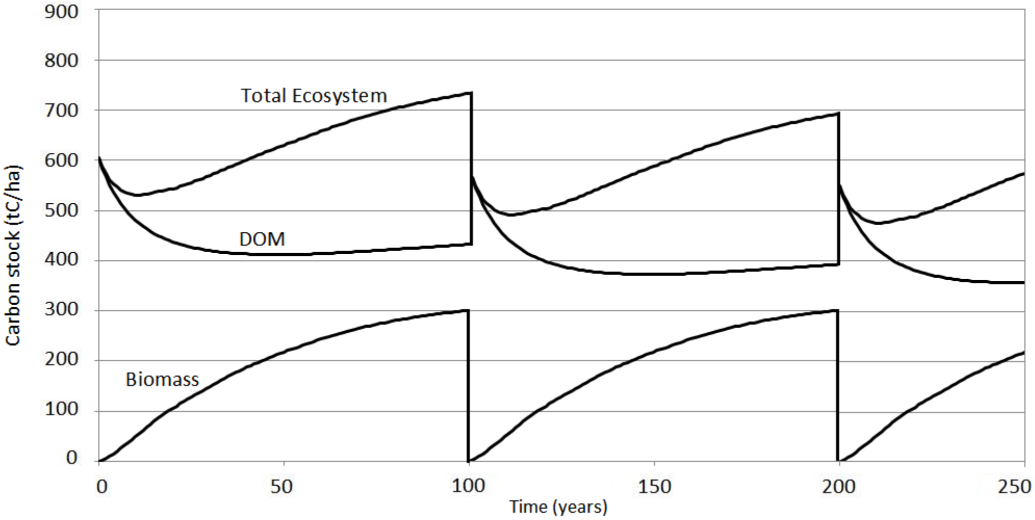
The annual change in total ecosystem carbon (TEC) is the sum of the changes in biomass and DOM carbon. With no harvest, the change in biomass is given by $\Delta B(a) = B9a + 1) - B(a)$ and the change in DOM is. With harvest, the change in biomass becomes $\Delta B(a) = B(1) - B(a)$ and the change in DOM is.

Figure 2 shows that carbon pool stocks for Turkish fir stand without harvest and with harvest on a 100-year rotation and the initial DOM stock of 605 tC/ha. TEC stocks increase by increasing biomass according to stand ages. DOM decreases slowly and then it changes when carbon is added to the DOM pool in the form of litterfall. Figure 2 part b shows the situation of DOM, biomass and TEC after fir stand harvest on a 100-year rotation age.

The price of carbon credits according to CD4CDM is ranged €3.5-€20 in 2007 (CD4CDM, 2007). Price for carbon credits traded on the Grantham Research Institute on Climate change and the Environment for 2011 is £30 (Grantham Research Institute, 2011). The price for carbon is $P^C = 3.67P^{CO_2}$ in this study because the molecular weight of CO_2 is approximately 3.67 times the atomic weight of C.



(a) No timber harvesting



(b) Timber harvest on 100 year rotation

Fig. 2. Projection of carbon pool development over time. (a) Without harvest, (b) With harvest

The model

We assume that the model is that the landlord paid for carbon sequestered by the forest and pays when carbon is released so a forest landlord is participating in carbon marketing. The forest is managed by using an even-aged silvicultural system. The rotation end is the same with beginning of the new rotation which continues eternally. The decision problem is presented with a dynamic program and used state variables to describe the system of the decision problem. The two-pool model of carbon that we use, carbon stock, in each of the more than 20 carbon pools modeled in CVM-CFS3, is formulated and solved by using dynamic programming.

The forest stand is described by the combination of the age of stand (years) and carbon stocks in the DOM pool (t C/ha). There are

501 discrete one-year age classes, *j*, and mid points a_j. There are 501 DOM classes, *i*, and mid points d_i. If stand harvest in stage *t*, the replanting occurs and stand age is set to 1 in stage *t+1*. If no harvesting in stage t, the stand age increases one year in stage *t+1*. The current DOM class I, age class j and harvest decision (harvest, *k=1*; no harvest, *k=0*) related to the change in TEC.

The change in total ecosystem carbon with no harvest,

$$\Delta C_{ij0} = B\left(\min\left(\left(a_j + 1\right), 500\right)\right) + \beta B\left(a_j\right) - \alpha d_j - B\left(a_j\right) \quad (3)$$

The change in total ecosystem carbon with harvest,

$$\Delta C_{ij1} = B(1) + \beta B\left(a_j\right) - \alpha d_j - \gamma V\left(a_j\right) \quad (4)$$

The net harvest revenue for age class j, (H_j) is,

$$H_j = (P^w - F^v)V\left(a_j\right) - F^a - E \quad (5)$$

The stage or periodic payoff (N_t) is

$$N(i, j, k) = \begin{cases} P^C \Delta C_{ij0} & : \quad k = 0 \\ P^C \Delta C_{ij1} + H_j & : \quad k = 1 \end{cases}$$

Equation (1) and (2) are converted to the proportion of the source DOM class area because discrete DOM classes is used. l_{ijk} express in equations represents the lower target class, u_{ijk} represents the upper target class. p_{ijk} represent the proportion that moves into the upper class and $([1 - p]_{ijk})$ represent the proportion that moves into the lower class.

$$l_{ij0} = \min\left(\left[(1 - \alpha)d_i + \beta B\left(a_j\right)\right], 500\right) \quad (7)$$

$$l_{ij1} = \min\left(\left[(1 - \alpha)d_i + \beta B\left(a_j\right) + B\left(a_1\right) - \gamma V\left(a_j\right)\right], 500\right) \quad (8)$$

$$u_{ijk} = \min\left(\left(l_{ijk} + 1\right), 500\right) \quad (9)$$

$$p_{ij0} = \left[(1 - \alpha)d_i + \beta B\left(a_j\right)\right] \quad (10)$$

$$p_{ij1} = \left[(1 - \alpha)d_i + \beta B\left(a_j\right) + B\left(a_1\right) - \gamma V\left(a_j\right)\right] \quad (11)$$

A weighted return from the target states is calculated associated with the harvest decision, *k*; and no harvest decision *k=0*. (Equation 12).

$$W_{ij0} = \left(1 - p_{ij0}\right)R_{t+1}\left\{l_{ij0}, \min\left((j + 1), 250\right)\right\} + p_{ij0}R_{t+1}\left\{u_{ij0}, \min\left((j + 1), 250\right)\right\}$$

For the harvest decision, *k=1*.

$$W_{ij1} = \left(1 - p_{ij1}\right)R_{t+1}\left\{l_{ij1,1}\right\} + p_{ij1}R_{t+1}\left\{u_{ij1,1}\right\} \quad (13)$$

The return for the last stage in the problem is initialized to zero,

$$R_T\{i, j\} = 0 \quad (14)$$

The assumption is justified on the basis that *T*, which is 500 years, and the discounted value of R_T for the reasonable discount rates for this problem is near zero. The discount rate, r (5% per annum) and discount factor, $\delta = (1 + r)^{-1}$ (for this analysis $\delta = 0.9528.$)

The recursive objective function is;

$$R_t\{i, j\} = max_k N\{i, j, k\} + \delta W_{ijk}, t = T - 1, T - 2, \dots, 0 \quad (15)$$

The recursive objective function calculates for each of the harvest decisions and selects the harvest decision that results in the maximum return as the optimal choice for the state combination in that stage.

The stage return at time zero for stands of age 0,

$$\forall i: \; R_0\{i, 0\} \leftarrow R_0\{i, 0\} - E \quad (16)$$

Results and discussion

Dynamic program determines the optimal harvest policy for maximizing landowner managing a forest stand for production of wood volume and sequestration of CO_2. The optimal policy is summarized by a decision rule. The combination of the stand age and DOM states for which the optimal decision is to defer harvest until at least the next period (Patrick, 2011). The results in this section were calculated by using the dynamic programming model which is programmed in MATLAB (Pratap, 2006).

The optimal harvest policies for different carbon prices are shown in Fig. 3. As P^{CO_2} increases, the optimal harvest age increases too. The optimal policy is sensitive to DOM stocks at the lower levels because the amount of CO_2 released to atmosphere through decomposition is lower with lower DOM stocks.

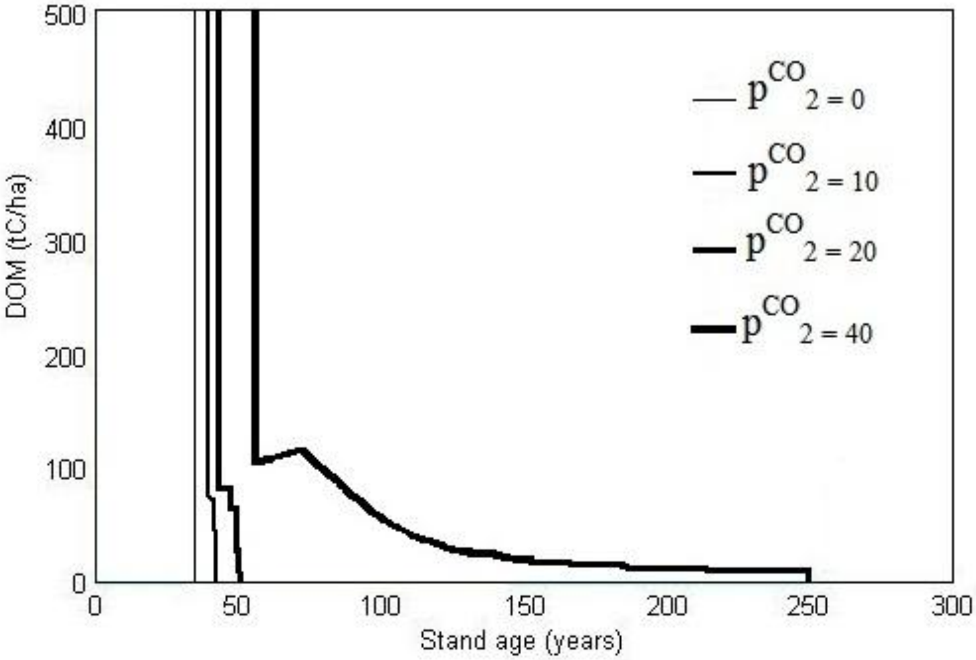
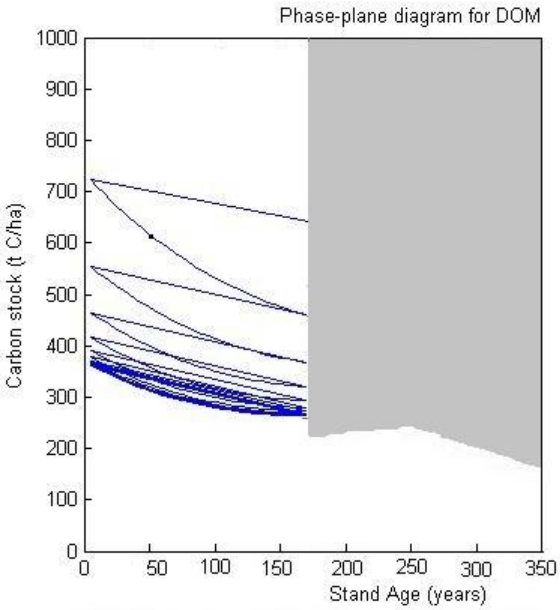


Fig. 3. Optimal harvest policies for different carbon prices

Fig. 4 shows that the decisions rule for $p^{CO_2} = \frac{40TL}{tC}$ with trajectories of stand development in state space. The grey polygon indicates the portion of the state space where the optimal decision is to harvest. The initial state in Fig. 4 panel (a) is a 50-year old stand with DOM stocks of 605 tC/ha. It grows and reaches 140 years, at which point it is harvested and regenerated on a continuing 140 year cycle.

Table 1 shows the summary of harvest age and carbon stock and mean annual increment equilibrium for different carbon prices. DOM and TEC are equal at age 0 because there is zero biomass at this point. DOM refers to amount of carbon stored in the dead organic matter pool. TEC refers to total ecosystem carbon or the sum of DOM and biomass pools. A higher p^{CO_2} gives a longer rotation. Because if the longer rotation age, carbon stocks are greater than age zero.



(a) Initial age: 50 years and 605 tC/ha

Fig. 4. The decision rules for $P^{CO_2} = \frac{40TL}{tC}$ as the shaded grey area

Table 1. **S**ummarizes the equilibrium conditions for different prices

$P^{CO_2}\left(\frac{TL}{tCO_2}\right)$	Rotation Age (years)	DOM and TEC at Age 0 (t C/ha)	DOM at rotation (t C/ha)	TEC at rotation (t C/ha)	MAI $\left(\frac{\frac{m^3}{ha}}{yr}\right)$
0	84	362	267	305	3.61
10	95	373	273	314	3.65
20	108	385	279	318	3.77
30	111	388	288	322	3.82
40	126	406	305	337	3.71
50	140	418	322	358	3.69

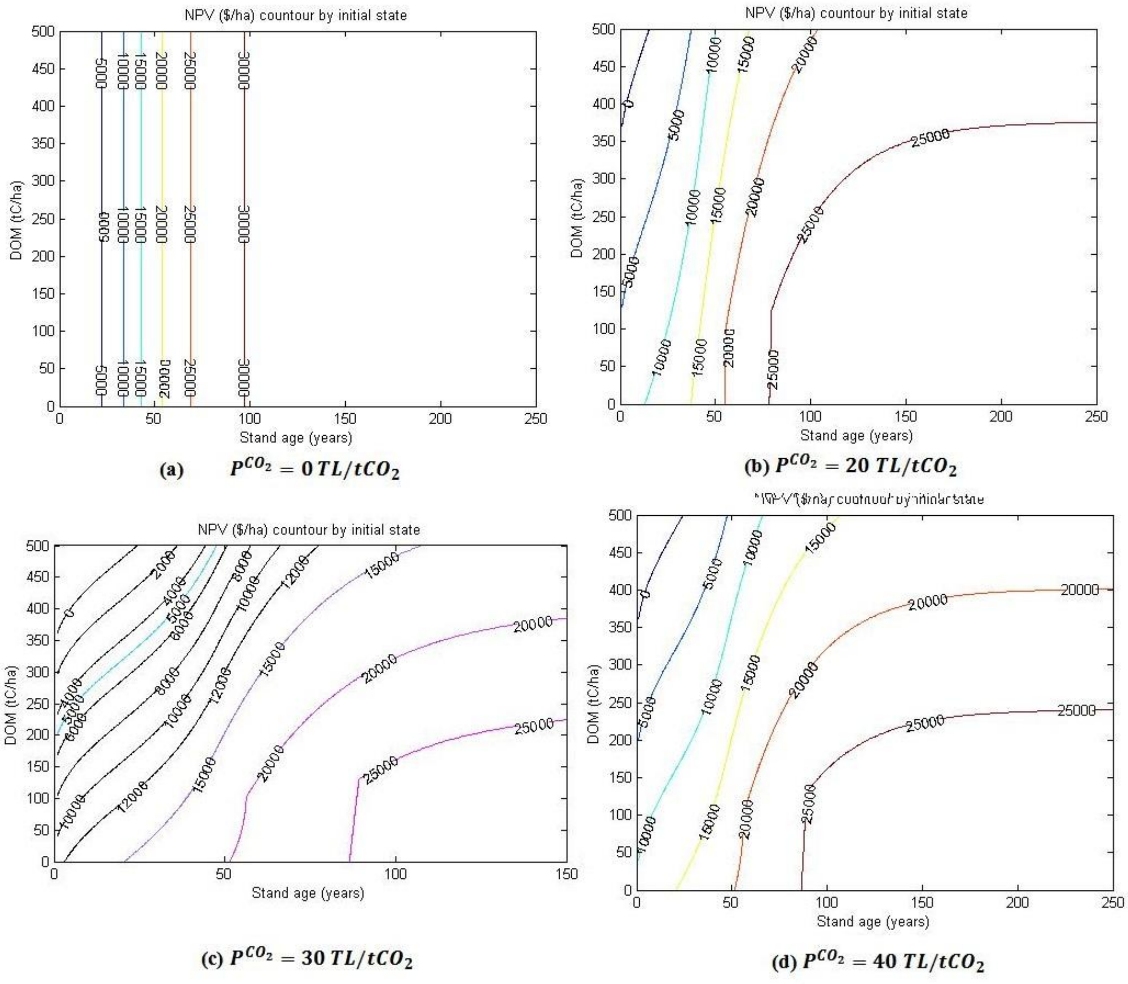
The mean annual increment MAI of the stand is shown in Table 1. MAI refers the productivity of stand in terms of average annual physical product output for different rotation ages. MAI is

$$MAI = \frac{V(R)}{R}$$

calculated as . According to the yield table used in this paper, the MAI is maximized at 105 years, which is close to rotation age when $P^{CO_2} = 20\frac{TL}{tCO_2}$.

The recursive objective function, $R_0\{d_i, a_i\}$, for this problem is shown in Fig. 5. The value of land carbon sequestration services for the all-state space, for carbon prices of 0, 20, 30, and 40 t C/ha.

The case where $P^{CO_2} = 0$ because carbon has no value, land timber and carbon sequestration services are independent of the amount of DOM stored. There is no stand age where its stand has positive land, timber and carbon values when DOM carbon stocks is above 410 t C/ha. Stands have less land, timber and carbon values with $P^{CO_2} = 0\ than$ young stands with low DOM stocks. The DOM stocks would have be less than 400 t C/ha to positive effects of participating in carbon market for decision makers. When the carbon prices is greater than 0, land, timber and carbon values decreases with increasing DOM stocks (Fig. 5). A large stock of DOM, which release greater CO_2 to atmosphere than smaller stocks, is an obligation for landowner represented in this model (Patrick, 2011).



(a) $P^{CO_2} = 0\,TL/tCO_2$

(b) $P^{CO_2} = 20\,TL/tCO_2$

(c) $P^{CO_2} = 30\,TL/tCO_2$

(d) $P^{CO_2} = 40\,TL/tCO_2$

Fig. 5. Land, timber and carbon values (TL/ha) by stand age and carbon stocks in the DOM pool for different carbon prices

In the model that we used, a landowner has the choice at each point in time to harvest a stand of a particular age and with a particular stock of DOM carbon, or not. The optimal harvest age increases from 84 to 95 years with 10 TL/tCO_2 increase in carbon price, so far the 11 years after harvest for the 0 TL/tCO_2 case, there is more TEC carbon in the 10 than 0 TL/tCO_2 projections. Higher carbon price will have more TEC over the projection period; there will be points in time where the zero carbon price case has more TEC. A result when Table 3 and Table 1 compared, the optimal harvest decision is independent of DOM stocks when changes in DOM do not affect the objective function. The optimal rotation age and carbon stocks are greater when DOM pool is not considered.

Table 2 presents that average differences in projection of TEC stocks given optimal policies when $P^{CO_2} = 0\frac{TL}{tCO_2}$ and $P^{CO_2} = 10\frac{TL}{tCO_2}$ for different projection.

Table 2. Average differences in projection of TEC stocks

Time (years)	Average Differences (tC/ha)
20	0.0
50	22.4
100	10.1
200	20.6
500	20.4
1000	20.6

Table 3. The summary of the harvest age and carbon stocks and mean annual increment equilibrium for different carbon prices when DOM is not considered

$p^{co_2}\left(\frac{TL}{tCO_2}\right)$	Rotation Age (years)	DOM and TEC at Age 0 (t C/ha)	DOM at rotation (t C/ha)	TEC at rotation (t C/ha)	MAI $\left(\frac{m^3}{\frac{ha}{yr}}\right)$
0	84	362	267	305	3.61
10	106	370	281	327	3.85
20	135	382	297	404	3.75
30	153	385	301	492	2.82
40	∞	400	313	543	0

Conclusion

In this study, a dynamic programming model was used to determine the optimal harvest decision for a forest stand used to provide both timber harvest volume and carbon sequestration services. Stand age and stocks of carbon stored in the DOM pools are used to describe the forest stand.

The model that we used examines optimal harvest decisions for a Turkish fir stand in Turkey. We draw the following conclusion from this study:

1. When carbon prices are high and initial DOM stocks in pool, the optimal decision is sensitive to current stocks of carbon in DOM pool.
2. Carbon price reduced the value of land, timber and carbon sequestration services relative to the zero case. Forest stand can be a net carbon source for several years after stand initiation because of tree decomposition (Fig.2).
3. Compared to the case where changes in carbon stock, optimal harvest age and equilibrium carbon stocks are lower when carbon stocks in the DOM pool are rewarded or penalized.

This study presents the model is that a forest landlord is participating in a carbon market where the landlord is paid for carbon sequestered by the forest and pays when carbon is released. A dynamic programming model of system, which captures the important elements of the system for an economic analysis, is able to develop by a detailed carbon budget simulation model. Alternative forms of carbon markets are able to explore by variations of this model.

Rotation ages quite increases and mean increment decreases when carbon prices are high enough. That is why there is a pressure for higher timber price. This analysis presents a model used for the optimal harvest age in the traditional Faustmann (1849) and Harman (1976). Because of the inter-period flow constraints effects, this study has differences with other studies such as McCarty et al. (2008). In this analysis, forest landlord is paid for carbon sequestered by the forest and pays when carbon is releases.

Acknowledgments

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