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Diameter Distribution of Bornmullerian Fir in Mixed Stands

***Oytun Emre SAKICI[1], Mete GÜLSUNAR[2]**
Kastamonu University, Faculty of Forestry, Kastamonu, TURKEY
Ondokuz Mayıs University, Vezirkopru Vocational High School, Samsun, TURKEY
*Corresponding author: oesakici@kastamonu.edu.tr

Abstract

The diameter distribution is one of the most powerful tools for depicting the properties of a stand. Diameter has generally strong correlations with other important stand variables including basal area, volume, etc. Quantification of the diameter distribution and its relationship to site, stand composition, age, and density is often valuable for both economic and biological purposes. In this case, the diameter distribution of stands should be determined with appropriate probability functions for all these purposes firstly. In this paper, we consider four probability density functions (Exponential, Exponential with two parameters, Weibull with two parameters and Weibull with three parameters) as diameter distribution models for Bornmullerian fir (*Abies nordmanniana* subsp. *bornmulleriana*) in mixed coniferous stands.

Keywords: Diameter distribution, Bornmullerian fir, Exponential density function, Weibull density function

Introduction

Fir is one of the most economically important tree species with a large distribution area (0.6 million hectares) in Turkey. There are five natural fir subspecies: *Abies nordmanniana* subsp. *nordmanniana*, *A. n.* subsp. *bornmulleriana*, *A. n.* subsp. *equi-trojani*, *Abies cilicica* subsp. *cilicia* and *A. c.* subsp. *isaurica*, within two species in Turkey's forests. They form pure and mixed stands with coniferous (*Pinus sylvestris*, *Pinus nigra*, *Picea orientalis* and *Cedrus libani*) and deciduous species (*Fagus orientalis*) in different regions of the country.

There is great a interest in knowing the number of trees/diameter class in a stand, because the diameter sizes determine the industrial use of wood and thus the price of different products. Diameter distributions also give information about stand structure, age structure, stand stability, etc. (Gorgoso et al., 2007). Stand dynamics were characterized by displaying present and predicted diameter distribution. The modeled diameter distribution is a valuable information that can be used to forecast the range of products which might be expected from a managed stand (Carus and Çatal, 2011).

Attempts to numerically analyze stem diameter distributions date from the early 1900s (Loetsch et al., 1973). Since then, many different procedures to represent the diameter distributions of stands have been tested all over the world. Various probability density functions such as Normal, Log-normal, Gamma, Beta, Johnson's SB and Weibull have been utilized to characterize the diameter frequency distributions of forest stands (Liu et al., 2002).

In Turkey, the study of diameter distributions started later than the United States and European countries. Saraçoğlu (1988) studied the diameter distribution by site classes in fir stands of Black Sea region. Carus (1996) compared various statistical distributions by describing different diameter distributions for even-aged *Fagus orientalis* stands. Yavuz et al. (2002) studied the diameter distributions of natural stands and plantations of *Fraxinus* spp. Carus and Çatal (2008) investigated diameter distributions in *Pinus brutia* stands. Sönmez et al. (2010) analyzed the diameter distributions of *Picea orientalis*. Ercanlı and Yavuz (2010) and Kahriman and Yavuz (2011) tested different probability density functions to display diameter distributions of *Picea orientalis*-*Pinus sylvestris* and *Pinus sylvestris*-*Fagus orientalis* mixed stands, respectively. Carus and Çatal (2011) studied *Pinus nigra* Stands.

In this study, the diameter distributions of Bornmullerian fir (*Abies nordmanniana* subsp. *bornmulleriana*) in mixed stands with

Pinus sylvestris were investigated. Four probability density functions (Exponential, Exponential with two parameters, Weibull with two parameters and Weibull with three parameters) were used to determine diameter distributions.

Material and Methods

The data from 180 sample plots were collected from mixed Bornmullerian fir-Scots pine (*Abies nordmanniana* subsp. *bornmulleriana*-*Pinus sylvestris*) stands located in Central Black Sea Region of Turkey (Figure 1). For each plot, the number of trees and diameter at breast height (DBH) of all trees were measured. All plots were rectangular (20 x 40 m) and equal-sized (0.08 ha).

Figure 1. Study area

To display the present diameter distributions in all sample plots, the measured diameters were divided into diameter classes of 4 cm starting from 8 cm, which was the minimum diameter included into the study. Totally, 8,682 Bornmullerian fir trees were sampled in the study. Summary statistics for sample plots are described in Table I. The distribution by diameter classes of all sampled Bornmullerian fir trees is shown in Figure 2.

Table 1. Summary statistics for sample plots

	Min	Max	Mean
Number of trees	17	93	48
Diameter at breast height (cm)	8,0	87,2	22,1

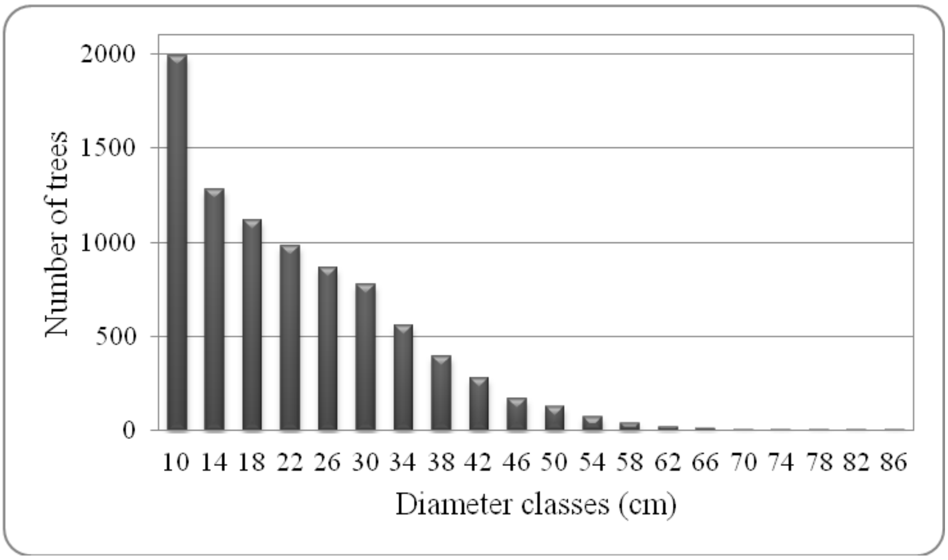

Figure 2. Diameter distribution of all trees in sample plots

The probability density functions used in the study are as follows:

Exponential:
$$F(x,\lambda) = \lambda \cdot \exp(-\lambda \cdot x)$$

Exponential with two parameters:
$$F(x,\lambda,\gamma) = \lambda \cdot \exp(-\lambda \cdot (x-\gamma))$$

Weibull with two parameters:
$$F(x,\alpha,\beta) = \frac{\alpha}{\beta} \cdot \left(\frac{x}{\beta}\right)^{\alpha-1} \cdot \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right)$$

Weibull with three parameters:
$$F(x,\alpha,\beta,\gamma) = \frac{\alpha}{\beta} \cdot \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \cdot \exp\left(-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)$$

Where;
x is DBH and α, β, and γ are parameters.

To fit the distribution models' parameters, EasyFit software was used. The statistical fitness of the models was analyzed with Kolmogorov-Smirnov test. The success of the models has been compared using the Error Index (e) improved by Reynolds et al. (1988). The rank representing the diameter distribution of the probability density functions used was determined by ranking the error index in ascending order. The smaller the error value, the better the fit. The error index (e) is described by the following formula:

$$e = \sum_{j=1}^{k} |n_p - n_o|$$

Where:
k is the number of diameter classes, n_p and n_o are predicted and observed number of trees in j^{th} diameter class.

Results and Discussion

According to Kolmogorov-Smirnov test results, probability density functions analyzed were statistically suitable for all the sample plots ($p>0.05$). Thus, all of the functions were compared for every sample plots. The comparisons using error index (e) showed that Exponential function with two parameters was the best for 77 sample plots, Weibull function with three parameters for 72 and Weibull function with two parameters for 31 (Table 2). Exponential function was the best function for none of the plots.

The results show that Exponential function with two parameters and Weibull function with three parameters nearly had equal number of sample plots with regard the best function. Exponential and Weibull (with two parameters) functions were not successful model for Bornmullerian fir in mixed stands. One more parameter for both has increased the representativeness of models.

Table 2. Results of ranks for distribution functions

Sample plot	Exponential		Exponential (2 p.)		Weibull (2 p.)		Weibull (3 p.)	
	e	rank	e	rank	e	rank	e	rank
1	483	*4*	454	*3*	432	*1*	445	*2*
2	448	*4*	395	*3*	395	*2*	394	*1*
3	394	*4*	345	*1*	352	*3*	348	*2*
4	497	*4*	448	*2*	451	*3*	445	*1*
5	526	*4*	476	*1*	489	*3*	479	*2*
6	503	*4*	449	*1*	458	*3*	452	*2*
7	569	*4*	506	*1*	518	*3*	513	*2*
8	488	*4*	436	*2*	433	*1*	441	*3*
9	453	*4*	408	*1*	416	*3*	411	*2*
10	469	*4*	419	*1*	422	*3*	419	*2*
11	472	*4*	417	*2*	417	*3*	417	*1*
12	552	*4*	500	*1*	517	*3*	501	*2*
13	657	*4*	582	*1*	585	*3*	583	*2*
14	575	*4*	516	*3*	515	*2*	513	*1*
15	601	*4*	536	*3*	524	*1*	525	*2*
16	653	*4*	566	*1*	586	*2*	587	*3*
17	776	*4*	689	*1*	716	*3*	697	*2*
18	697	*4*	617	*1*	646	*3*	626	*2*
19	548	*4*	498	*1*	509	*3*	508	*2*
20	804	*4*	707	*1*	746	*2*	752	*3*
21	451	*4*	407	*2*	418	*3*	405	*1*
22	631	*4*	563	*1*	573	*2*	589	*3*
23	756	*4*	671	*1*	690	*3*	686	*2*
24	775	*4*	668	*1*	696	*2*	716	*3*
25	991	*4*	844	*1*	870	*2*	909	*3*
26	896	*4*	759	*1*	777	*2*	793	*3*
27	745	*4*	647	*1*	654	*2*	665	*3*
28	611	*4*	555	*3*	552	*2*	550	*1*
29	576	*4*	521	*3*	515	*2*	509	*1*
30	718	*4*	641	*1*	664	*2*	687	*3*
31	510	*4*	453	*1*	455	*3*	455	*2*
32	607	*4*	517	*1*	533	*3*	521	*2*
33	681	*4*	594	*1*	598	*3*	596	*2*
34	897	*4*	688	*1*	695	*2*	695	*3*
35	879	*4*	760	*1*	785	*3*	779	*2*
36	734	*4*	642	*1*	647	*2*	647	*3*
37	706	*4*	620	*2*	638	*3*	616	*1*
38	427	*4*	387	*2*	388	*3*	384	*1*
39	768	*4*	682	*1*	689	*3*	687	*2*
40	831	*4*	731	*3*	714	*2*	712	*1*
41	852	*4*	736	*1*	749	*3*	739	*2*
42	775	*4*	669	*1*	670	*2*	671	*3*
43	596	*4*	518	*3*	500	*2*	499	*1*
44	634	*4*	569	*3*	551	*1*	556	*2*
45	579	*4*	516	*3*	491	*1*	492	*2*
46	608	*4*	537	*3*	528	*2*	527	*1*
47	561	*4*	496	*3*	488	*2*	487	*1*
48	662	*4*	585	*3*	570	*2*	570	*1*

Table 2. (*Continued*)

Sample	Exponential		Exponential (2 p.)		Weibull (2 p.)		Weibull (3 p.)	
plot	e	rank	e	rank	e	rank	e	rank
49	469	*4*	426	*3*	411	*1*	411	*2*
50	514	*4*	460	*3*	457	*2*	454	*1*
51	338	*4*	301	*3*	296	*2*	296	*1*
52	410	*4*	368	*3*	366	*2*	366	*1*
53	516	*4*	456	*1*	458	*2*	463	*3*
54	470	*4*	418	*2*	417	*1*	427	*3*
55	766	*4*	683	*1*	693	*3*	691	*2*
56	466	*4*	415	*2*	417	*3*	412	*1*
57	718	*4*	630	*1*	639	*3*	635	*2*
58	826	*4*	737	*3*	713	*2*	712	*1*
59	475	*4*	417	*1*	429	*2*	434	*3*
60	601	*4*	527	*2*	528	*3*	526	*1*
61	356	*4*	340	*3*	332	*1*	334	*2*
62	622	*4*	550	*2*	559	*3*	550	*1*
63	543	*4*	468	*1*	476	*3*	469	*2*
64	624	*4*	543	*2*	552	*3*	541	*1*
65	479	*4*	416	*2*	428	*3*	413	*1*
66	523	*4*	455	*3*	454	*2*	451	*1*
67	516	*4*	456	*1*	457	*2*	461	*3*
68	370	*4*	324	*2*	333	*3*	322	*1*
69	484	*4*	440	*3*	424	*2*	415	*1*
70	415	*4*	361	*1*	365	*3*	361	*2*
71	585	*4*	519	*1*	528	*3*	527	*2*
72	519	*4*	461	*2*	460	*1*	464	*3*
73	430	*4*	386	*3*	371	*2*	371	*1*
74	610	*4*	540	*3*	537	*2*	534	*1*
75	584	*4*	518	*1*	524	*2*	525	*3*
76	439	*4*	386	*3*	375	*1*	376	*2*
77	673	*4*	594	*3*	586	*2*	582	*1*
78	584	*4*	511	*1*	529	*3*	520	*2*
79	768	*4*	672	*3*	664	*2*	663	*1*
80	777	*4*	679	*1*	691	*3*	684	*2*
81	790	*4*	693	*1*	708	*3*	707	*2*
82	578	*4*	513	*3*	505	*2*	504	*1*
83	278	*4*	261	*2*	262	*3*	257	*1*
84	346	*4*	327	*1*	328	*2*	329	*3*
85	625	*4*	551	*1*	557	*3*	555	*2*
86	476	*4*	458	*3*	415	*1*	441	*2*
87	585	*4*	514	*1*	525	*2*	549	*3*
88	473	*4*	419	*1*	428	*2*	437	*3*
89	381	*4*	368	*2*	342	*1*	375	*3*
90	470	*4*	411	*1*	414	*2*	415	*3*
91	261	*4*	215	*1*	232	*2*	234	*3*
92	297	*4*	266	*3*	265	*2*	264	*1*
93	340	*4*	301	*3*	298	*1*	299	*2*
94	378	*4*	337	*3*	327	*2*	327	*1*
95	516	*4*	451	*3*	449	*2*	447	*1*
96	428	*4*	379	*3*	372	*1*	374	*2*

Table 2. (*Continued*)

Sample	Exponential		Exponential (2 p.)		Weibull (2 p.)		Weibull (3 p.)	
plot	e	rank	e	rank	e	rank	e	rank
97	460	*4*	395	*1*	399	*2*	401	*3*
98	562	*4*	504	*2*	512	*3*	501	*1*
99	488	*4*	441	*2*	446	*3*	436	*1*
100	759	*4*	668	*1*	704	*3*	686	*2*
101	649	*4*	591	*1*	624	*3*	598	*2*
102	598	*4*	516	*1*	533	*2*	543	*3*
103	519	*4*	450	*2*	451	*3*	444	*1*
104	556	*4*	489	*3*	478	*2*	477	*1*
105	574	*4*	502	*3*	496	*2*	494	*1*
106	519	*4*	457	*3*	439	*2*	438	*1*
107	522	*4*	459	*3*	447	*2*	445	*1*
108	420	*4*	376	*3*	359	*2*	357	*1*
109	360	*4*	321	*3*	314	*2*	314	*1*
110	397	*4*	351	*3*	340	*1*	341	*2*
111	391	*4*	342	*2*	353	*3*	341	*1*
112	424	*4*	377	*3*	370	*2*	370	*1*
113	473	*4*	419	*3*	403	*2*	402	*1*
114	476	*4*	413	*3*	409	*2*	408	*1*
115	541	*4*	475	*1*	477	*2*	477	*3*
116	506	*4*	445	*1*	455	*2*	466	*3*
117	510	*4*	443	*1*	444	*2*	444	*3*
118	639	*4*	553	*2*	555	*3*	548	*1*
119	718	*4*	562	*1*	585	*3*	567	*2*
120	593	*4*	519	*3*	517	*2*	513	*1*
121	493	*4*	438	*3*	428	*2*	427	*1*
122	371	*4*	333	*2*	334	*3*	331	*1*
123	416	*4*	372	*2*	375	*3*	371	*1*
124	482	*4*	429	*2*	444	*3*	428	*1*
125	381	*4*	339	*1*	344	*3*	340	*2*
126	493	*4*	436	*2*	437	*3*	435	*1*
127	455	*4*	401	*3*	401	*2*	399	*1*
128	393	*4*	354	*3*	341	*1*	342	*2*
129	351	*4*	286	*1*	304	*3*	312	*2*
130	621	*4*	535	*1*	541	*3*	540	*2*
131	316	*4*	287	*3*	285	*1*	285	*2*
132	404	*4*	357	*1*	362	*2*	366	*3*
133	395	*4*	350	*3*	340	*2*	338	*1*
134	415	*4*	369	*1*	386	*3*	372	*2*
135	377	*4*	329	*3*	321	*2*	320	*1*
136	405	*4*	343	*1*	348	*3*	346	*2*
137	389	*4*	328	*3*	328	*2*	327	*1*
138	421	*4*	364	*2*	368	*3*	363	*1*
139	541	*4*	472	*3*	471	*2*	466	*1*
140	456	*4*	406	*1*	410	*3*	408	*2*
141	498	*4*	440	*1*	460	*3*	454	*2*
142	457	*4*	419	*1*	424	*3*	420	*2*
143	425	*4*	374	*2*	383	*3*	373	*1*
144	595	*4*	518	*3*	517	*2*	515	*1*

Table 2. (*Continued*)

Sample plot	Exponential		Exponential (2 p.)		Weibull (2 p.)		Weibull (3 p.)	
	e	rank	e	rank	e	rank	e	rank
145	354	*4*	312	*3*	308	*1*	309	*2*
146	363	*4*	321	*2*	319	*1*	324	*3*
147	293	*4*	255	*1*	258	*3*	256	*2*
148	198	*4*	190	*2*	190	*1*	191	*3*
149	284	*4*	255	*3*	250	*2*	241	*1*
150	327	*4*	290	*3*	287	*1*	288	*2*
151	327	*4*	290	*1*	294	*3*	291	*2*
152	293	*4*	277	*3*	256	*1*	260	*2*
153	367	*4*	323	*1*	326	*2*	331	*3*
154	374	*4*	331	*3*	329	*1*	329	*2*
155	257	*4*	222	*1*	224	*2*	230	*3*
156	214	*4*	189	*1*	192	*3*	189	*2*
157	435	*3*	485	*4*	380	*2*	379	*1*
158	546	*4*	482	*1*	488	*2*	489	*3*
159	475	*4*	413	*1*	429	*3*	414	*2*
160	558	*4*	481	*3*	468	*1*	468	*2*
161	441	*4*	358	*3*	352	*1*	356	*2*
162	422	*4*	369	*3*	360	*2*	359	*1*
163	481	*4*	404	*2*	415	*3*	404	*1*
164	266	*4*	237	*3*	231	*1*	236	*2*
165	362	*4*	301	*1*	308	*3*	306	*2*
166	280	*4*	249	*3*	244	*1*	245	*2*
167	278	*4*	236	*2*	245	*3*	234	*1*
168	335	*4*	293	*1*	308	*3*	294	*2*
169	426	*4*	373	*3*	367	*2*	365	*1*
170	405	*4*	360	*1*	363	*2*	364	*3*
171	588	*4*	519	*3*	503	*1*	503	*2*
172	526	*4*	456	*2*	455	*1*	467	*3*
173	339	*4*	298	*1*	309	*3*	302	*2*
174	487	*4*	430	*2*	431	*3*	429	*1*
175	321	*4*	295	*3*	286	*1*	286	*2*
176	410	*4*	356	*1*	364	*3*	359	*2*
177	468	*4*	414	*1*	431	*2*	432	*3*
178	286	*4*	282	*3*	231	*1*	232	*2*
179	345	*4*	318	*3*	310	*2*	300	*1*
180	354	*4*	333	*3*	306	*2*	305	*1*

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