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Solitary Wave Solutions of the (3+1)-dimensional Khokhlov–Zabolotskaya–Kuznetsov Equation by Using the $(G'/G, 1/G)$-Expansion Method

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Abstract

In this study, the (3+1)-dimensional Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation, which is a mathematical model of non-absorption and dispersion in the non-linear medium, which sheds light on the sound beam phenomenon, which has a physically important place, is examined. In order to find the exact solution of this equation, an effective and reliable method, $(G'/G, 1/G)$-expansion method, is used among analytical methods. The purpose of this method is to obtain more than one traveling wave solution classes depending on the conditions of the λ parameter. These classes are categorized into hyperbolic, trigonometric, complex trigonometric and rational forms. The graphics of the solitary waves represented by these successfully obtained solution classes are presented as 2-dimensional, 3-dimensional and contours. This article makes use of ready-made package programs for complex arithmetic operations and graphic drawings.

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Keywords: $(G'/G, 1/G)$-expansion method; (3+1)-dimensional Khokhlov–Zabolotskaya–Kuznetsov equation; Traveling wave solution.

(3+1)-Boyutlu Khokhlov–Zabolotskaya–Kuznetsov Denkleminin $(G'/G, 1/G)$-Açılım Metodu Yardımıyla Solitary Dalga Çözümleri

Öz

Bu çalışmada, fiziksel olarak önemli bir yere sahip olan ses ışını (sound beam) olayına ışık tutan, özellikle lineer olmayan ortamda dağılım ve soğurma olmayan durumların matematiksel modeli olan (3+1)-boyutlu Khokhlov–Zabolotskaya–Kuznetsov (KZK) denklemi incelendi. Bu denklemin tam çözümünü bulmak için analitik metotlar arasında yer alan etkili ve güvenilir bir yöntem olan $(G'/G, 1/G)$-açılım metodu kullanıldı. Bu metodun seçilme amacı λ parametresinin durumlarına bağlı olarak birden fazla yürüyen dalga çözüm sınıfları elde edilmesidir. Bu sınıflar hiperbolik, trigonometrik, kompleks trigonometrik ve rasyonel formda kategorize edilir. Başarılı bir şekilde elde edilen bu çözüm sınıflarının temsil ettiği solitary dalgaların grafikleri 2-boyutlu, 3-boyutlu ve kontur olarak sunuldu. Bu makalede karmaşık aritmetik işlemler ve grafik çizimleri için hazır paket programlardan faydalanıldı.

Anahtar Kelimeler: $(G'/G, 1/G)$-açılım metodu; (3+1)-boyutlu Khokhlov–Zabolotskaya–Kuznetsov Denklemi; Solitary dalga çözümleri.

1. Introduction

The debates about the wave theory that started in the 18th century have been brought to a considerable level. The wave theory we are discussing today and discussed in the future can be divided into two groups, linear and nonlinear. However, nonlinear wave discussions are more valuable because life is not linear. For this reason, the traveling wave solutions of partial differential equations shed light on many events in nature, bringing mathematical models to the fore. Along with these mathematical models, many researchers have discussed the solution methods of these models. Generally, the methods that generate the solutions of nonlinear mathematical models are of the oscillating traveling wave type. In applied science, studies about perceiving the traveling wave as a signal and processing these signals have become popular today. Mathematical models, called NPDEs include quantum mechanics, plasma physics, hydro-dynamic molecular biology, sheet water wave, nonlinear optics, optical fibers, chemistry, biological science, etc. as seen in various fields of nonlinear science. Investigating NPDEs provides a clearer understanding of complex events. Lately, many new mathematical models used by experts all over the world to describe real-life problems of today have attracted attention.

In this sense, some methods are trial equation method, modified simple equation method, modified extended tanh method, generalized hyperbolic-function method, sub equation method, complex method, auxiliary equation method, the homogeneous balance method, the improved Bernoulli sub-equation function method and many more methods [1-29].

We consider the following Zabolotskaya and Khokhlov (ZK) equation [30],

$$(u_t + uu_x)_x + nu_{yy} + mu_{zz} = 0. \quad (1)$$

This equation was first proposed by Zabolotskaya and Khokhlov in 1969 [31]. The physical interpretation of this equation shows the propagation of the sound beam in a non-linear medium with no dispersion or absorption [32]. This nonlinear medium in particular is not strong. This non-linear medium in particular is not strong. With the term added to Eq. (1), the following (3+1)-dimensional KZK equation is obtained [32]:

$$u_{xt} + (u_x)^2 + uu_{xx} + ru_{xxx} + nu_{yy} + mu_{zz} = 0, \quad (2)$$

where *r, n* and *m* are constant and $r \neq 0$. In addition, in Eqn. (2), which is the mathematical model of the sound beam phenomenon, the function that represents acoustic pressure and sought is $u(x, y, z, t)$. Here *t* represents time and $(x, y, z) \in R^3$ [33]. This equation was first proposed by Kuznetsov with the help of Eqn. (1) in 1971 [34]. The term adsorption is defined as thermo-viscous. A higher-order NPDEs have been defined by adding this term. Traveling wave solutions were investigated for Eqn. (2) by Akçagil and Aydemir in 2016 with the help of the tanh–coth method [32]. On the other hand, new exact solutions were reached by Ray with the help of Kudryashov methods for the time fractional KZK equation [35]. In 2019, analytical solutions of the (3+1) dimensional time fractional KZK equation were produced with the help of modified Riemann-Liouville derivative and (G′/G)-expansion method by Zhang et al. [36]. In addition, the effect of diffraction in these solutions was investigated. In 2021, traveling wave solutions were produced in trigonometric function and dark optical soliton solution format by applying the modified exp $(-\Omega(\xi))$-expansion function method for Eqn. (2) by Demiray and Kastal [37]. The main theme of this study is to obtain the traveling wave solutions of Eqn. (2) with the help of the $(G'/G, 1/G)$- expansion method [38].

The most important reason for using this method is to produce different types of traveling wave solutions from the literature for the (3+1)-dimensional KZK equation. One of the most important advantages of this method is that it produces traveling wave solutions in three different forms. In this study, information about the methodology of the method discussed in Section 2 is given. In the Section 3, the application of the method to the Eqn. (2) and finally in the Section 4, important results are given.

2. Method

2.1. $(G'/G, 1/G)$-expansion method

In this section, we present analysis of the $(G'/G, 1/G)$-expansion method [38].

$$Z(u, u_x, u_y, u_z, u_t, u_{xx}, u_{tt}, \dots) = 0. \tag{3}$$

If $u = U(\xi) = u(x, y, z, t)$, $\xi = x + y + z - ct$ classical wave transformation is applied in Eqn. (3) while c is a constant, Eqn. (3) is converted into a nODE and this can be written as:

$$W(U, UU', U'', \dots) = 0. \tag{4}$$

Reduced complexity by integrating Eqn. (4). $G(\xi)$ function is a quadratic function ODE solution,

$$G''(\xi) + \lambda G(\xi) = \mu. \tag{5}$$

Also to ensure operational aesthetics as $\frac{G'}{G} = \phi = \phi(\xi)$and $\psi = \psi(\xi) = \frac{1}{G(\xi)}$. Here, the derivatives of the defined functions can be written

$$\phi' = -\phi^2 + \mu\psi - \lambda,\ \psi' = -\phi\psi. \tag{6}$$

By considering the equations given by Eqn. (6), we can present the behavior of the solution function Eqn. (5) with respect to the λ state.

i) If $\lambda < 0$

$$G(\xi) = c_1\, sinh\big(\sqrt{-\lambda}\xi\big) + c_2\, cosh\big(\sqrt{-\lambda}\xi\big) + \frac{\mu}{\lambda}, \tag{7}$$

where $c_2$and c_1 are real numbers. Considering Eqn. (7);

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma + \mu^2}(\phi^2 - 2\mu\psi + \lambda),\ \sigma = c_1^2 - c_2^2, \tag{8}$$

written in this form.

ii) If $\lambda > 0$

$$G(\xi) = c_1\, sin\big(\sqrt{\lambda}\xi\big) + c_2\, cos\big(\sqrt{\lambda}\xi\big) + \frac{\mu}{\lambda}, \tag{9}$$

where c_2 and c_1 are real numbers. Eqn. (9), there is following equation;

$$\psi^2 = \frac{\lambda}{\lambda^2\sigma - \mu^2}(\phi^2 - 2\mu\psi + \lambda), \quad \sigma = c_1^2 + c_2^2, \tag{10}$$

iii) If $\lambda = 0$

$$G(\xi) = \frac{\mu}{2}\xi^2 + c_1\xi + c_2, \tag{11}$$

where c_2 and c_1 are real numbers. Eqn. (11), there is following equation;

$$\psi^2 = \frac{1}{c_1^2 - 2\mu c_2}(\phi^2 - 2\mu\psi). \tag{12}$$

The solution of Eqn. (3) in terms of ψ and ϕ polynomials is

$$U(\xi) = \sum_{i=0}^{n} a_i \phi^i + \sum_{i=1}^{n} b_i \phi^{i-1}\psi, \tag{13}$$

where in b_i $(i = 1, \ldots, n)$ and a_i $(i = 0, 1, \ldots, n)$ are constants to calculate. *n* is a positive integer to be calculated according to the balance principle for Eqn. (4). The corresponding derivatives of Eqn. (13) are calculated. These derivatives are substituted in Eqn. (4). Next, the polynomial is connected to ψ and ϕ are formed. Equating the coefficients of the ψ and ϕ in the obtained polynomial to zero, a system of equations is constructed. The built equation system is solved with the help of a computer software program. The values of the calculated constants are written in their place in Eqn. (13). Solutions of Eqn. (4) are obtained. Thus, we find the solutions in relation to the hyperbolic functions for λ<0, the trigonometric functions for λ>0 and the rational functions for λ=0.

3. Solutions of the (3+1)-dimensional KZK Equation via $(G'/G, 1/G)$-expansion Method

We consider Eqn. (2). Additionally, let us consider traditional wave transform as below:

$$u = U(\xi) = u(x, y, z, t), \quad \xi = x + y + z - ct. \tag{14}$$

We write Eqn. (14) into system Eqn. (2) to attain nonlinear ODEs

$$(m + n - c)U + \frac{1}{2}U^2 + rU' = 0. \tag{15}$$

By use of balance principle in Eqn. (15), we get $n = 1$ and in Eqn. (13) the following situation is attained:

$$U(\xi) = a_0 + a_1\ \phi[\xi] + b_1\ \psi[\xi], \tag{16}$$

where $a_0,\ a_1, b_1$ then the constants to be determined are unknown. If Eqn. (16) is written in Eqn. (15) and the coefficients of the Eqn. (2) equal zero, we can set up the following systems of an algebraic equation

$$\begin{aligned}
(\phi[\xi])^0 &: \quad -ca_0 + ma_0 + na_0 + \frac{a_0^2}{2} - r\lambda a_1 - \frac{\lambda^2 b_1^2}{2(\mu^2 + \lambda^2\sigma)} = 0,\\
\phi[\xi] &: \quad -ca_1 + ma_1 + na_1 + a_0 a_1 = 0,\\
(\phi[\xi])^2 &: \quad -ra_1 + \frac{a_1^2}{2} - \frac{\lambda b_1^2}{2(\mu^2 + \lambda^2\sigma)} = 0,\\
\psi[\xi] &: \quad r\mu a_1 - cb_1 + mb_1 + nb_1 + a_0 b_1 + \frac{\lambda\mu b_1^2}{\mu^2 + \lambda^2\sigma} = 0,\\
\phi[\xi]\psi[\xi] &: \quad -rb_1 + a_1 b_1 = 0.
\end{aligned} \tag{17}$$

With the software program, we reached the solutions of the system (17) and the following situations.

If $\lambda < 0$,

Case 1.

$$a_0 = -2ir\sqrt{\lambda}, \quad a_1 = 2r, \quad b_1 = 0, \quad \mu = 0, \quad c = m + n - 2ir\sqrt{\lambda}, \tag{18}$$

where $i = \sqrt{-1}$,replacing Eqn. (18) into Eqn. (16), the following complex hyperbolic solution is attained

$$u_1(x,y,z,t) = -2ir\sqrt{\lambda} + \frac{\left(2r\left(c_2\sqrt{-\lambda}\,cosh\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]+c_1\sqrt{-\lambda}\,sinh\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]\right)\right)}{\left(c_1\,cosh\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]+c_2\,sinh\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]\right)}. \tag{19}$$

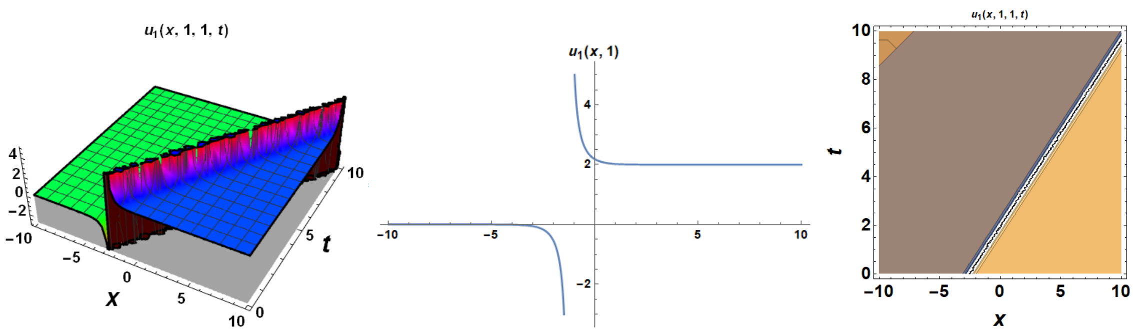

Figure 1: 3D, 2D and contour graphs for $c_2 = 2,\ c_1 = 1,\ \lambda = -1,\ r = 0.5,\ m = 0.2,\ n = 0.1,\ y = 1,\ z = 1$ of Eqn. (19)

There is $\sqrt{\lambda}\sqrt{-\lambda}$ in u that we have presented as a solution. Since $\lambda < 0$, we have presented the solution consists only of the real part.

Case 2.

$$a_0 = ir\sqrt{\lambda}, \quad a_1 = r, \quad b_1 = \frac{\sqrt{-r^2\mu^2 - r^2\lambda^2\sigma}}{\sqrt{\lambda}}, \quad c = m + n + ir\sqrt{\lambda}, \tag{20}$$

where $i = \sqrt{-1}$, replacing Eqn. (20) into Eqn. (16), the following hyperbolic solution is attained

$$u_2(x,y,z,t) = ir\sqrt{\lambda} + \frac{\sqrt{-(-c_1{}^2 + c_2{}^2)r^2\lambda^2 - r^2\mu^2}}{\sqrt{\lambda}\left(\frac{\mu}{\lambda} + c_1\,cosh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right] + c_2\,sinh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]\right)}$$

$$+\frac{r\left(c_2\sqrt{-\lambda}\,cosh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]+c_1\sqrt{-\lambda}\,sinh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]\right)}{\frac{\mu}{\lambda}+c_1\,cosh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]+c_2\,sinh\left[\left(x+y+z-t\left(m+n+ir\sqrt{\lambda}\right)\right)\sqrt{-\lambda}\right]}. \quad (21)$$

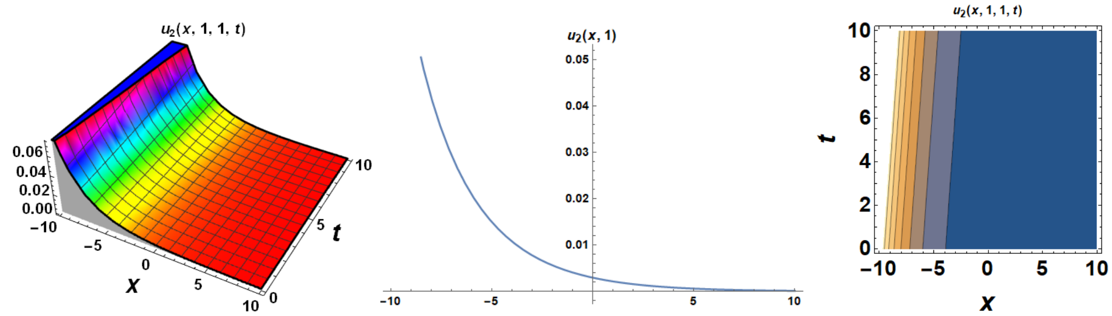

Figure 2: 3D, 2D and contour graphs for$c_2 = 2,\ c_1 = 1,\ \lambda = -0.1,\ \mu = -3,\ r = 0.5,\ m = 0.2,\ n = 0.1,\ y = 1,\ z = 1$ values of Eqn. (21)

There is $\sqrt{\lambda}\sqrt{-\lambda}$ in u that we have presented as a solution. Since $\lambda < 0$, we have presented the solution consists only of the real part.

If $\lambda > 0$,

Case 1.

$$a_0 = -2ir\sqrt{\lambda}, \quad a_1 = 2r, \quad b_1 = 0, \quad \mu = 0, \quad c = m + n - 2ir\sqrt{\lambda}, \quad (22)$$

where $i = \sqrt{-1}$, replacing Eqn. (22) in Eqn. (16), the following trigonometric solution is attained

$$u_3(x, y, z, t) = -2ir\sqrt{\lambda} + \frac{2r\left(c_2\sqrt{\lambda}\,cos\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{\lambda}\right]-c_1\sqrt{\lambda}\,sin\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{\lambda}\right]\right)}{c_1\,cos\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{\lambda}\right]+c_2\,sin\left[\left(x+y+z-t\left(m+n-2ir\sqrt{\lambda}\right)\right)\sqrt{\lambda}\right]}. \quad (23)$$

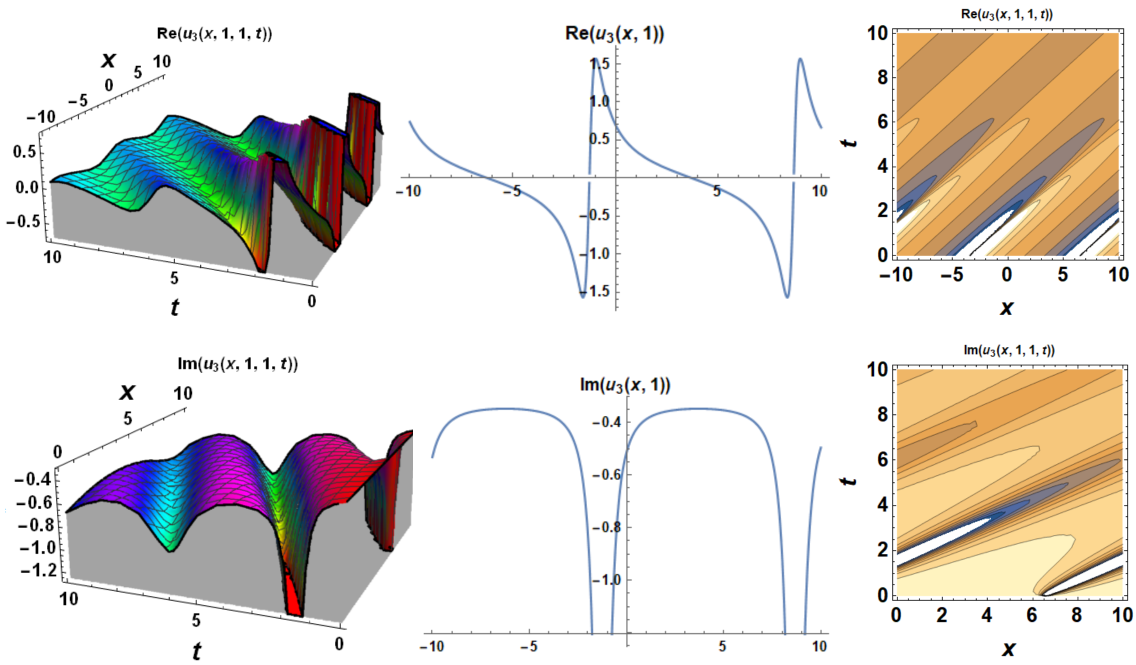

Figure 3: Real and imaginary parts of 3D, 2D and contour graphs for $c_2 = 2,\ c_1 = 1,\ \lambda = 0.1,\ r = 0.5,\ m = 1,\ n = 1.2,\ y = 1,\ z = 1$ of Eqn. (23)

If $\lambda = 0$,

Case 1.

$$a_0 = 0, \quad a_1 = 2r, \quad b_1 = 0, \quad \mu = 0, \quad c = m + n, \tag{24}$$

replacing Eqn. (24) in Eqn. (16), the following rational solution is attained

$$u_4(x, y, z, t) = \frac{2c_2 r}{c_1 + c_2(-(m+n)t + x + y + z)}. \tag{25}$$

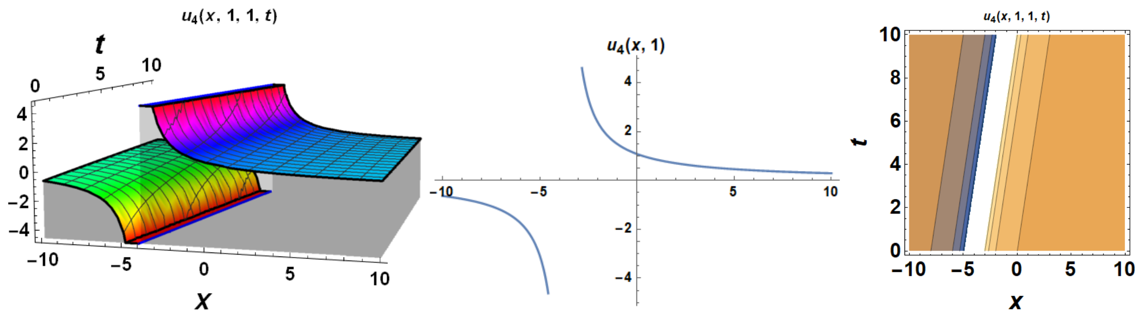

Figure 4: 3D, 2D and contour graphs for $c_2 = 0.5,\ c_1 = 1,\ \lambda = 0,\ r = 2,\ m = 0.2,\ n = 0.1,\ y = 1,\ z = 1$ of Eqn. (25)

Case 2.

$$a_0 = 0, \quad a_1 = r, \quad \mu = \frac{{c_2}^2 r^2 - b_1^2}{2c_1 r^2}, \quad c = m + n, \tag{26}$$

replacing Eqn. (26) in Eqn. (16), the following rational solution is attained

$$u_5(x,y,z,t) = \frac{b_1}{c_1 + c_2(-(m+n)t + x + y + z) + \frac{(-(m+n)t + x + y + z)^2(c_2{}^2r^2 - b_1^2)}{4c_1r^2}} + \frac{r\left(c_2 + \frac{(-(m+n)t+x+y+z)(c_2{}^2r^2-b_1^2)}{2c_1r^2}\right)}{c_1 + c_2(-(m+n)t+x+y+z) + \frac{(-(m+n)t+x+y+z)^2(c_2{}^2r^2-b_1^2)}{4c_1r^2}}. \quad (27)$$

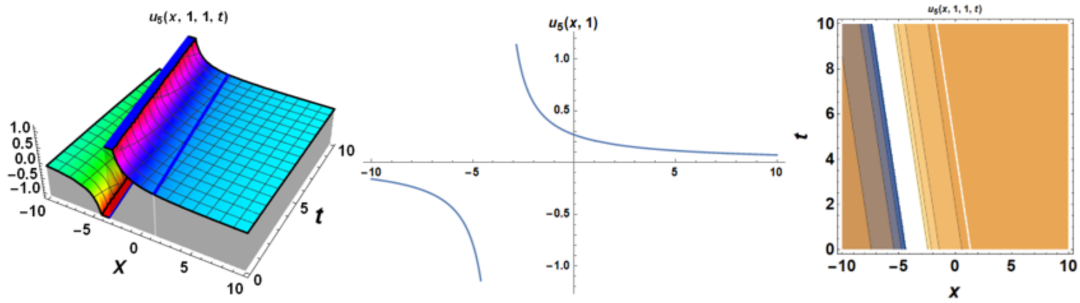

Figure 5: 3D, 2D and contour graphs for $c_2 = 0.4,\ c_1 = 1,\ b_1 = 0.5,\ \lambda = 0,\ r = 0.5,\ m = -0.2,\ n = -0.1,\ y = 1,\ z = 1$ of Eqn. (27)

Traveling wave solutions play an important role in physically transporting energy from one place to another. The traveling wave solutions obtained in this study can offer a different perspective to the acoustic theory. The graphs presented in Figs. 1-5 illustrate the wave behaviour of traveling wave solutions at any instant, which we can call a standing wave. While drawing these graphs, the *y* and *z* dimensions are considered fixed.

4. Conclusion

In this study, we have proposed hyperbolic, trigonometric, complex trigonometric and rational traveling wave solutions with the help of $(G'/G, 1/G)$-expansion method of Eqn. (2) which is the mathematical model of the sound beam in a non-linear medium without physical dispersion and absorption. The method is generally categorized into three different classes depending on the λ parameter. The equation was checked with the help of a ready-made package program that the traveling wave solutions obtained for each class provided. In the traveling wave solutions obtained, solitary wave solutions were obtained by giving arbitrary constants to the parameters and the graphics were presented as 3D, 2D, and contour. The solution of the algebraic equation system discussed in this study, complex operations and the graphics of these solutions were obtained using a ready-made package program. It has been concluded that this method we have used is useful and reliably applicable in equations with strong nonlinearity.

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