COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME 23 A ANNÉE 1974

A Numerical Integration of the Lunar Orbit

by

Zeki TÜFEKÇİOĞLU

1

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

---

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifiques représentées à la Faculté: Mathématiques pures et appliquées, Astronomie, Physique et Chimie théorique, expérimentale et technique, Géologie, Botanique et Zoologie.

La Revue, à l'exception des tomes I, II, III, comprend trois séries

Série A : Mathématiques, Physique et Astronomie.

Série B : Chimie.

Série C : Sciences naturelles.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adresse: Fen Fakültesi Tabliğler Dergisi, Fen Fakültesi, Ankara, Turquie.

A Numerical Integration of the Lunar Orbit

Zeki TÜFEKÇİOĞLU

Department of Astronomy, University of Ankara, Ankara Turkey.

"Applying the elliptic restricted problem of three bodies to the sun-earth-moon system, the lunar orbit has been obtained by numerical integration. The positions and velocities of the moon have been found for different points in the orbit. The Jacobi constant at each point is also calculated for check."

Among the various types of the restricted three body problem, the elliptic case is of a particular interest. In this case, the motion of an infinitesimal particle moving under the influence of the finite point masses is studied. It is assumed that the two bodies move about their common center of mass in elliptic orbits and that these orbits are not disturbed by the third body.

let ξ, η be the dimensionless coordinates of the third body; ρ_1, ρ_2 its dimensionless distances from the primaries; μ, $1-\mu$ the dimensionless masses of the primaries (The distances and masses are made dimensionless dividing them by the distance between the primaries and the total mass of the primaries, respectively).

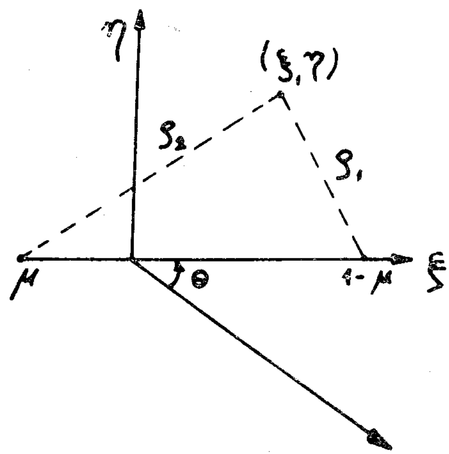
Then the equations of motion of the third body are:

$$\frac{d^2\xi}{d\theta^2} - 2\frac{d\eta}{d\theta} = \frac{1}{1+e\cos\theta}\left[\xi - (1-\mu)\frac{\xi-\mu}{\rho_1^3} - \mu\frac{\xi-\mu+1}{\rho_2^3}\right]$$

$$\frac{d^2\eta}{d\theta^2} + 2\frac{d\xi}{d\theta} = \frac{1}{1+e\cos\theta}\left[\eta - (1-\mu)\frac{\eta}{\rho_1^3} - \mu\frac{\eta}{\rho_2^3}\right]$$

where θ and e is the true anomaly and the eccentricity of the smaller primary around the other.

Now suppose that μ is the sun and $1 - \mu$ the earth. If we make a coordinate transformation from (ξ, η) to (x,y) whose origin is at the earth and let $\theta = N\tau$ where $N = \sqrt{1 + M}$, $M = \dfrac{m\oplus}{m\odot}$, ant τ is the dimensionless time, the equations of motion of the moon around the earth will be:



$$\frac{d^2x}{d\tau^2} - 2N\frac{dy}{d\tau} = \frac{1}{1 + e\cos N\tau}\left[(x-1)\left(1 - \frac{1}{\rho_1^3}\right) + Mx\left(1 - \frac{1}{\rho_2^3}\right)\right]$$

$$\frac{d^2y}{d\tau^2} + 2N\frac{dx}{d\tau} = \frac{1}{1 + e\cos N\tau}\left[y\left(1 - \frac{1}{\rho_1^3}\right) + My\left(1 - \frac{1}{\rho_2^3}\right)\right]$$

These, with the Jacobi constant $C = M\left(\rho_2^2 + \dfrac{2}{\rho_2}\right) + \left(\rho_1^2 + \dfrac{2}{\rho_1}\right) - (\dot{x}^2 + \dot{y}^2)$, are the equations to be integrated. For numerical integration we use the Runge-Kutta fourth order method. To find the initial conditions, we assume that the lunar orbit is circular. Then the centrifugal force equals the gravitational force and the velocity of the moon is $v = \sqrt{\frac{k^2M}{r}}$ where k is the gaussian constant and r is the mean distance of the moon from the earth. On the other hand, since (x, y) is a coordinate system rotating with mean motion n, the components of the velocity of the moon referred to this system are:

$$\frac{dx}{dt} = v_x + ny \qquad \text{and} \qquad \frac{dy}{dt} = v_y - nx$$

To make them dimensionless we divide by n and set $\tau = nt$ where t is the dimensional time. So we have

$$\frac{dx}{d\tau} = \frac{v_x}{n} + y \qquad \text{and} \qquad \frac{dy}{d\tau} = \frac{v_y}{n} - x$$

At the point where the moon is in conjunction with the earth and sun, we have $v_x = 0$, $v_y = v$, $x = r$ and $y = 0$. taking $r = 0.002571$ A. U., $k = 0.01720209$, $M = 0.0000030$, $n = 2\pi/365.25$ we get for $\tau = 0$: $x = 0.002571$, $y = 0$,

$$\frac{dx}{d\tau} = 0 \text{ and } \frac{dy}{d\tau} = 0.0315812 \ .$$

These are the initial conditions to start the integration. The integration has been carried out with the step $\tau = 0.005$ which is about 7 hours. To save space the results are given in the following table with the step $\tau = 0.010$. As can be seen from the table the moon gets very close to the starting point after one revolution. The residuals are of the order of 10^{-5} A. U. $\tau = 0.5233$ divided by n gives about 30 days for the synodic period of the moon. The last column gives the Jacobi constant. It has been calculated for check. In spite of the fact that the eccentricity of the earth orbit is taken into account, the constancy of C is quite satisfactory.

τ 0.	x 0.00	y 0.00	ẋ 0.0	ẏ 0.0	C 3.00135
000	257100	000000	0000000	3158120	761
010	255223	31502	374893—	3134483	769
020	249622	62533	743733—	3064032	761
030	240388	92631	1100641—	2948130	754
040	227668	121352	1440074—	2788971	747
050	211662	148276	1756952—	2589479	739
060	192619	173019	2046757—	2353182	724
070	170830	195223	2305584—	2084077	717
080	146621	214605	2530163—	1786497	702
090	120350	230881	2717840—	1465001	694
100	92395	243841	2866546—	1124271	680
110	63154	253318	2974750—	769051	680
120	33038	259189	3041404—	404098	672
130	2466	261383	3065907—	34165	672
140	28138—	259872	3048072—	336016—	672
150	58354—	254678	2988112—	701723—	672
160	87762—	245868	2886646—	1058242—	680
170	115952—	233559	2744718—	1400877—	687
180	142527—	217913	2563838—	1724959—	694
190	167105—	119914	2346019—	2025889—	709
200	189333—	177487	2093825—	2299192—	717
210	208878—	153259	1810403—	2540600—	724
220	225445—	126795	1499501—	2746162—	739
230	238791—	98468	1165455—	2912355—	739
240	248697—	68689	813147—	3036215—	747
250	255011—	37892	447923—	3115458—	754
260	257631—	6533	75475—	3114857—	754
270	256516—	24923—	298299	3134919—	754
280	251681—	56009—	667462	3074711—	754
290	243202—	86266—	1026194	2969054—	747
300	231210—	115246—	1368957	2819871—	739
310	215893—	142527—	1690618	2629818—	732
320	197484—	167717—	1986552	2402172—	724
330	176226—	190457—	2252702	2140707—	717
340	152541—	210432—	2485611	1849575—	709
350	126669—	227365—	2682415	1533188—	694
360	99020—	241027—	2840817	1196131—	694
370	69987—	251234—	2959051	843087—	680
380	29977—	257851—	3035841	478794—	680
390	9411—	260788—	3070356	108017—	680
400	21287	260006—	3062189	264459	680
410	51690	255509—	3011346	633834	680
420	81373	247355—	2918252	995286	694
430	109917	235646—	2783773	1343978	694
440	136915	220534—	2609262	1675076	709
450	161975	202219—	2396602	1983787	709
460	184728	180949—	2148255	2265418	724
470	204832	157016—	1867307	2515466	739
480	221978	130759—	1557487	2729729	747
490	235899	102554—	1223164	2904435	754
500	246376	72813—	869299	3036379	761
510	253239	41977—	501368	3123056	761
520	256376	10508—	125234	3162778	769
5233	256583	35—	7—	3165420	761

REFERENCES

[1] Brouwer D. and Clemence G.M. Methods of Celestial Mechanics, Academic Press, New York and London, 1961.

[2] Szebehely V. and Giacaglia G.E.O. On the Elliptic Restricted Problem of Three Bodies, A. J. Vol. 69, 1964.

[3] Tüfekçioğlu Z. On the Librational Orbits in the Elliptic Restricted Three Body Problem, Communications de la Faculte des Sciences de l'Universite d'Ankara, TOME 18A, 1969.

ÖZET

Eliptik halde şartlı üç cisim problemini Güneş-Yer-Ay sistemine uygulayarak Ay'ın yörüngesi, sayısal integrasyonla elde edilmiştir. Ay'ın yeri ve hızı yörünge üzerindeki değişik noktalarda bulunmuştur. Keza, her noktadaki Jacobi sabiti de kontrol gayesiyle hesap edilmiştir.

Prix de l'abonnement annuel

Turquie : 15 TL; Étranger: 30 TL.

Prix de ce numéro : 5 TL (pour la vente en Turquie).

Prière de s'adresser pour l'abonnement à : Fen Fakültesi

Dekanlığı Ankara, Turquie.

Ankara Üniversitesi Basımevi–1974