Automatic extraction of the polygon obtained by the intersection of parcels with the geometric algorithms

Emirhan Ozdemir [1] , **Faruk Yıldırım** [2] , **Fatih Kadi** *[2]

¹Iğdır University, Vocational School of Higher Education for Technical Sciences, Iğdır, Turkey
²Karadeniz Technical University, Engineering Faculty, Department of Geomatics Engineering, Trabzon, Turkey

ABSTRACT

Today, many mapping techniques (development plan implementation, expropriation, urban and rural arrangement, etc.) are applied geometrically. One of the situations that will be encountered in these applications is the relative status of the parcels. Firstly, point-plot relationships for arbitrary points were examined in this study. It was investigated whether these points were inside or outside the parcel. The aim is to determine the positions of the points relative to the parcels and to observe whether there is an intersection. If there is an intersection between the main parcel and the test parcel as a result of the point-parcel examination, the intersection points of these two parcels are found. Parcel corner points and intersection points are ordered according to the specified criteria and the intersection parcel is obtained. The area of the intersection parcel obtained is calculated. All possible special cases of the two parcels are examined relative to each other. With the proposed method, the results for each of the special cases are obtained and presented. The accuracy of the results was compared with CAD and GIS software and their advantages and disadvantages were examined. The proposed method is designed to be two or more parcels in many surveying techniques and gives high accuracy results. In addition, it is foreseen that the proposed method will provide great convenience in calculations functionally and temporally.

1. INTRODUCTION

The intersection of two or more parcels is an important situation in surveying and mapping applications (expropriation, urban area design, land consolidation, GIS applications). Today, there are many algorithms that determine the relationships between the parcel and the point. The important thing is to examine the intersection situations of two or more parcels by using these algorithms. These algorithms differ in terms of processing times and applicability of special cases. The proposed method aims to automatically select the corners and intersection points of the parcels, and to create the intersection parcel and obtain its area without error.

There are many mathematical and geometric methods that examine whether any point whose coordinate is known is inside or outside of another parcel whose coordinates are known to all vertices. These methods are classified according to the shape of the plots (convex and concave) (Kitay 1985; Yomralıoğlu 2000; Yanalak and İpbüker 2003). The Proposed method first recognizes the corner points of at least two plots. The location of any selected corner point is examined relative to the other parcel. This is repeated for all vertices. The aim is to determine whether the selected corner point is inside or outside the other plot. Thus, this situation will be taken into account in the order to be made while finding the intersection plot. Then, the intersection situations of the parcels are examined. If there is an intersection, the coordinates of the intersection points are determined (Figure 1). All obtained points are ranked according to the criteria determined by the proposed method (Figure 4). Finally, as a result of the sorting, the intersection plot is obtained and its area is calculated correctly.

There are different methods to obtain the intersection plot obtained by the intersection of multiple plots. However, these methods may fail to give good results in all situations. Therefore, each special condition encountered while developing the method was examined separately and the most suitable algorithm was determined by considering the processing time. As a result, the accuracy evaluation of the proposed method was compared with CAD and GIS software. Thus, it has been determined that the proposed method obtains correct results in all special cases.

2. Material and METHODS

2.1. Point in Polygon Test for Concave and Convex Parcels

There are 3 methods for point in polygon test in both types of parcels. These are Ray Intersection, Sum of Angles and Swath methods. Among these methods, ray intersection and swath methods do not provide solutions for some special cases (passing through the corner point, being on the edge). The sum of angles method is more disadvantageous than the other two methods, due to both the processing time and the rounding errors caused by the sum of angles. Instead of these three methods, an algorithm that takes special cases into account has been developed.

There is more than one type of representation of a line in the plane. The vector equality of a line segment defined by the coordinates of the starting (x_0, y_0) and ending (x_1, y_1) point is expressed by the following equation.

$$L = \{(x_0, y_0) + (r[x_1 - x_0], r[y_1 - y_0]|\, 0 < r < 1\} \qquad (1)$$

In the plane, the line segments L and L′ are defined by the points P_0, P_1 and P'_0, P'_1 respectively (Figure 1). In order for both line segments to intersect with each other, Equation 2 must be provided.

$$P_0 + r(P_1 - P_0) = P'_0 + r'(P'_1 - P'_0) \qquad (2)$$

Using the coordinate values of the points, r and r' values are calculated with the help of equations 3, 4, 5, 6, 7, 8 and 9.

$$x_0 + r(x_1 - x_0) = x'_0 + r'(x'_1 - x'_0) \qquad (3)$$

$$y_0 + r(y_1 - y_0) = y'_0 + r'(y'_1 - y'_0) \qquad (4)$$

$$D = \begin{vmatrix} (y_1 - y_0) & -(y'_1 - y'_0) \\ (x_1 - x_0) & -(x'_1 - x'_0) \end{vmatrix} \qquad (5)$$

$$D_1 = \begin{vmatrix} (y'_0 - y_0) & -(y'_1 - y'_0) \\ (x'_0 - x_0) & -(x'_1 - x'_0) \end{vmatrix} \qquad (6)$$

$$D_2 = \begin{vmatrix} (y_1 - y_0) & (y'_0 - y_0) \\ (x_1 - x_0) & (x'_0 - x_0) \end{vmatrix} \qquad (7)$$

$$r = D_1/D, \quad r' = D_2/D \qquad (8)$$

If r and r′ values are between 0 and 1, the segments L and L′ intersect at a single point. But if the D determinant is equal to zero, these line segments are parallel. If the determinants D, D_1, and D_2 are equal to zero, these line segments overlap.
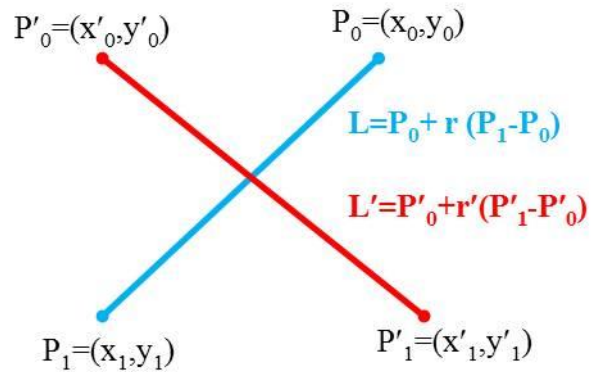

Figure 1. Line segments L and L′

The coordinates of the Q point (x_0, y_0) given for the inquiry are the first point of the beam. The ray is defined as vectorial in equation 9.

$$R = \{(x_0, y_0) + r(m_x, 2M_y)|\, r > 0\} \qquad (9)$$

In Equation 9, when the coordinates of the end point of the ray are defined as r = 1, equation number 10 is obtained.

$$(x', y') = (x_0 + m_x, y_0 + 2M_y) \qquad (10)$$

In Eq. 10, m_x and M_y are used to draw a direction to the beam. Thus, the beam will not be like the special cases in figure 1 and figure 2. For the M_y value, $|y_0 - y_i|$ the largest value of the absolute value difference is taken. $|x_0 - x_i|$ The smallest value of the absolute value difference other than zero is taken for m_x (Figure 2).
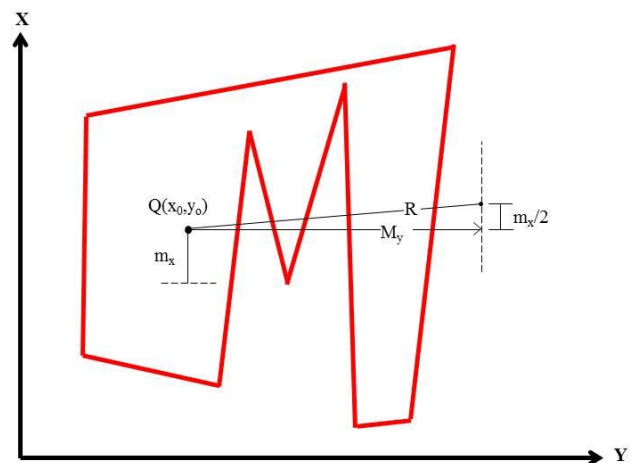

Figure 2. M_y and m_x rays

A number of determinant calculations are required for the calculation of the number of points where the R ray whose starting point is Q (x_0, y_0) intersects the parcel (Equations 11, 12 and 13). The total number of intersections is calculated from the product of the determinants $D_i'(D_i - D_i')$ and $D_i''D_i$.

D_i, D_i' and D_i'' parameters are calculated using the following equations.

$$D_i = \begin{vmatrix} 2M_y & -(y_{i+1} - y_i) \\ m_x & -(x_{i+1} - x_i) \end{vmatrix} \qquad (11)$$

$$D_i' = \begin{vmatrix} 2M_y & (y_i - y_0) \\ m_x & (x_i - x_0) \end{vmatrix} \qquad (12)$$

$$D_i'' = \begin{vmatrix} (y_i - y_0) & -(y_{i+1} - y_i) \\ (x_i - x_0) & -(x_{i+1} - x_i) \end{vmatrix} \qquad (13)$$

If the determinant products are positive, the R ray cuts the S_i edge of the parcel. In other cases, it doesn't. Thus, the number of intersections is calculated. If this number is odd, point Q is inside the parcel, if it is even, the point is outside the parcel (Taylor 1994; Huang and Shih 1997; Jianqiang et al. 2018).

2.2. The Intersection of Parcels and the Production of the Parcel using the Corner Points of the Common Area

The corner points of the area obtained from the intersection of the two parcels are complex and these points are not numbered clockwise. First of all, these complex points should be given point numbers in the clockwise direction for calculation of the parcel area. Nowadays there are a lot of methods that have different geometric algorithms for this process. The main goal of these methods is to be applicable to both concave and convex parcels (Sutherland and Hodgman 1974; Weiler and Atherton, 1977). In addition, the algorithm of intersection of convex polygons has been developed (Cyrus and Beck 1978). Today, there are algorithms that have different special cases for the area calculation of the polygon obtained by the intersection of two parcels (Barsky and Liang 1984; Nicholl et al. 1987; Maillot 1992; Day 1992; Möller 1997; Greiner and Hormann 1998; Huang and Liu 2002).

Special cases are the numbering of the corner points of the parcels in clockwise direction, being applicable for concave and convex parcels, and processing time. First of all, corner points of the polygon obtained by the intersection of many parcels should be numbered clockwise. Then, by selecting any of the polygons, the correct equations of each side are generated with the help of consecutive vertices. The same process step is applied to all polygons to obtain the correct equations of all sides. It is checked whether each line intersect. If there are lines that intersect, a new name is given to the point where they intersect (Figure 3a.). And this point is added between those two vertices in the table where the vertices are written sequentially (Figure 4). This is done for all sides until the sequence is complete. This sequence starts from the left and continues until the first intersection point. This point is defined as the first point of the common region and all points in between up to the other intersection point are written as the edges of the common region. When the second intersection is reached, it stops. If the second intersection point exists in the initial ranking, it is accepted at that time and the same process is continued based on that order. This process continues until it reaches the point defined as the first

point of the polygon formed as a result of the intersection. Thus, the coordinates of all corner points of the common area have been determined (Figure 3a, 3b). All sides of the 1st polygon (A → B → C) and the 2nd polygon (D → E → F → G → H) are defined in the clockwise direction. Check whether the first edge (AB) of the 1st polygon intersects all sides of the second polygon. Edge AB and sides EF and HD of the second parcel intersect. Those intersecting points are given the names a and b, respectively. The same process steps are then applied for the BC edge. This edge intersects the FG edge and the GH edge. These points are given the names c and d, respectively. Finally, the same process steps are applied for the CA edge. Thus, the final names of polygons after intersection are obtained by the 1st polygon [A → b → a → B → c → d → C] and the 2nd polygon [D → E → a → F → c → G → d → H → b] (Figure 3b).
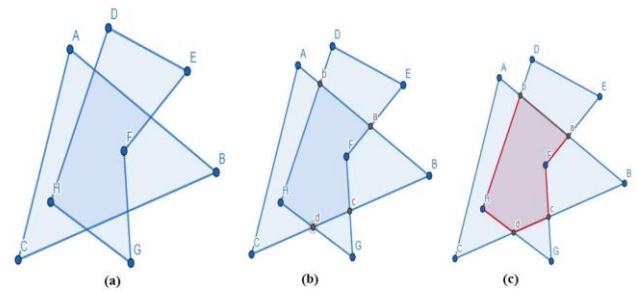

Figure 3. Two intersecting polygons and intersection points

It has been started from point b, which is the first intersection point of the 1st polygon, and continued until the second intersection point a. Point a is in the order in the 2nd polygon and has been continued from there to c. If point c was not in the order in the 1st polygon, the process would stop at that point. But point c is present in the order in the 1st polygon. Then it is continued from point c in the 1st polygon to point d, which is the next intersection point. Likewise, point d is present in the order in the 2nd polygon. Finally, considering the order in the 2nd polygon, the intersection point after point d is point b. This point is the point originally defined. Thus, the corner coordinates of the polygon formed have been determined (Figure 4).
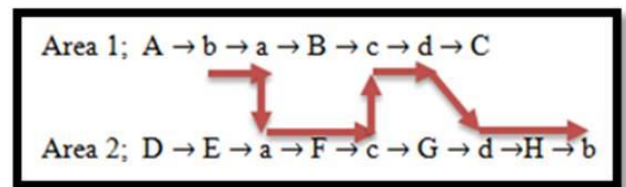

Figure 4. Two intersecting polygons and intersection points

When these points are joined, an intersection polygon has been formed (Figure 3c).

2.2. Proposed Method

The aim of this study is to calculate the corner coordinates and area calculation of the generated common area as a result of the coincidence of any two

convex or concave parcels (polygons) in Geomatics Engineering applications with the help of MATLAB software. While calculating this common area, it will be sufficient to give the coordinates of only two parcels with the help of developed algorithms. In addition, unlike GIS and CAD software, it is aimed to develop algorithm faster and eliminating user errors without the need for the user to determine intersection points and intersection zones. In the developed algorithm, the vertex coordinates of the polygon obtained from the intersection of two parcels are determined automatically by the algorithm without the need for the user. Because, while the intersection point of two lines is determined, any of these two direct lines can intersect with the other edges of the parcel whose coordinates are known. Another advantage of the proposed algorithm is that when determining the corner coordinates of the Polygon formed after the intersection step, it automatically queries whether other points of the intersection parcel other than the intersection points are the corner points of the parcel (Figure 5).
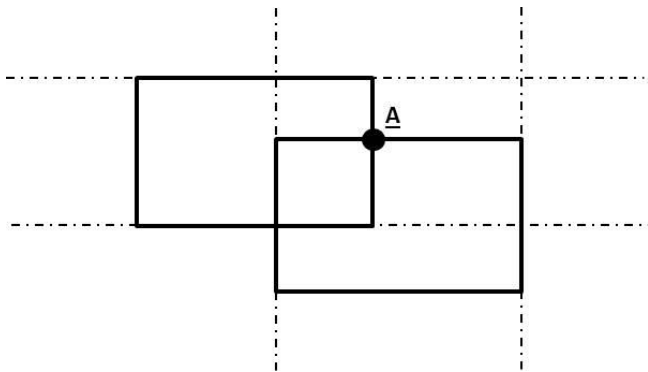

Figure 5. Determining the intersection point

2.3. Application

A large number of application parcels have been selected in order to examine special cases. These special cases are discussed in detail below. The calculated areas and corner coordinates are controlled by GIS software and the advantages and disadvantages are examined. A main parcel (P) has been identified, covering all special cases that may occur for cartography activities.

Table 1. Coordinates of the corner points of the main parcel

Corner Points	Y(m)	X(m)	Corner Points	Y(m)	X(m)
1	20	140	12	270	100
2	70	200	13	230	90
3	90	160	14	280	60
4	140	190	15	220	30
5	140	230	16	160	50
6	180	220	17	130	20
7	200	180	18	120	80
8	240	210	19	60	40
9	310	180	20	70	110
10	250	140	21	40	80
11	290	120			

10 parcels with different special conditions to intersect with this main parcel have been determined. The coordinate information of these parcels, which were determined to examine special cases, is presented in Table 2.

Table 2. Coordinates of the corner points of the parcels to be intersected

Type of Parcel	Corner Points	Y(m)	X(m)	Type of Parcel	Corner Points	Y(m)	X(m)
A	A1	220	50	F	F1	280	170
	A2	250	20		F2	270	100
	A3	230	10		F3	170	100
	A4	200	20	G	G1	280	170
B	B1	220	220		G2	250	95
	B2	250	230		G3	170	100
	B3	280	210	H	H1	150	70
	B4	260	180		H2	200	50
C	C1	300	80		H3	175	25
	C2	300	50		H4	110	20
	C3	240	50	I	I1	290	70
D	D1	120	200		I2	280	60
	D2	160	200		I3	220	30
	D3	160	160	J	J1	40	220
	D4	120	160		J2	200	220
E	E1	280	170		J3	110	30
	E2	270	100		J4	20	100
	E3	170	100				

2.3.1. Case number 1

A parcel that intersects with the main plot at two different points has been used. In the figure 6, the positions of the two parcels with respect to each other and the common area has been generated as a result of

the intersection of these parcels have been shown in detail with the corner coordinates (Figure 6).

2.3.2. Case number 2

Another parcel that intersects the main plot at two different points has been taken and the area where both

of them have been intersected is shown in detail (Figure 7).

2.3.3. Case number 3

A parcel intersecting at two different points to the right of the main plot has been selected. These parcels, which intersect at two different points, have been selected in different parts of the main parcel (Figure 8).

2.3.4. Case number 4

An edge of the main plot and an edge of the selected parcel intersect vertically. This situation has been examined as a special case (Figure 9).

2.3.5. Case number 5

The difference of this case from other cases is that the main parcel and the selected parcel have more than 2 intersections. In the light of the obtained results, it has been seen that the algorithm developed in MATLAB software has been worked in this case as well (Figure 10).

2.3.6. Case number 6

One of the special cases is that a corner point of the main parcel and the selected parcel is the same. It is seen that this situation works flawlessly even when the corner points of the main parcel and selected parcel are the same (Figure 11).

2.3.7. Case number 7

The intersection points of the main parcel and the selected parcel and the corner point of the selected parcel is as follows (Figure 12.).

2.3.8. Case number 8

The case that at least one side of the main parcel and the selected parcel are parallel to each other has been shown in the figure (Figure 13.).

2.3.9. Case number 9

The common side of the main parcel and the selected parcel has been shown below (Figure 14.).

2.3.10. Case number 10

In this case, many special cases coexist. These special cases are that the corner points of the two parcels are the same, one is that the edges of these parcels are perpendicular to each other and the number of intersections of the sides is more than two on the same side. This option sets an example for most situations in the field. It works flawlessly according to the mentioned algorithm (Figure 15).
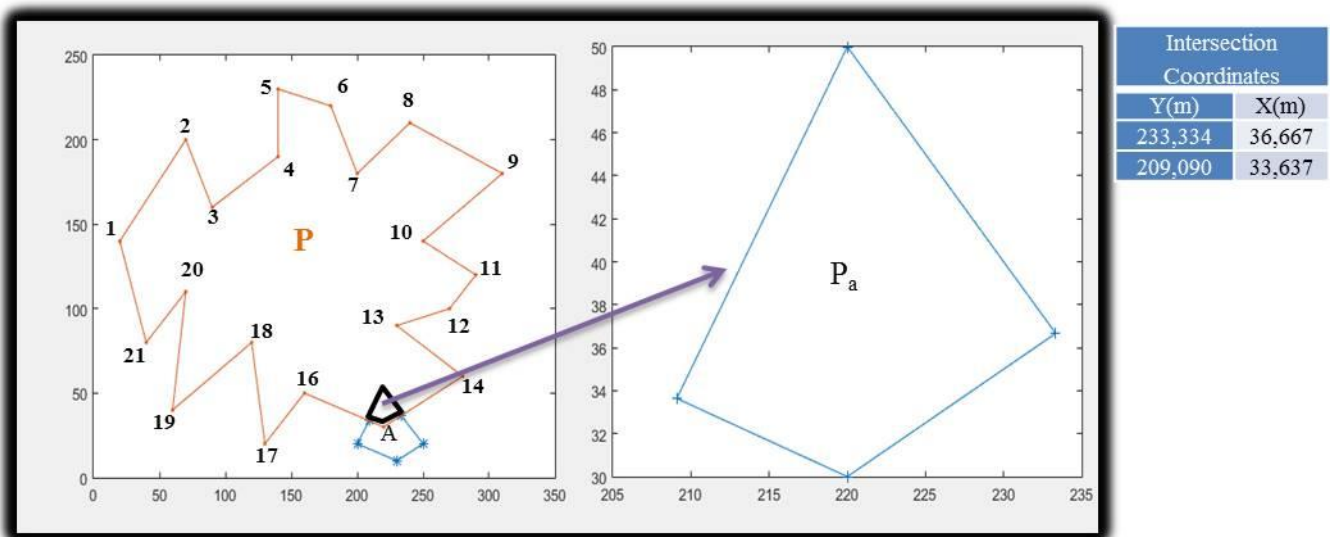

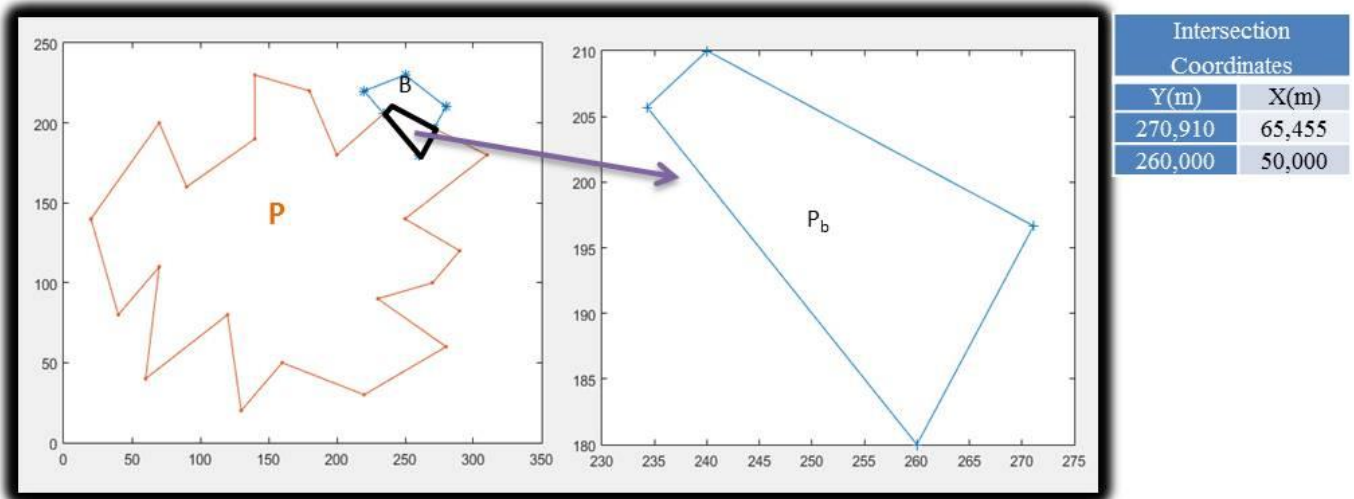**Figure 6.** New P_a parcel produced after intersection with P and A parcels

Figure 7. New P_b parcel produced after intersection with P and B parcels
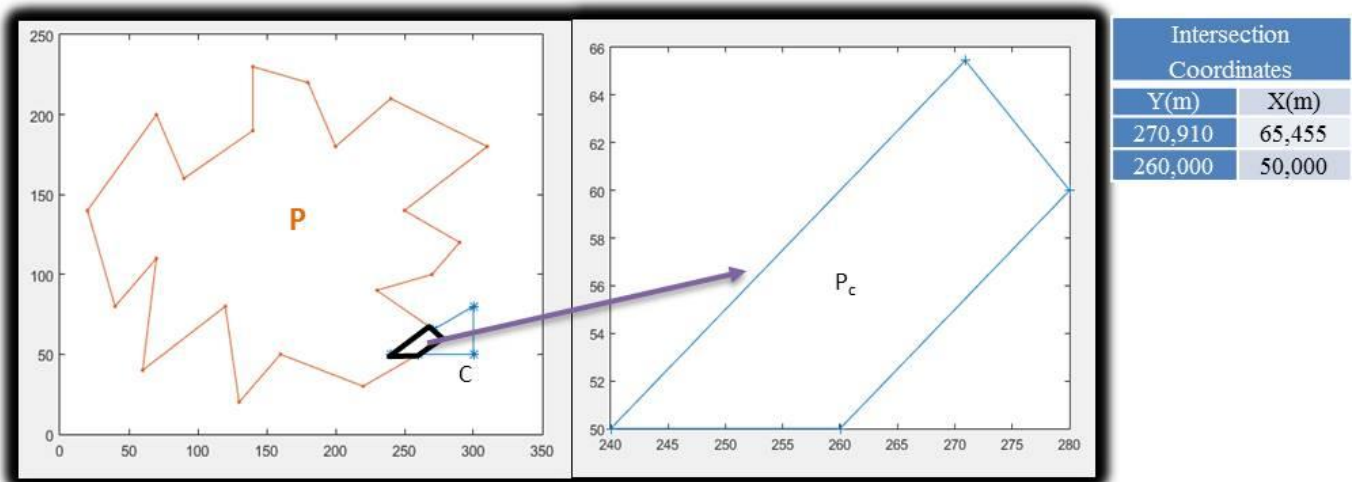

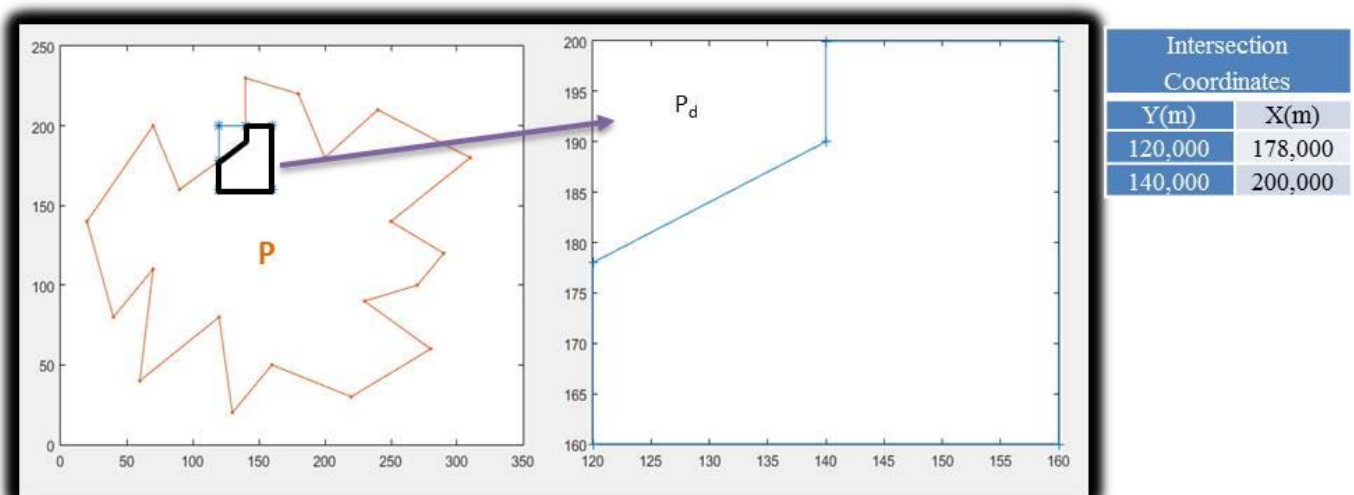**Figure 8.** New P_c parcel produced after intersection with P and C parcels

Figure 9. New P_d parcel produced after intersection with P and D parcels

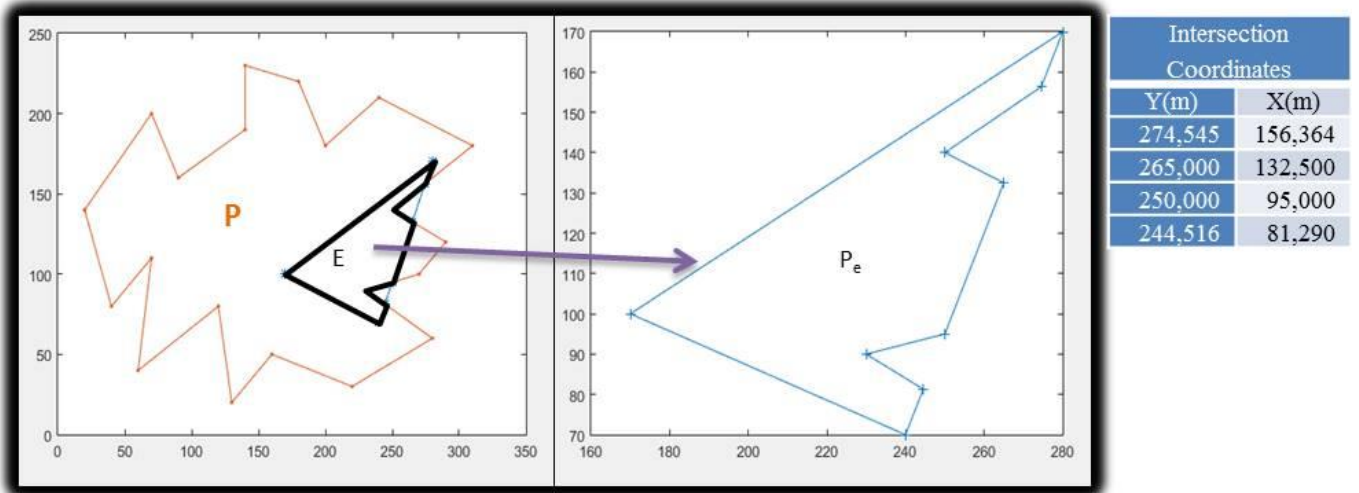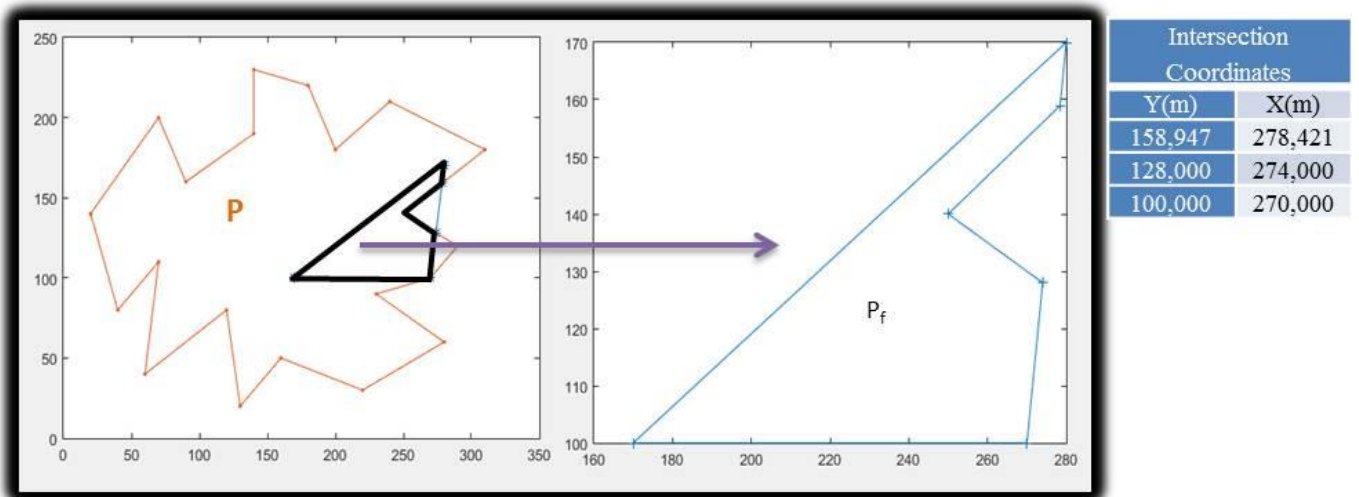**Figure 10.** New P_e parcel produced after intersection with P and E parcels

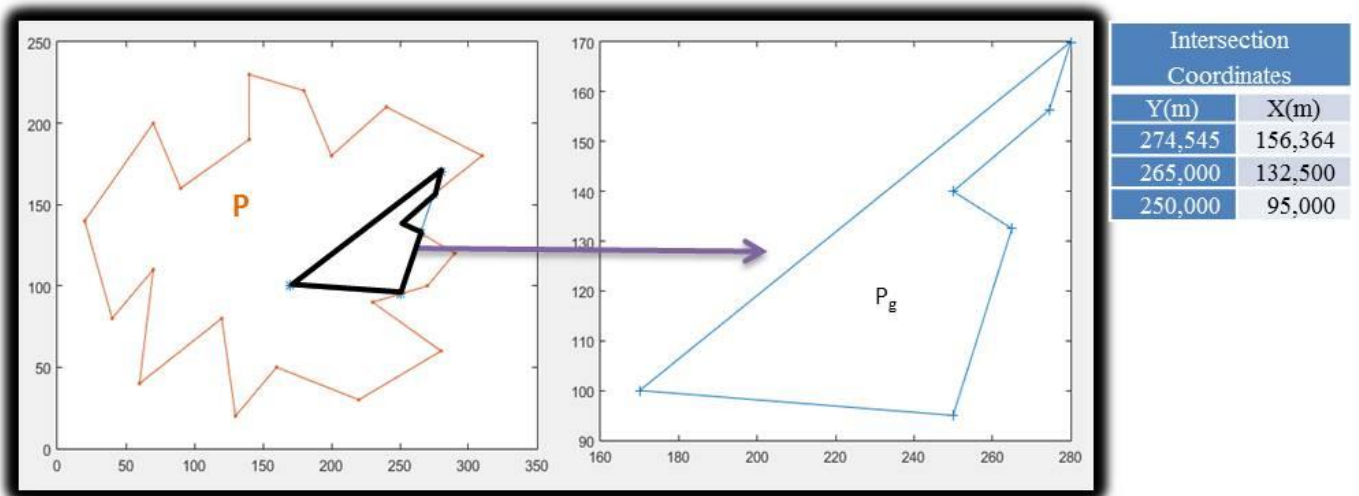**Figure 11.** New P_f parcel produced after intersection with P and F parcels

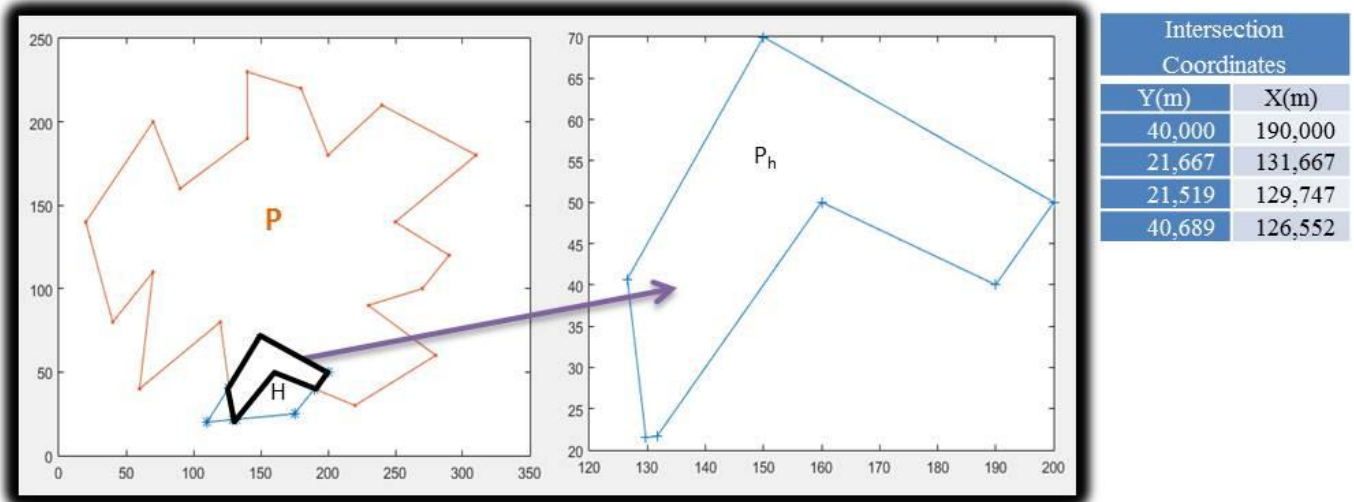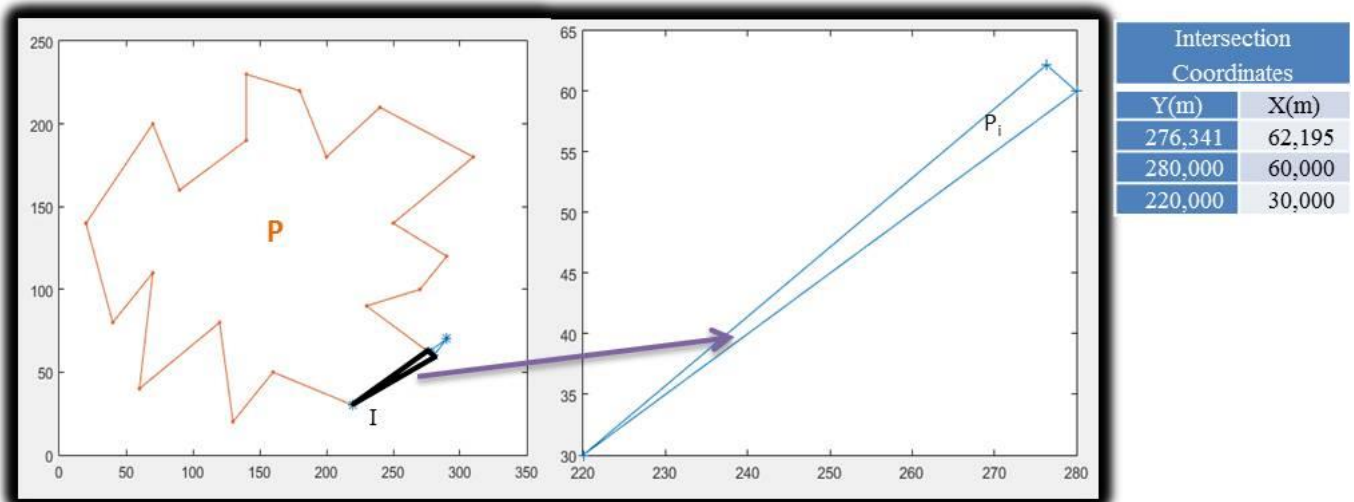**Figure 12.** New P_g parcel produced after intersection with P and G parcels

Intersection Coordinates	
Y(m)	X(m)
40,000	190,000
21,667	131,667
21,519	129,747
40,689	126,552

Figure 13. New P$_h$ parcel produced after intersection with P and H parcels

Intersection Coordinates	
Y(m)	X(m)
276,341	62,195
280,000	60,000
220,000	30,000

Figure 14. New P$_h$ parcel produced after intersection with P and H parcels

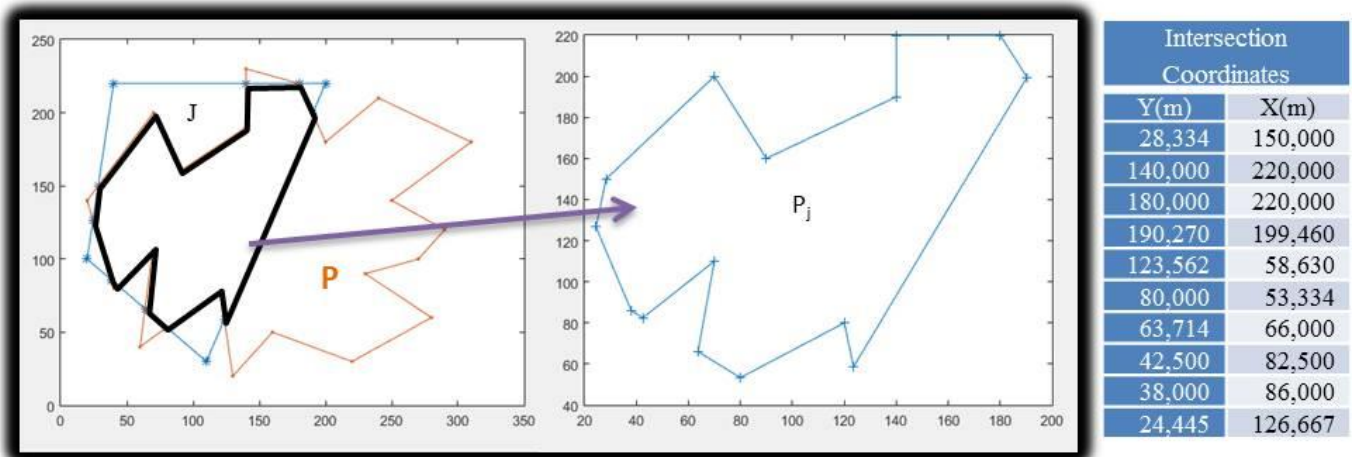| Intersection Coordinates	
Y(m)	X(m)
28,334	150,000
140,000	220,000
180,000	220,000
190,270	199,460
123,562	58,630
80,000	53,334
63,714	66,000
42,500	82,500
38,000	86,000
24,445	126,667

Figure 15. New P$_j$ parcel produced after intersection with P and J parcels

2.4. Comparison with GIS Software

In this study, the operation steps for the area calculation of the polygon formed as a result of the intersection of two parcels have been also applied in the ArcGIS software. Applications on all selected parcels in the special cases described above have been made using ArcGIS (clip command) software. The results have been compared (Figure 16).

In this study, the operation steps for the area calculation of the common polygon formed as a result of the intersection of two parcels were also performed in

ArcGIS software (Table 3). Both methods give error-free results for the special cases. While area and intersection coordinates are calculated in ArcGIS software, the corner coordinates of the parcel consisting of intersection are not automatically presented to the user in an ordered manner. However, with the effort of the user, the ordered corner coordinates of the parcel can be obtained. This causes time loss and user errors. It is sufficient to give the corner coordinates of only two parcels as input data to the algorithm developed in this study. With the help of the developed algorithm, the intersection points of the main parcel and the selected parcel are automatically obtained and presented to the user.

Table 3. Coordinates of the corner points of the parcels to be intersected

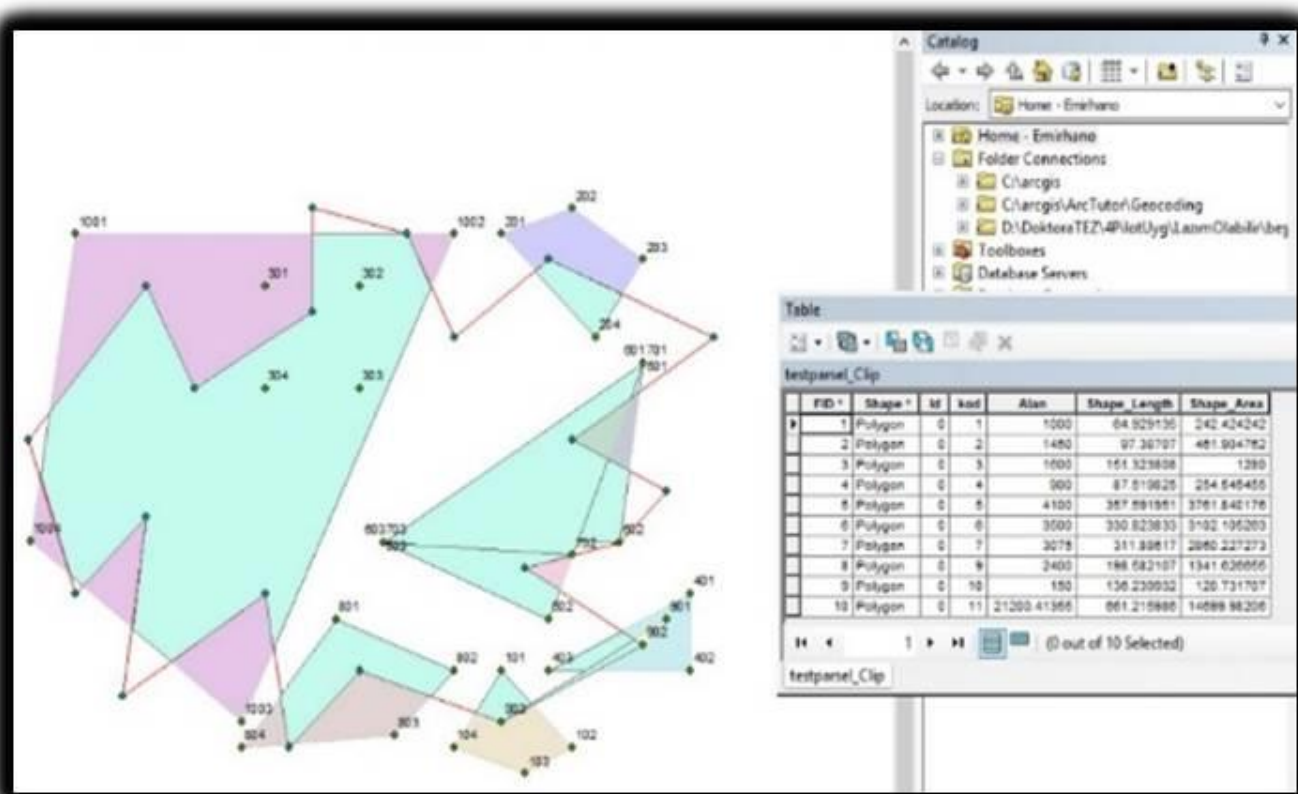| Parcels	Area (m²)
A	242,424
B	461,905
C	254.546
D	1280,000
E	3761,841
F	3102,107
G	2860,229
H	1341,628
I	120,730
J	14726,000

Figure 16. Representation and analysis of the application made in ArcGIS software

3. RESULTS and DISCUSSIONS

It is aimed to determine the common polygon formed by two overlapping convex or concave parcels and to calculate the corner coordinates and area of this polygon in this study. In this context, an algorithm has been developed considering special variables. With the help of the developed algorithm, the intersection of a main parcel selected with different parcels with ten different special conditions was examined. The corner points and area of the common parcel formed as a result of the intersection were found. The results obtained were compared with the results obtained from the ArcGIS software by applying the same process steps. Areas and intersection coordinates gave consistent results with millimeter precision. Corner points should be numbered consecutively for the area calculation of the parcel formed as a result of the intersection. This process step is determined by the user in CAD software. A method that can produce results by sequentially ordering the corner points of concave and convex parcels without the need for the user is presented to the users.

The dynamic nature of CAD and GIS software used in surveying and mapping applications allows users to make a patch program to fix a defect detected in these programs or a later error. It provides great contributions to the users in determining the corner point coordinates and area values without the need for the user, especially during the expropriation and zoning applications, which are among the survey and map applications, during the determination of the current status of the parcels before and after the implementation and the calculation of the area values. Using the algorithm developed in this context will minimize both time loss and user errors caused by point and parcel selection.

Author contributions

Emirhan Özdemir: Conceptualization, Methodology, Software**; Faruk Yıldırım:** Data curation, Writing-Original draft preparation, Software, Validation; **Fatih Kadı:** Visualization, Investigation, Writing-Reviewing and Editing.

Conflicts of interest

The authors declare no conflicts of interest.

REFERENCES

Barsky B A & Liang Y D (1984). A New Concept and Method for Line Clipping, ACM Transactions on Graphics, 3(1), 1–22.

Cyrus M & Beck J (1978). Generalized Two- and Three-Dimensional Clipping, Computers &Graphics, 3(1), 23–28.

Day J D (1992). A New Two-Dimensional Line Clipping Algorithm for Small Windows, Computer Graphics Forum, 11(4), 241–245.

Greiner G & Hormann K (1998). Efficient Clipping of Arbitrary Polygons, ACM Transactions on Graphics, 17(2), 71–83.

Huang C W & Shih T Y (1997). On The Complexity of Point-in-Polygon Algorithms, Computers & Geosciences, 23(1), 109-118.

Huang Y Q & Liu Y K (2002). An Algorithm for Line Clipping against a Polygon Based on Shearing Transformation, Computer Graphics Forum, 21(4), 683–688.

Jianqiang H, Jianzhi S, Yi C, Qiang C & Li T (2018). Tan Optimal Reliable Point-in-Polygon Test and Differential Coding Boolean Operations on Polygons Symmetry, 10, 477-503.

Kitay M G (1985). Land Acquisitionin Developing Countries, Lincolh İnstiue, Boston, Usa.

Maillot P G (1992). A New, Fast Method for 2D Polygon Clipping: Analysis and Software Implementation, ACM Transactions on Graphics, 11(3), 276–290.

Möller T (1997). A Fast Triangle-Triangle Intersection Test, Journal of Graphics Tools, 2(2), 25–30.

Nicholl T M, Lee D T & Nicholl R A (1987). An Efficient New Algorithm for 2-D Line Clipping: Its Development and Analysis, ACM SIGGRAPH Computer Graphics, 21(4), 253–262.

Taylor G (1994). Point in Poligon Test, Survey Review, 32(254), 479-484.

Sutherland I E & Hodgman G W (1974). Reentrant Polygon Clipping, Communications of the ACM, 17(1), 32–42.

Weiler K & Atherton P (1977). Hidden surface removal using polygon area sorting, ACM SIGGRAPH Computer Graphics, 11(2), 214–222.

Yanalak M & İpbüker C (2003). Hesaplamalı Geometri, Harita Dergisi, 129, 50-62.

Yomralıoğlu T (2000). Coğrafi Bilgi Sistemleri: Temel Kavramlar ve Uygulamaları, 5.Baskı (2009), s.480, ISBN 975-97369-0-X, İstanbul.