Uluslararası Mühendislik Araştırma ve Geliştirme Dergisi
International Journal of Engineering Research and Development

UMAGD, (2021) 13(2), 521-530.

10.29137/umagd.887624

Cilt/Volume:13 Sayı/Issue:2 Haziran/June 2021

Araştırma Makalesi / Research Article

An Improved Algorithm for Minimizing Makespan on Flowshops with Uncertain Processing Times

Ali Allahverdi[*1], **Muberra Allahverdi**[2]

[1]*Department of Industrial and Management Systems Engineering, Kuwait University, P.O. Box 5969, Safat, Kuwait*
[2]*Department of Mathematical Sciences, School of Natural Sciences, Kean University, 1000 Morris Ave, Union, NJ 07083, USA*

Başvuru/Received: *01/03/2021* ***Kabul / Accepted***: *27/03/2021* ***Çevrimiçi Basım / Published Online:*** *23/05/2021*

Son Versiyon/Final Version: *18/06/2021*

Abstract
We address the four-machine flowshop scheduling problem with the objective of minimizing makespan with uncertain processing times The problem was investigated in the literature (RAIRO Operations Research 54, 529-553) and different algorithms were proposed. In this paper, we propose a new algorithm for the problem. The new proposed algorithm is compared with the best existing algorithm in the literature by using extensive computational experiments. Computational experiments show that the new proposed algorithm performs much better than the best existing algorithm in the literature in terms of error while both have the same computational time. Specifically, the new proposed algorithm reduces the error of the best existing algorithm in the literature about 40%. This result has been confirmed by using hypotheses testing with a significance level of 0.01.

Key Words
"Makespan, scheduling, algorithm, uncertainty"

***Corresponding Author:** *ali.allahverdi@ku.edu.kw*

1. Introduction

About seventy percent of the real life of scheduling problems fall in the category of flowshops, Fuchigami and Rangel (2018). There are several real life flowshop scheduling problems with four stages, i.e., with four machines, e.g., Stefansson (2011). In the current paper, we study the flowshop scheduling problem (having four-machine) with makespan performance measure, which helps in decreasing production costs.

One of the assumptions made in the literature is that processing times of jobs are deterministic. On the other hand, often real world manufacturing environments are most of the times subject to an extensive uncertainties, Gonzalez-Neira et al. (2017), Mahjoub et al. (2011). Therefore, the assumption of deterministic processing times is not realistic for all flowshop environments. Moreover, assuming processing times follow certain distributions is not legitimate for some flowshop scheduling environments, e.g., Kouvelis and Yu (1997). Therefore, processing times need to be considered as uncertain.

In an uncertain environment, job processing times are considered to be random variables but their probability distributions are unidentified. If $t_{j,k}$ indicates job j's processing time (j=1,2, …,n) on machine k (k =1,2, …,m}). Similarly, if LBt_j shows the lower bound and $UBt_{j,k}$ shows the upper bound of $t_{j,k}$. Then, $t_{j,k}$ satisfies $LBt_{j,k} \leq t_{j,k} \leq UBt_{j,k}$.

Allahverdi and Sotskov (2003) and Allahverdi and Aydilek (2010a) investigated the problem of $F2|LBt_{j,k} \leq t_{j,k} \leq UBt_{j,k}|C_{max}$. Allahverdi and Aydilek (2010a) presented several algorithms and Allahverdi and Sotskov (2003) presented some dominance relations for the problem. Moreover, Allahverdi and Aydilek (2010b) provided different algorithms for the problem of $F2|LBt_{j,k} \leq t_{j,k} \leq UBt_{j,k}|L_{max}$. Also, Aydilek and Allahverdi (2010) provided several heuristics while Sotskov et al. (2004) presented several dominance relations for the problem of $F2|LBt_{j,k} \leq t_{j,k} \leq UBt_{j,k}|\sum C_i$. On the other hand, the problem of $F3|LBt_{j,k} \leq t_{j,k} \leq UBt_{j,k}|\sum C_i$ was investigated by Sotskov et al. (2004) where they provided few dominance relations for problem. For some other uncertain scheduling environments, the problem was investigated as well in the literature, e.g., Allahverdi and Allahverdi (2018), Aydilek et al. (2013, 2015, 2017), Lai and Sotskov (1999), Lai et al. (1997).

The $F4|\text{prmu}, Lt_{j,k} \leq t_{j,k} \leq Ut_{j,k}|C_{max}$ problem was recently investigated by Allahverdi and Allahverdi (2020). They established some dominance relations and presented different algorithms. They indicated that one of their proposed algorithms performs as the best. In the current paper, we propose a new algorithm and indicated that the new algorithm performs better than the best algorithm of Allahverdi and Allahverdi (2020).

The proposed new algorithm is explained in the succeeding section. Next, computational experiments are given in the third section. Finally, conclusions along with some possible extensions to the problem are provided in Section 4.

2. Proposed New Algorithm

The investigated problem was first studied by Allahverdi and Allahverdi (2020). They presented 12 algorithms and compared the 12 algorithms with each other. They showed that one of the algorithms, called Algorithm A7, performed as the best algorithm. We represent their Algorithm A7 by OA. In this section, we propose a new algorithm, which we call it New Algorithm (NA). The steps of NA are given below.

NA Statements

The $LBt_{i,k}$ and $UBt_{i,k}$ values are given where i denotes the number of jobs and k denotes the number of machine

For for $i=1,..,n$ let

$pfm_i=l_1((LBt_{i,1}+UBt_{i,1})/2)+l_2((LBt_{i,2}+UBt_{i,2})/2)$

$psm_i= l_3((LBt_{i,3}+UBt_{i,3})/2)+ l_4((LBt_{i,4}+UBt_{i,2})/2)$

End For

Let s_u and s_a denote unassigned and assigned sequences where initially, $s_u=\{1, 2, …, n\}$ and $s_a=\phi$

Let $poz_1=1$, and $poz_2=poz_3=zero$

While poz_3<n,

Let $pfm_m=minimum\{pfm_i\}$ and $psm_m=minimum\{psm_i\}$ where $i \in s_u$

If $pfm_m \leq psm_m$, put that job in the sequence s_a, in position poz_1

and let $poz_1=poz_1+1$

Else put that job in the sequence s_a, in position $n-poz_2$

and let $poz_2=poz_2+1$

Delete the job from the sequence s_u

Let $poz_3=poz_3+1$

End While

Assign the only job remaining in s_u to the last remaining position in s_a. The sequence s_a is the solution of the algorithm NA.

The values of l_1, l_2, l_3, and l_4, used to compute pfm_i and psm_i in the algorithm NA, are investigated by fine-tuning their values. The values from 0 to 1 with the increments of 0.1 were investigated for each of l_1, l_2, l_3, and l_4, It was found that the values of l_1=0.8, l_2=0.2, $l3$=0.2 and l_4=0.8 were the best for the algorithm. Hence, these values are used for the NA algorithm in the computational section.

3. Algorithm Evaluations

Our newly proposed algorithm NA and the best existing algorithm OA are compared in this section. For the computational experiments, we use the same parameters that were used in evaluating algorithm OA for a fair comparison. Aydilek et al. (2017) used similar experimental parameters that we use in this paper.

$UBt_{i,k}$*'s* are generated from a uniform distributions U($D+1$, 100). On the other hand, $LBt_{i,k}$'s are taken from a uniform distribution U(1, $UBt_{i,k} - D$) where D is set at the values of 40, 30, 20, or 10. D indicates the difference between bounds (lower and upper) on processing times. These four values of gaps were also used in the evaluation of OA. Once the lower bounds and upper bounds are generated, instances of processing times are required to be generated for computational purposes. To generate instances of processing times only using a single distribution is misleading since processing times are uncertain. Therefore, they are generated using normal, positive linear, uniform, and negative linear distributions. The uniform and normal distributions are representative for symmetric distributions while the other two distributions are representative for skewed distributions.

We consider six values for n as 300, 400, 500, 600, 700, 800. Furthermore, we consider four values of 40, 30, 20, and 10 for D. Moreover, there are four considered distributions. Hence, we have in total 96 (4*4*6) combinations. 1000 replications were generated for each combination, which resulted in a total of 96,000 generated problems.

The algorithms OA and NA were compared by both performance measures of Error (average error) and Std (standard deviation). The error was expressed as (algorithm's makespan – minimum makespan) / minimum makespan. The computational results are provided in Table 1-4, for uniform, normal, negative linear, and positive linear distribuions. In the tables, the first column indicates the number of jobs, the second column denotes the difference between the upper and lower bounds of processing time, the third and fourth columns show the errors of OA and NA, respectively. The fifth and sixth columns represent the standard deviations of the OA and NA. Finally, the last column shows the percentage improvement in error of NA over OA.

The summarized computational results are presented in Figure 1-4. It is clear from the figures that the new algorithm NA performs much better than the existing algorithm OA. The overall percentage improvement of the proposed algorithm NA over the existing algorithm OA is 38.61 %. It is clear from the figures that the algorithm's performance does not alter much either with the number of jobs or the difference of the upper and lower bounds of job processing times. Figure 5 summarizes the results in terms of the distributions and delta while Figure 6 summarizes the results in terms of the distributions and job numbers.

The new algorithm NA and the existing algorithm in the literature OA were also statistically compared by utilizing a two-sample t test. The next hypotheses were performed to compare the performances of the both algorithms OA and NA.

$$H_0: \mu(NA) = \mu(OA)$$
$$H_1: \mu(NA) < \mu(OA)$$

where μ(Algorithm-h) symbolizes Algorithm-h's average error. The null hypothesis H_0 was rejected at a significance level (α) of 0.01 for all n and Δ combinations.

Table 1. Experimental results - Normal Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
300	10	0.1048	0.0639	0.0021	0.0015	39.0115
300	20	0.1000	0.0596	0.0020	0.0014	40.4189
300	30	0.0757	0.0469	0.0016	0.0011	38.0926
300	40	0.0765	0.0464	0.0016	0.0011	39.2996
400	10	0.0855	0.0487	0.0017	0.0011	43.0434
400	20	0.0717	0.0421	0.0015	0.0010	41.2755
400	30	0.0667	0.0398	0.0013	0.0010	40.2993
400	40	0.0605	0.0397	0.0011	0.0009	34.3207

Table 1 (cont). Experimental results - Normal Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
500	10	0.0711	0.0474	0.0016	0.0011	33.3315
500	20	0.0640	0.0330	0.0013	0.0008	48.3936
500	30	0.0624	0.0340	0.0012	0.0008	45.4520
500	40	0.0538	0.0330	0.0011	0.0007	38.6690
600	10	0.0748	0.0386	0.0015	0.0009	48.3881
600	20	0.0633	0.0317	0.0012	0.0008	49.9877
600	30	0.0532	0.0312	0.0011	0.0008	41.4104
600	40	0.0486	0.0290	0.0010	0.0007	40.3749
700	10	0.0592	0.0374	0.0013	0.0008	36.8286
700	20	0.0504	0.0322	0.0010	0.0007	36.1635
700	30	0.0480	0.0286	0.0010	0.0007	40.3868
700	40	0.0485	0.0294	0.0010	0.0007	39.3579
800	10	0.0557	0.0293	0.0012	0.0007	47.3078
800	20	0.0512	0.0275	0.0011	0.0007	46.3253
800	30	0.0447	0.0235	0.0009	0.0006	47.4913
800	40	0.0423	0.0225	0.0008	0.0005	46.7650

Table 2. Experimental results - Uniform Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
300	10	0.1113	0.0683	0.0023	0.0015	38.5890
300	20	0.1012	0.0863	0.0021	0.0020	14.7158
300	30	0.0986	0.0826	0.0020	0.0018	16.1901
300	40	0.1049	0.0953	0.0019	0.0020	9.1123
400	10	0.0883	0.0544	0.0017	0.0013	38.4200
400	20	0.0913	0.0516	0.0019	0.0011	43.5368
400	30	0.0805	0.0545	0.0016	0.0012	32.2359
400	40	0.0909	0.0661	0.0016	0.0014	27.2762
500	10	0.0831	0.0439	0.0017	0.0011	47.2228
500	20	0.0757	0.0470	0.0015	0.0011	37.9654
500	30	0.0727	0.0455	0.0015	0.0010	37.3814
500	40	0.0753	0.0592	0.0014	0.0012	21.3773
600	10	0.0727	0.0416	0.0014	0.0010	42.8160
600	20	0.0671	0.0412	0.0013	0.0010	38.6796
600	30	0.0646	0.0474	0.0013	0.0011	26.6545
600	40	0.0721	0.0414	0.0014	0.0009	42.6536
700	10	0.0674	0.0349	0.0014	0.0009	48.2614
700	20	0.0627	0.0344	0.0013	0.0008	45.1832
700	30	0.0698	0.0359	0.0013	0.0008	48.5508
700	40	0.0615	0.0401	0.0012	0.0009	34.7529

Table 2 (cont.). Experimental results - Uniform Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
800	10	0.0579	0.0382	0.0013	0.0009	34.0963
800	20	0.0589	0.0306	0.0011	0.0007	48.0392
800	30	0.0555	0.0312	0.0010	0.0007	43.7571
800	40	0.0589	0.0353	0.0011	0.0008	40.1509

Table 3. Experimental results - Positive Linear Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
300	10	0.0909	0.0599	0.0020	0.0014	34.1019
300	20	0.0787	0.0568	0.0016	0.0013	27.8308
300	30	0.0815	0.0584	0.0016	0.0012	28.3393
300	40	0.0801	0.0542	0.0016	0.0012	32.3999
400	10	0.0723	0.0513	0.0016	0.0012	29.0390
400	20	0.0751	0.0431	0.0015	0.0010	42.5667
400	30	0.0615	0.0390	0.0011	0.0008	36.6554
400	40	0.0630	0.0418	0.0012	0.0010	33.7081
500	10	0.0641	0.0431	0.0014	0.0011	32.7232
500	20	0.0676	0.0392	0.0013	0.0009	41.9713
500	30	0.0581	0.0408	0.0012	0.0009	29.7391
500	40	0.0650	0.0318	0.0012	0.0007	51.0430
600	10	0.0651	0.0361	0.0014	0.0009	44.5162
600	20	0.0634	0.0340	0.0012	0.0008	46.3306
600	30	0.0507	0.0363	0.0010	0.0008	28.4212
600	40	0.0533	0.0300	0.0010	0.0007	43.6218
700	10	0.0567	0.0320	0.0012	0.0008	43.5140
700	20	0.0502	0.0311	0.0011	0.0007	38.1106
700	30	0.0451	0.0277	0.0009	0.0006	38.5470
700	40	0.0448	0.0281	0.0008	0.0006	37.1593
800	10	0.0607	0.0312	0.0012	0.0008	48.6142
800	20	0.0510	0.0265	0.0010	0.0006	48.0716
800	30	0.0474	0.0217	0.0009	0.0005	54.3399
800	40	0.0406	0.0247	0.0007	0.0005	39.2330

Table 4. Experimental results - Negative Linear Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
300	10	0.1242	0.0744	0.0026	0.0017	40.0813
300	20	0.1118	0.0720	0.0022	0.0016	35.6125
300	30	0.1132	0.0787	0.0021	0.0018	30.4638
300	40	0.1127	0.0819	0.0021	0.0017	27.2850

Table 4 (cont.). Experimental results - Negative Linear Distribution

n	D	Error of OA	Error of NA	Std of OA	Std of NA	% Improvement
400	10	0.0998	0.0550	0.0021	0.0014	44.8716
400	20	0.1063	0.0525	0.0021	0.0012	50.5669
400	30	0.0880	0.0610	0.0017	0.0013	30.6426
400	40	0.0999	0.0637	0.0017	0.0015	36.2614
500	10	0.0864	0.0418	0.0017	0.0010	51.5902
500	20	0.0761	0.0493	0.0016	0.0011	35.1628
500	30	0.0748	0.0560	0.0014	0.0012	25.1616
500	40	0.0804	0.0651	0.0015	0.0014	19.1128
600	10	0.0807	0.0449	0.0017	0.0011	44.4261
600	20	0.0805	0.0431	0.0016	0.0011	46.5258
600	30	0.0851	0.0427	0.0016	0.0010	49.8085
600	40	0.0754	0.0524	0.0014	0.0011	30.5661
700	10	0.0792	0.0419	0.0016	0.0010	47.0908
700	20	0.0670	0.0375	0.0013	0.0009	44.1186
700	30	0.0826	0.0395	0.0015	0.0010	52.2542
700	40	0.0712	0.0456	0.0015	0.0010	35.9916
800	10	0.0640	0.0398	0.0013	0.0010	37.7042
800	20	0.0582	0.0364	0.0012	0.0008	37.4284
800	30	0.0574	0.0373	0.0011	0.0008	34.9051
800	40	0.0627	0.0382	0.0011	0.0009	39.1360

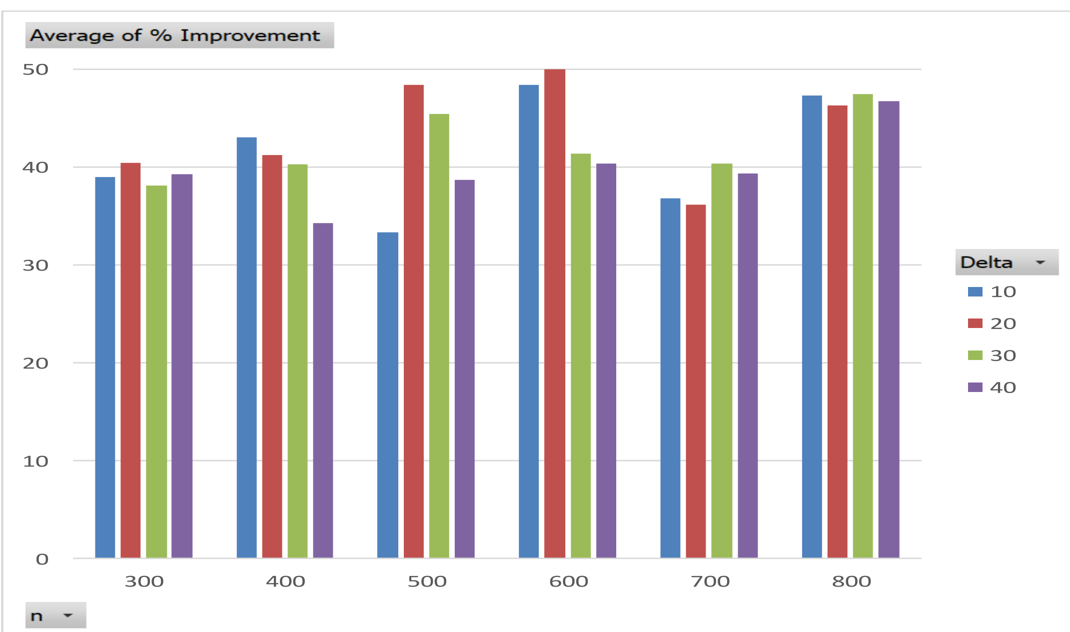

Figure 1. Percentage improvement of NA over OA – Normal Distribution

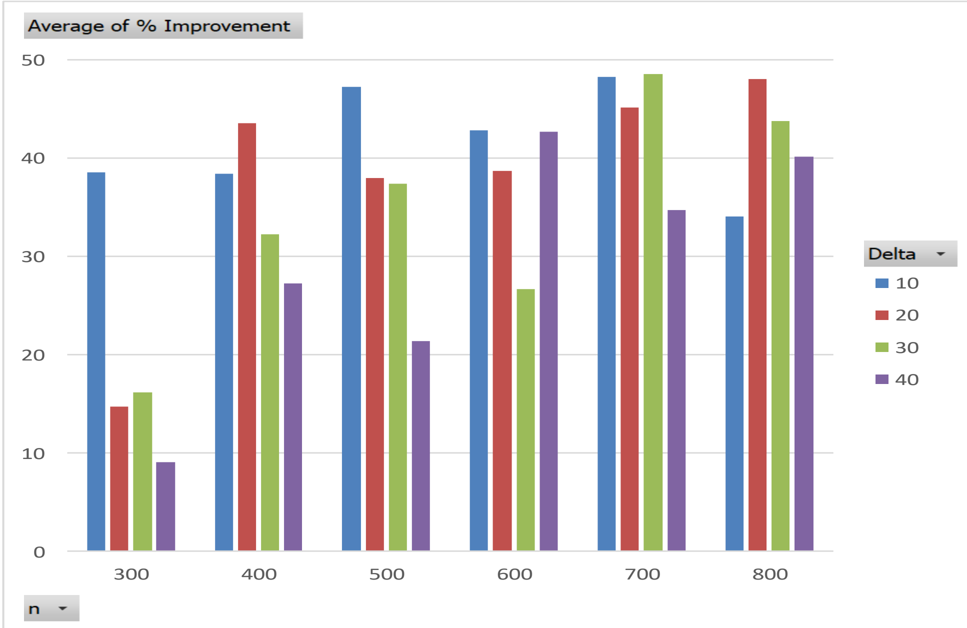

Figure 2. Percentage improvement of NA over OA – Uniform Distribution

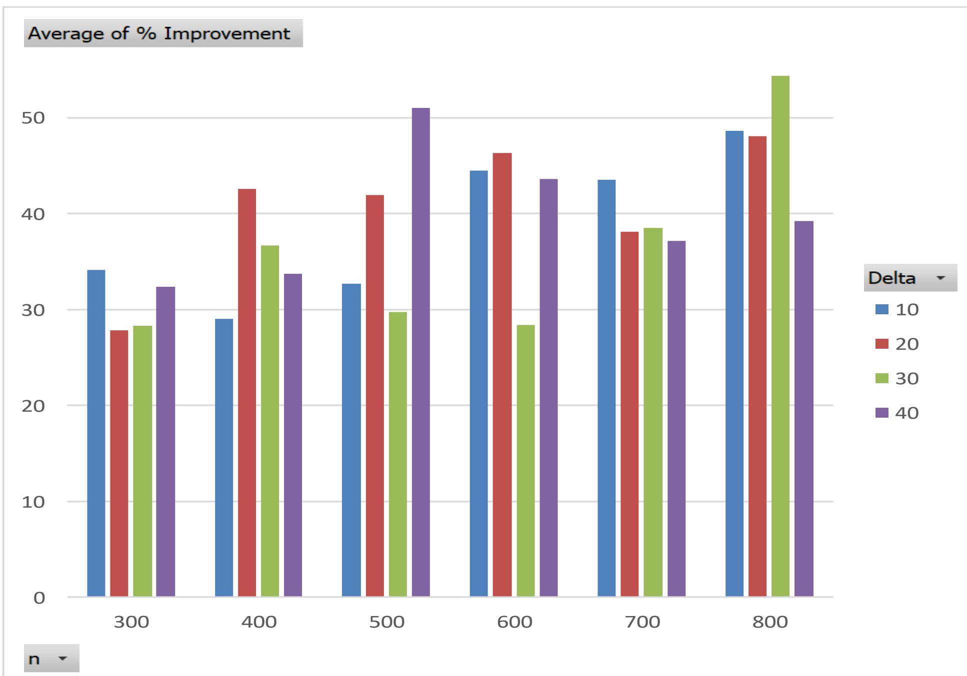

Figure 3. Percentage improvement of NA over OA – Positive Linear Distribution

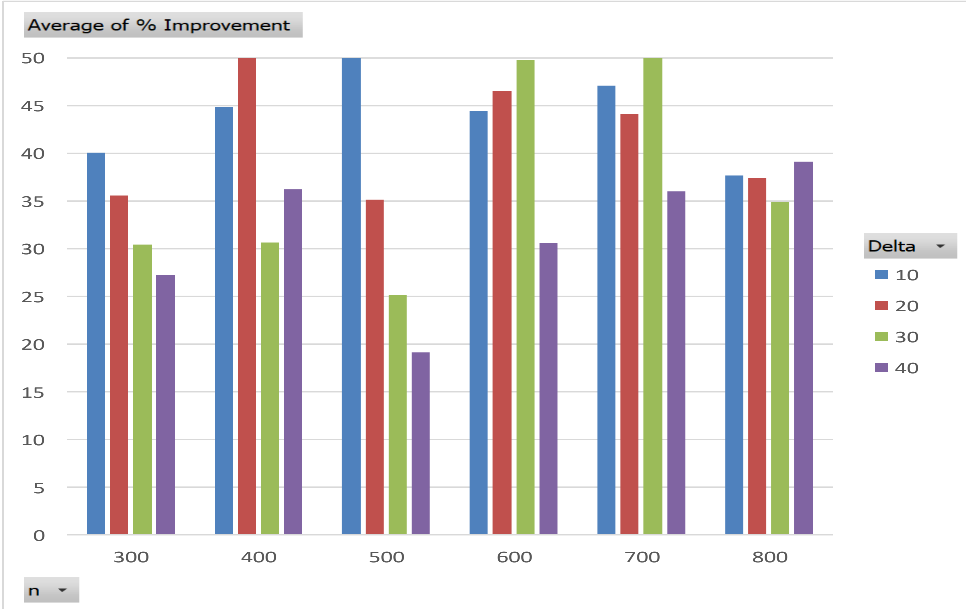

Figure 4. Percentage improvement of NA over OA – Negative Linear Distribution

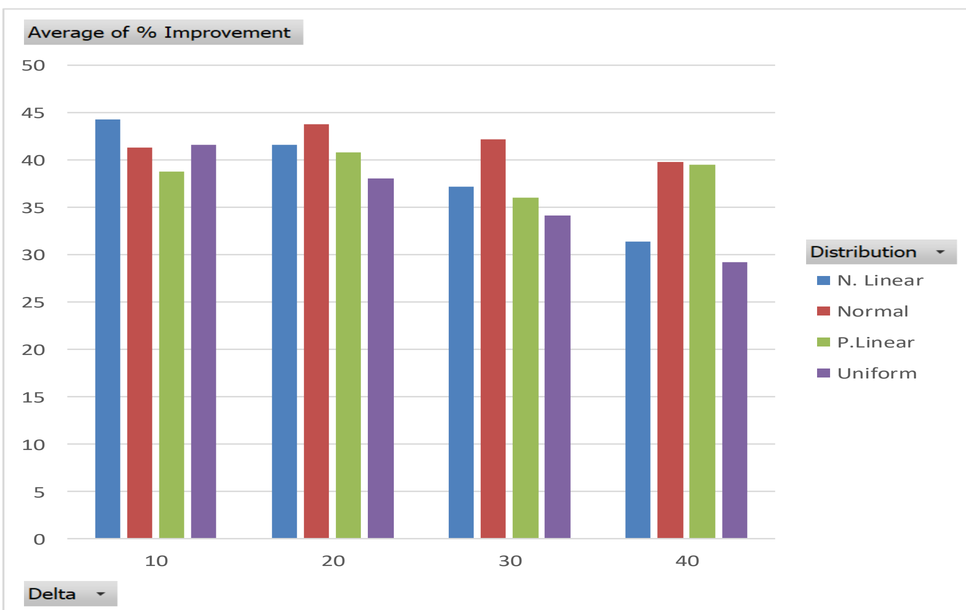

Figure 5. Percentage improvement of NA over OA over Distributions and Delta

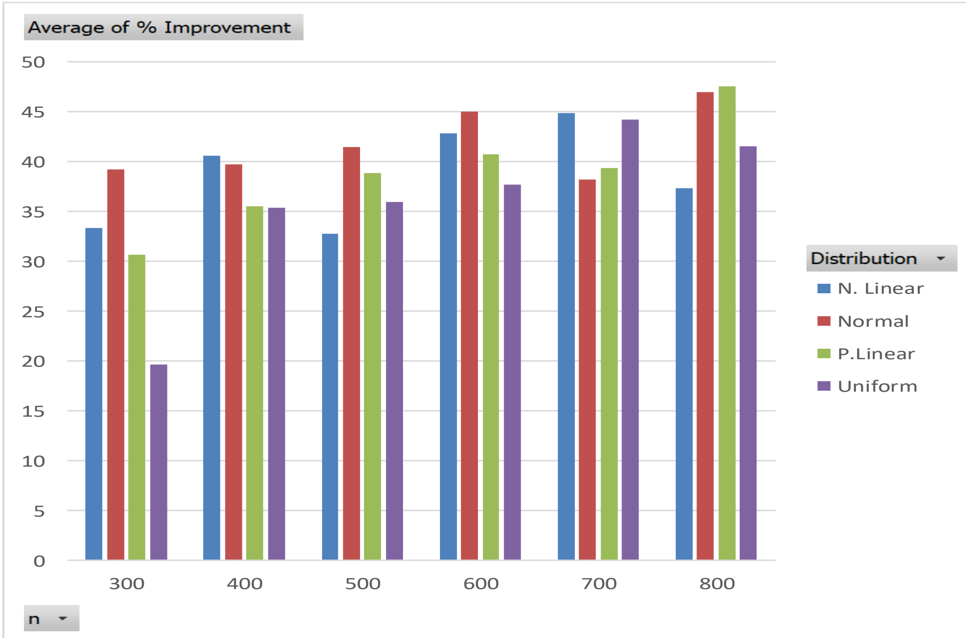

Figure 6. Percentage improvement of NA over OA over Distributions and Number of Jobs

4. Conclusion

The flowshop scheduling problem with four-machine where processing times are uncertain is addressed. The objective is to minimize makespan. This problem was earlier investigated in the scheduling literature and several algorithms were presented. It was shown that the algorithm OA in the literature was the best. In the current paper, we propose a new algorithm (NW). We show that the proposed new algorithm NW significantly reduces the error of the best existing algorithm OA. In other words, the algorithm NW reduces the error of the best existing algorithm OA about 40%. It should be noted that both algorithms OA and NW have the same computational times. This result was statistically verified by conducting test of hypothesis with a significance level of 0.01. Therefore, the newly proposed algorithm NW is recommended.

One of the assumptions made in this paper is that there are no setup times. This may be true for majority of manufacturing systems while it may not be appropriate for some other manufacturing systems. Thus, an extension of the addressed problem is to consider the flowshop scheduling problem with four-machine for minimizing makespan where setup times are separate from processing times and processing times are uncertain. Another extension is to investigate the considered problem with a due date related performance measure such as total tardiness or number of tardy jobs.

References

Allahverdi, A., 2001. The tricriteria two-machine flowshop scheduling problem. International Transactions in Operational Research 8, 403-425.

Allahverdi, A., 2004. A new heuristic for m-machine flowshop scheduling problem with bicriteria of makespan and maximum tardiness. Computers & Operations Research 31, 157-180.

Allahverdi, A., Allahverdi, M., 2018. Two-machine no-wait flowshop scheduling problem with uncertain setup times to minimize maximum lateness. Computational and Applied Mathematics 37, 6774-679.

Allahverdi, M., Allahverdi, A., 2020. Algorithms for Four-machine flowshop scheduling problem with uncertain processing times to minimize makespan. RAIRO Operations Research 54, 529-553.

Allahverdi, A., Aydilek, H., 2010a. Heuristics for two-machine flowshop scheduling problem to minimize makespan with bounded processing times. International Journal of Production Research 48, 6367- 6385.

Allahverdi, A., and Aydilek, H., 2010b. Heuristics for two-machine flowshop scheduling problem to minimize maximum lateness with bounded processing times. Computers and Mathematics with Applications 60, 1374-1384.

Allahverdi, A., and Sotskov, Y.N., 2003. Two-machine flowshop minimum length scheduling problem with random and bounded processing times. International Transactions in Operational Research 10, 65-76.

Aydilek, H., and Allahverdi, A., 2010. Two-machine flowshop scheduling problem with bounded processing times to minimize total completion time. Computers and Mathematics with Applications 59, 684-693.

Aydilek, A., Aydilek, H., Allahverdi, A., 2013. Increasing the profitability and competitiveness in a production environment with random and bounded setup times. International Journal of Production Research 51, 106- 117.

Aydilek, A., Aydilek, H., Allahverdi, A., 2015. Production in a two-machine flowshop scheduling environment with uncertain processing and setup times to minimize makespan. International Journal of Production Research 53, 2803-2819.

Aydilek, A., Aydilek, H., Allahverdi, A., 2017. Algorithms for minimizing the number of tardy jobs for reducing production cost with uncertain processing times. Applied Mathematical Modelling 45, 982-996.

Dumaine, B., 1989. How managers can succeed through speed. Fortune 12, 54-59.

Fuchigami, H.Y., Rangel, S., 2018. A survey of case studies in production scheduling: Analysis and perspectives. Journal of Computational Science 25, 425-436.

Garey, M.R., Johnson, D.S., Sethi, R., 1976. The complexity of flowshop and jobshop scheduling. Mathematics of Operations Research 1, 117-129.

Gonzalez-Neira, E.M., Ferone, D., Hatami, S., Juan, A.A. 2017. A biased-randomized simheuristic for the distributed assembly permutation flowshop problem with stochastic processing times. Simulation Modelling Practice and Theory 79, 23-36.

Kouvelis, P., Yu, G. 1997. Robust Discrete Optimization and its Applications, Kluwer Academic Publisher.

Mahjoub, A., Sanchez, J. E. P. 2011. Trystram, D. Scheduling with uncertainties on new computing platforms. Computational Optimization and Applications 48, 369-398.

Sotskov, Y.N., Allahverdi, A., Lai, T.C., 2004. Flow shop scheduling problem to minimize total completion time with random and bounded processing times. Journal of Operational Research Society 55, 277-286.

Stefansson, H., Sigmarsdottir, S., Jensson, P., Shah, N., 2011. Discrete and continuous time representations and mathematical models for large productions scheduling problems: a case study from the pharmaceutical industry. European Journal of Operational Research 215, 383-392.