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THE VARIATION OF THE EXTRAPOLATED ENDPOINT FROM BACKWARD TO FORWARD SCATTERING FOR SPECIFIED ISOTROPIC SCATTERING

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ABSTRACT

The Milne problem is studied for one speed, time independent and plane symmetric linear neutron transport equation. The variation of the extrapolated endpoint from backward to forward scattering for specified isotropic scattering is obtained. The zeroth and first order expressions for the critical slab thickness which have been derived previously are expressed in terms of the extrapolated endpoint. Hence, the numerical values af both extrapolated endpoint and the critical slab thickness are verified.

INTRODUCTION

The expression for the extrapolated endpoint for the FBIS (Forward-Backward-Isotropic-Scattering) model, using the method of elementary solutions, have been obtained as (Tezcan and Sever, 1986).

$$cZ_0 = \frac{c\nu_0}{2} \ln \left[\frac{B + A \exp\left(-\frac{2Z'_0}{\nu'_0}\right) - B\nu'_0 (1-c') \cdot \exp\left(-\frac{2Z'_0}{\nu'_0}\right) I}{-B\exp\left(-\frac{2Z'_0}{\nu'_0}\right) - A + \frac{B^2}{A} \nu'_0 c'(1-c') \cdot \exp\left(-\frac{2Z'_0}{\nu'_0}\right) I} \right], \quad c<1 \tag{1}$$

and

$$cZ_{oa} = c\nu_o \tan^{-1}\left[(A\text{-}B)\tan\frac{Z'_o}{\nu_o'}\right], \qquad (2)$$

$$c > 1$$

$$cZ_{ob} = c\nu_o \tan^{-1}\left[(A-B)\,\frac{2\tan\frac{Z_o'}{\nu_o'} - \gamma}{\gamma\tan\frac{Z_o'}{\nu_o'} + 2}\right] \qquad (3)$$

where

$$\gamma = \frac{B}{A}\,\nu'_o c'\,(1-c')\,I,$$

$$I = \int_o^1 \frac{\nu' X'^2(-\nu)\,d\nu'}{(1-c'\nu'\tanh^{-1}\nu')^2 + \left(\frac{c'\pi\nu'}{2}\right)^2}$$

$$c' = \frac{c\,(1-\alpha-\beta)}{1-\alpha\,c-c\beta}, \qquad \nu'_o = q\nu_o, \qquad (4)$$

$$q = [(1-\alpha c)^2 - c^2\beta^2]^{1/2},$$

$$\left\{\begin{array}{c} A \\ B \end{array}\right\} = \frac{1}{2}\left[1 \pm \left(\frac{1-\alpha c-c\beta}{1-\alpha c+c\beta}\right)^{1/2}\right]. \qquad (5)$$

To study the variation of the extrapolated endpoint or the critical slab thickness from backward to forward scattering for specified isotropic scattering, the following parameters α and β should be replaced by

$$\alpha = k + \gamma, \qquad \beta = k - \gamma. \qquad (6)$$

The zeroth and first order expressions for the critical slab thickness have been obtained as (Tezcan and Sever, 1985).

$$t_o = \frac{\nu'_o}{q}\tan^{-1}\left[\frac{1}{A\text{-}B}\tan\left(\frac{\pi}{2} - \frac{Z'_o}{\nu'_o}\right)\right], \qquad (7)$$

$$t_1 = \frac{\nu'_o}{q}\tan^{-1}\left[\frac{1}{A\text{-}B}\tan\left(\frac{\pi}{2} - \frac{Z'_o}{\nu'_o}\tan^{-1}\nu'_o\,n\,(c', a)\right)\right] \qquad (8)$$

where

$$n(c', a) = \frac{(c'/2)(c'-1) I_1}{1-(c'/2)(c'-1) I_2}, \tag{9}$$

$$I_j = \int_0^1 \frac{\nu'^{j+1} x'^2(-\nu)}{N(\nu')} \frac{A\exp\left(-\frac{qa}{\nu'}\right) + B\exp\left(+\frac{qa}{\nu'}\right)}{A\exp\left(+\frac{qa}{\nu'}\right) + B\exp\left(-\frac{qa}{\nu'}\right)} \tag{10}$$

$$N(\nu') = \frac{\nu'}{g'(c, \nu)},$$

$$g'(c,\nu) = g(c'\nu') = \frac{1}{(1-c'\nu'\tanh^{-1}\nu')^2 + \left(\frac{c'\pi\nu'}{2}\right)^2}, \tag{11}$$

$$x'(-\nu) = \exp\left[-\frac{c'}{2}\int_0^1 g'(c, \nu)\left(1+\frac{c'\mu^2}{1-\mu^2}\right)\ln(\mu+\nu')\,d\mu\right]. \tag{12}$$

THE CRITICALITY EXPRESSIONS

The time independent linear transport equation for the FBIS model is

$$\mu \frac{\partial \psi}{\partial x} + \psi(x, \mu) = \frac{c(1-\alpha-\beta)}{2} \int \psi(x, \mu')\,d\mu' + \alpha c\,\psi(x, \mu) + c\beta\,\psi(x, -\mu) \tag{13}$$

Using the change of variables (İnönü 1973), equation (5) and

$$\psi'(x', \mu) = \frac{1}{A^2-B^2}\left[A\,\psi(x, \mu) - B\,\psi(x, -\mu)\right], \tag{14}$$

Equation (13) turns to be

$$\mu \frac{\partial \psi'(x', \mu)}{\partial x'} + \psi'(x', \mu) = \frac{c'}{2}\int_{-1}^{1} \psi'(x', \mu')\,d\mu'. \tag{15}$$

Combining the two expressions given in equations (7) and (2) for the linear transport equation (15), the zeroth order expression for the linear transport equation (13) can be written as

$$t_{oa} = \tilde{n}\nu_o - 2\, Z_{oa}, \tag{16}$$

where ν_o and Z_{oa} are the eigenvalues and the extrapolated enpoint of equation (13) respectively. Similarly when equation (7) and equation (3), equation (8) and equations (2) and (3), are expressed in terms of unprimed variables of the linear transport equation (13), the following expressions may be obtained

$$t_{ob} = 2\nu_o \tan^{-1}\left[\frac{1}{A\text{-}B}\;\frac{2(A\text{-}B) - \gamma\tan\dfrac{Z_{ob}}{\nu_o}}{2\tan\dfrac{Z_{ob}}{\nu_o} + (A\text{-}B)\,\gamma}\right], \tag{17}$$

$$t_{1a} = 2\nu_o\tan^{-1}\left\{\frac{1}{A\text{-}B}\tan\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{A\text{-}B}\tan\frac{Z_{oa}}{\nu_o}\right) - \tan^{-1} q\nu_o n\right]\right\}, \tag{18}$$

$$t_{1b} = 2\nu_o\tan^{-1}\left\{\frac{1}{A\text{-}B}\tan\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2\tan\dfrac{Z_{ob}}{\nu_o} + (A\text{-}B)\gamma}{2(A\text{-}B) - \gamma\tan\dfrac{Z_{ob}}{\nu_o}}\right) - \tan^{-1} q\nu_o n\right]\right\}. \tag{19}$$

The numerical values of the critical slab thickness, t_o and t_1 calculated from the zeroth and first order approximate expressions are in good agreement with the previous numerical results. This justifies: i) the numerical values of the extrapolated endpoint for the FBIS (Forward-Backward-Isotropic-Scattering) model ii) the numerical values of the critical slab thickness for the FBS (Forward-Backward-Scattering for specified isotropic scatering) model which can be obtained replacing α and β in all of the expressions by $k + \gamma$ and $k - \gamma$respectively iii) the values of the extrapolated endpoint calculated in this work for $c<1$ as γ assumes values in the range $-k < \gamma < k$. The increase in the extrapolated endpoint from backward to forward scattering, for specified isotropic scattering comes from the increase in the number of neutrons in a finite region.

Table 1 The extrapoldted endpoint cZ_0 vs γ for $c < 1$

γ	c = 0.3, c′ = 0.1	c = 0.7, c′ = 0.1	c = 0.9, c′ = 0.1
-k	0.336	0.452	0.486
-k / 2	0.391	0.577	0.640
O	0.471	0.789	0.936
k / 2	0.613	1.309	1.743
3k / 4	0.757	1.965	3.079
5k / 6	0.839	2.395	4.154
9k / 10	0.942	2.961	5.811
19k / 20	1.080	3.741	8.482
k	3.294	17.93	69.17

Table 2 The extrapolated endpoint cZoa, cZob vs γ for $c > 1$

γ	c = 1.1	c′ = 2.0	c = 1.2	c′ = 2.0
	cZoa	cZob	cZoa	cZob
–k	0.268	0.293	0.302	0.323
-k / 2	0.349	0.379	0.384	0.408
O	0.500	0.538	0.527	0.554
k / 2	0.883	0.931	0.842	0.869
3k / 4	1.435	1.486	1.204	1.227
5k / 6	1.816	1.863	1.408	1.427
9k / 1	2.308	2.348	1.630	1.644
19k / 20	2.903	2.930	1.852	1.860
k	3.933	3.933	2.145	2.145

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