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MOLAR MOLECULAR HEAT AND MASS TRANSFER IN A CAPILLARY POROUS BODY OF SPHERICAL GEOMETRY

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ABSTRACT

A boundary value problem governing the process of molar-molecular heat- and mass transfer in a capillary porous body of spherical geometry in presence of heat and mass sources and sinks under tbe generalized boundary conditions is transferred into the three boundary value problems for tbe heat conduction equations. Using the Laplace transform, the boundary value problems of partial differential equations are converted into boundary value problems af ordinary differential equations and they are solved by Galerkin's method. As an illustration the problem af molar molecular heat and mass transfer under specific boundary condition of the third kind is solved and discussed in detail.

NOMENCLATURE

x	dimensionless space coordinate
Fo	Fourier number
ϵ	phase change criterion
Pn	Posnov number
Ko	Kossovich number
Lu	Luikov number
Bu	Bulygin number
B_i	Blot number
θ_1 (x, Fo)	dimensionless temperature
θ_2 (x, Fo)	dimensionless potential for mass transfer
θ_3 (x, Fo)	dimensionless potential for filtration transfer

Subscript

m	matter
q	heat

p	filtration
i	assuming three values: i = 1,2,3
j	assuming n values: j = 1,2,3 ... n
l	assuming two values: l = 1,2

INTRODUCTION

Molar-molecular heat and mass transfer taking place in a capillary porous body have wide applications in separation processes, food technology, space engineering etc. [1,2,3]. Mathematical model describing this process was first established by Luikov [4,5,6]. The differential equations governing this process in a spherical porous body in presence of heat and mass sources and sinks in dimensionless form may be expressed as follows:

$$\frac{\partial\theta_1}{\partial Fo} = \frac{\partial^2\theta_1}{\partial x^2} + \frac{2}{x}\frac{\partial\theta_1}{\partial x} - \epsilon K_o \frac{\partial\theta_2}{\partial Fo} + w_1(x, Fo),$$

$$\frac{\partial\theta_2}{\partial Fo} = Lu_m\left[\frac{\partial^2\theta_2}{\partial x^2} + \frac{2}{x}\frac{\partial\theta_2}{\partial x}\right] - Lu_m Pn\left[\frac{\partial^2\theta_1}{\partial x^2} + \frac{2}{x}\frac{\partial\theta_1}{\partial x}\right]$$

$$- \frac{Lu_p Bu}{Ko}\left[\frac{\partial^2\theta_3}{\partial x^2} + \frac{2}{x}\frac{\partial\theta_3}{\partial x}\right] + w_2(x, Fo),$$

$$\frac{\partial\theta_3}{\partial Fo} = Lu_p\left[\frac{\partial^2\theta_3}{\partial x^2} + \frac{2}{x}\frac{\partial\theta_3}{\partial x}\right] + \frac{\epsilon Ko}{Bu}\frac{\partial\theta_2}{\partial Fo} + w_3(x, Fo), \quad (1)$$

where $w_i(x, Fo)$ $(i = 1,2,3)$ are the functions accounting for the heat and mass sources and sinks. System (1) may be considered subjects to the initial conditions

$$\theta_i(x, o) = f_i(x), \quad (2)$$

symmetry conditions

$$\frac{\partial\theta_i}{\partial x}(o, Fo) = 0 \quad (3)$$

and the generalized boundary conditions

$$A_o \frac{\partial\theta_1}{\partial x}(1, Fo) + A_1\theta_1(1, Fo) + A_2\theta_2(1, Fo) = g_1(Fo),$$

$$B_o \frac{\partial \theta_2}{\partial x}(1, Fo) + B_1 \frac{\partial \theta_1}{\partial x}(1, Fo) + B_2 \frac{\partial \theta_3}{\partial x}(1, Fo) + B_3\theta_2 = g_2(F_o)$$

$$C_o \frac{\partial \theta_3}{\partial x}(1, Fo) + C_1\theta_3(1, Fo) = g_3(Fo), \qquad (4)$$

where A_o, A_1, A_2, B_o, B_1, B_2, B_3, C_1 and C_o are aggregate of known dimensionless thermophysical coefficients and g_i (Fo) are prescribed fluxes to be determined by the experiment.

For many types of boundary conditions, there exists analytical solution of the equations (1) expressed in terms of infinite series [7], the use of which in practice is found to be difficult in a number of cases. Therefore, the use of approximate analytical solution are necessary. The approximate analytical solution is also required in practical engineering where the general validity of a solution is not as important as a fast and simple treatment of the problem.

In this paper the author has given an approximate analytical solution of the system of equations (1) — (4). The method of solution consists of the following steps:

(i) transformation of the boundary-value problem into three boundary value problems for the heat conduction equations;

(ii) use of Laplace transform on the transformed equations to convert them into the boundary value problems of ordinary differential equations,

(iii) use of Galerkin's method to solve them.

This gives a simpler and inexpensive approximate solution which is closed form and analytically fully-determined. As an illustration the problem of molar-molecular heat and mass transfer under specific boundary conditions of the third kind is solved. Tables 1-9 show the influence of dimensionless variables x, Fo internal criteria ϵ, Ko, Pn, Lu_m, Lu_p, Bu and surface criteria Bi_m, Bi_q on dimensional transfer potentials. Numerical results have been obtained using a computer [8].

SOLUTION

To obtain approximate solution, the system of equations (1) may be written in an equivalent form of the system of three nonconnected parabolic type equations with respect to combined potentials.

$$z_i(x, Fo) = D_i\theta_1(x, Fo) + E_i\theta_2(x, Fo) + F_i\theta_3(x, Fo), \qquad (5)$$

where the constant coefficient of the right hand side of (5) are of the form

$$D_1 = 1,\ E_1 = \frac{v^2_1 - 1}{Pn}, \quad F_1 = \frac{(v^2_1 - 1)\ Lu_p Bu}{Lu_m KoPn\ (v^2_1\ Lu_p - 1)}$$

$$D_2 = \frac{Pn}{v^2_2 - 1},\ E_2 = 1, \quad F_2 = \frac{Lu_p Bu}{(v^2_2 Lu_p - 1)\ Lu_m Ko},$$

$$D_3 = \frac{(v^2_3 Lu_p - 1)\ KoPnLu_m}{(v^2_3 - 1) Lu_p Bu},\ E_3 = \frac{(v^2_3 Lu_p - 1)\ KoLu_m}{Lu_p Bu},\ F_3 = 1.$$

Here v_i are the roots of the equation

$$v^6 - \beta_1 v^4 + \beta_2 v^2 - \beta_3 = 0,$$

where

$$\beta_1 = 1 + \epsilon\ KoPn + \frac{1-\epsilon}{Lu_m} + 1 / Lu_p,$$

$$\beta_2 = (1-\epsilon) / Lu_m + (1 + \epsilon\ KoPn + 1 / Lu_m) / Lu_p,$$

$$\beta_3 = 1 / Lu_m Lu_p.$$

Substitution of (5) transforms the system of equations (1) into three non-connected equations of the heat conduction type

$$\frac{\partial z_i}{\partial Fo} = k_i \left[\frac{\partial^2 z_i}{\partial x^2} + \frac{2}{x} \frac{\partial z_i}{\partial x} \right] + M_i(x, Fo) \qquad (6)$$

where

$$k_i = 1 / v_i^2,$$

$$M_i(x,Fo) = k_i \left[D_i w_1 + E_i \left(Pnw_1 + \frac{w_2}{Lu_m} + \frac{Buw_3}{Lu_m Ko} \right) + \frac{w_3}{Lu_p} F_i \right].$$

The initial, symmetry and boundary conditions acquire the form

$$z_i(x, o) = \psi_i(x), \qquad (7)$$

$$\frac{\partial z_i}{\partial x}(o, Fo) = o \qquad (8)$$

and

$$\frac{\partial z_i}{\partial x} (1, Fo) + h_i z_i (1, Fo) = \varnothing_i (Fo) \qquad (9)$$

respectively. Here h_i are the roots of the equation

$$\begin{vmatrix} B_o(A_1-A_oh_i) & -A_1B_1 & 0 \\ Z_2B_o & -A_2B_1+A_o(B_3-B_oh_i) & 0 \\ 0 & -C_1B_2 & B_o(C_1-C_oh_i) \end{vmatrix} = 0,$$

$$\psi_i (x) = D_i f_1 (x) + E_i f_2 (x) + F_i f_3(x)$$

and

$$\varnothing_i (Fo) = \frac{1}{A_o} (D_i - \frac{B_1}{B_o} E_i) g_1(Fo) + \frac{E_i}{B_o} g_2(Fo)$$

$$+ \frac{1}{C_o} (F_i - \frac{B_2}{B_o} E_i) g_3(Fo).$$

With the solutions, $z_i (x, Fo)$ of the system of equations (6)-(9) konwn, the transfer potentials $\theta_i (x, Fo)$ are expressed in terms of these solutions as:

$$\theta_i (x, Fo) = \frac{\triangle\theta_i}{\triangle}, \qquad (10)$$

where

$$\triangle = \begin{vmatrix} 1 & E_1 & F_1 \\ D_2 & 1 & F_2 \\ D_3 & E_3 & 1 \end{vmatrix} \neq 0,$$

$$\triangle\theta_1 = \begin{vmatrix} z_1 & E_1 & F_1 \\ z_2 & 1 & F_2 \\ z_3 & E_3 & 1 \end{vmatrix},$$

$$\triangle\theta_2 = \begin{vmatrix} 1 & z_1 & F_1 \\ D_2 & z_2 & F_2 \\ D_3 & z_3 & 1 \end{vmatrix},$$

$$\triangle\theta_3 = \begin{vmatrix} 1 & E_1 & z_1 \\ D_2 & 1 & z_2 \\ D_3 & E_3 & z_3 \end{vmatrix},$$

Introducing te Laplace transform

$$\bar{z}_i (x,s) = \int_0^\infty z_i (x, Fo) \exp (-sFo)\, dFo \qquad (11)$$

and reducing the system of equations (6)—(9) to:

$$\left(\frac{d^2}{dx^2} + \frac{2}{x} \frac{d}{k_i} - \frac{s}{k_i}\right) \bar{z}_i (x,s) = - \frac{1}{k_i} (\psi_i (x) + \bar{M}_i (x,s)) \quad (12)$$

$$\frac{d\bar{z}_i}{dx} (o,s) = o, \quad (13)$$

$$\frac{d\bar{z}_i}{dx} (1,s) + h_i \bar{z}_i (1, s) = \bar{\varnothing}_i(s) \quad (14)$$

Introducing new independent variable by the relation

$$\bar{y}_i (x, s) = \bar{z}_i (x, s) - \frac{1}{h_i} \bar{\varnothing}_i (s) \quad (15)$$

Using (15) equations (12)—(14) become

$$\left(\frac{d^2}{dx^2} + \frac{2}{x} \frac{d}{dx} - \frac{s}{k_i}\right) \bar{y}_i (x, s) = - \frac{1}{k_i} (\psi_i(x) + \bar{M}_i (x, s)) + \frac{s}{h_1 k_i} \bar{\varnothing}_i (s), \quad (16)$$

$$\frac{d\bar{y}_i}{dx} (o, s) = 0, \quad (17)$$

$$\frac{d\bar{y}_i}{dx} (1, s) + h_i \bar{y}_i (1, s) = 0. \quad (18)$$

Using the Galerkin's method [9] which makes it possible to find the approximate solution of the system of equations (16)—(18) in the form of an analytic expression. Let nth order approximation for $\bar{y}_i (x, s)$ be denoted by $\bar{y}_{in}(x, s)$. Then, in the present case

$$\bar{y}_{in}(x, s) = \sum_{j=1}^{n} \bar{C}_{ij}(s) u_{ij}(x), \quad (19)$$

where

$$u_{ij}(x) = 1 - \frac{h_i}{2j + h_i} x^{2j} \quad (20)$$

The coefficients $\overline{C}_{ij}(s)$ ($j = 1,2, \dots n$) are obtained if we solve the following system of linear algebraic equations

$$\sum_{j=1}^{n} \overline{C}_{ij}(s) \int_0^1 x^2 u_{ik}(x) L [u_{ij}(x)] dx = \int_0^1 x^2 u_{ik}(x) \Big[- \frac{1}{k_i} (\psi_i(x)$$

$$+ \overline{M}_i (x, s) - \frac{s}{h_i} \overline{\varnothing}_i (s) \Big] dx \qquad k = 1,2,3, \dots n \tag{21}$$

Thus, $\bar{y}_{in}(x, s)$ is determined and hence $\bar{z}_{in}(x, s)$. Then z_{in} (x, Fo), which denotes inverse Laplace transform of $\bar{z}_{in}$ (x, s), is obtained by the application of Heaviside theorm. Obviously, z_{in} (x, Fo) is the nth order approximation of $z_i(x, Fo)$ and is given by

$$z_{in}(x, Fo) = \frac{\varnothing_i (Fo)}{h_i} + \sum_{j=1}^{n} C_{ij}(Fo) \Big[1 - \frac{h_i}{2j + h_i} x^{2j} \Big] \tag{22}$$

To save space, we present only the first approximation $z_{i1}(x, Fo)$ as follows:

$$z_{i1} (x, Fo) = \frac{\varnothing_i (Fo)}{h_i} + C_{i1} (Fo) \Big[1 - \frac{h_i}{2 + h_i} x^2 \Big] , \tag{23}$$

where

$$C_{i1} (Fo) = \frac{R_{io}}{Q_{io}} \varnothing_i(Fo) - R_{io}P_{io} \int_0^{Fo} \varnothing_i (Fo-u)$$

$$\times \exp \left(- \frac{P_{io}}{Q_{io}} u \right) du + \frac{1}{Q_{io}} \exp \left(- \frac{P_{io}}{Q_{io}} Fo \right)$$

$$\times \int_0^1 x^2 \left(\frac{h_i}{2 + h_i} x^2 - 1 \right) \psi_i(x) dx$$

$$+ \int_0^1 x^2 \left(\frac{h_i}{2 + h_i} x^2 - 1 \right) \Big\{ \frac{1}{Q_{io}} \int_0^{Fo} M_i (x, Fo-u)$$

$$\times \exp \left(- \frac{P_{io}}{Q_{io}} u \right) du \Big\} dx,$$

$$P_{io} = \left(\frac{2h_i}{2 + h_i} \right) \Big[-1 + \frac{3}{5} \left(\frac{h_i}{2 + h_i} \right) \Big] k_i,$$

$$Q_{io} = -\frac{1}{3} + \left(\frac{h_i}{2+h_i}\right)\left[\frac{2}{5} - \frac{1}{7}\left(\frac{h_i}{2+h_i}\right)\right],$$

$$R_{io} = \left(\frac{1}{h_i}\right)\left[\frac{1}{3} - \frac{1}{4}\left(\frac{h_i}{2+h_i}\right)\right].$$

A PARTICULAR CASE

Consider the molar molecular heat and mass transfer in a spherical porous body under boundary conditions of the third kind. The problem is symmetrical and the initial distributions of potentials throughout the material are uniform. In this case, we have

$$A_o = 1,\ A_1 = Bi_q,\quad A_2 = -(1-\epsilon)\, Lu_m Ko\ \ Bi_m.$$

$$B_o = -1,\ B_1 = Pn,\ B_2 = \frac{BuLu_p}{KoLu_m},\ B_3 = -Bi_m,$$

$$C_o = o,\ C_1 = 1,\quad g_1(Fo) = Bi_q - (1-\epsilon)\, Lu_m KoBi_m,$$

$$g_2(Fo) = -Bi_m,\ g_3(Fo) = o,\ f_i(x) = o,\ w_i(x, Fo) = o.$$

After necessary algebraic manipulations, solution (23) for $i = 1, 2$ takes the following form:

$$z_{11}(x, Fo) = \frac{\varnothing_1}{h_i} + \left(1 - \frac{h_1}{2+h_1}\, x^2\right) C_{11}(Fo), \tag{24}$$

$$z_{21}(x, Fo) = \frac{\varnothing_2}{h_2} + \left(1 - \frac{h_2}{2+h_2}\, x^2\right) C_{21}(Fo), \tag{25}$$

where

$$\varnothing_1 = [Bi_q - (1-\epsilon)\, Lu_m\, Ko\, Bi_m] + [Bi_m + (Bi_q - (1-\epsilon) \times Lu_m\, Ko\, Bi_m)\, Pn]\left(\frac{v^2{}_1 - 1}{Pn}\right),$$

$$\varnothing_2 = [Bi_q - (1-\epsilon)\, Lu_m\, Ko\, Bi_m]\,(Pn/(v^2{}_2 - 1)) + [Bi_m + (Bi_q - (1-\epsilon)\, Lu_m\, Ko\, Bi_m)\, Pn],$$

$$h_l = \tfrac{1}{2}\,[Bi_m + Bi_q - (1-\epsilon)\, Lu_m\, Ko\, Bi_m\, Pn + (-1)^l\, \{(Bi_m + Bi_q - (1-\epsilon)\, Lu_m Ko Bi_m Pn)^2 - 4\, Bi_m Bi_q\}^{1/2}],$$

$$C_{11}(Fo) = \frac{R_{10}\varnothing_1}{Q_{10}} \exp\left(-\frac{P_{10}}{Q_{10}} Fo\right),$$

$$C_{21}(Fo) = \frac{R_{20}\varnothing_2}{Q_{20}} \exp\left(-\frac{P_{20}}{Q_{20}} Fo\right),$$

$$P_{lo} = \left(\frac{2h_l}{2+h_l}\right)\left[-1 + \frac{3}{5}\left(\frac{h_l}{2+h_l}\right)\right] k_l,$$

$$Q_{lo} = -\frac{1}{3} + \left(\frac{h_l}{2+h_l}\right)\left[\frac{2}{5} - \frac{1}{7}\left(\frac{h_l}{2+h_l}\right)\right],$$

$$R_{lo} = \left(\frac{1}{h_l}\right)\left[\frac{1}{3} - \frac{1}{5}\left(\frac{h_l}{2+h_l}\right)\right].$$

To determine solution of the system of equations (12)—(14) for $i = 3$, we choose following as the system of basis functions:

$$u_{30}(x) = (x^3 - x^2)(\varnothing_3 / s)$$

$$u_{3j}(x) = x^{j+1}\left(x - \frac{j+2}{j+1}\right)$$

Repeating the same procedure as done in the general solution, we obtain

$$z_{3n}(x, Fo) = (x^3 - x^2)\varnothing_3 + \sum_{j=1}^{n} C_{3j}\, x^{j+1}\left(x - \frac{j+2}{j+1}\right) \tag{26}$$

To a first approximation

$$z_{31}(x, Fo) = (x^3 - x^2)\varnothing_3 + C_{31}(Fo)\, x^2\left(x - \tfrac{3}{2}\right), \tag{27}$$

where

$$\varnothing_3 = \left[\frac{(v_3^2 Lu_p - 1)\, KoPnLu_m}{(v_3^2 - 1)\, Lu_p Bu} + Pn\, \frac{(v_3^2 Lu_p - 1)\, KoLu_m}{Lu_p Bu}\right]$$

$$\times\ [Bi_q - (1-\epsilon)\, Lu_m Ko Bi_m] + \frac{(v_3^2 Lu_p - 1)\, KoLu_m Bi_m}{Lu_p Bu},$$

$$C_{31} = \frac{1}{3}\varnothing_3\left[17 - \frac{947}{58}\exp\left(-\frac{216}{145} k_3\right)\right].$$

The effect of variability of dimensionless variables x, Fo, internal criteria ϵ, Ko, Pn, Lu_m, Lu_p, Bu and surface criteria Bi_m, Bi_q over non-dimensional transfer potentials are given in Tables 1 to 9. On the basis of these tables we can propose simplified dimensionless equations for the description of molar and molecular heat and mass transfer. Such equations are:

$$\theta_1 = \theta_1\,(x, Fo, \in, Ko, Pn, Lu_m, Bi_m, Bi_q).$$

$$\theta_2 = \theta_2\,(x, Fo, \in, Ko, Pn, Lu_m, Bi_m, Bi_q),$$

$$\theta_3 = \theta_3\,(x, Fo, \in, Ko, Pn, Lu_m, Lu_p, Bu, Bi_m, Bi_q). \quad (28)$$

TABLE 1

$\in = 0.1$ $Lu_m = 0.3$ $Ko = 3$ $Pn = 0.25$ $Lu_p = 100$
$Bu = 0.05$ $Bi_m = 5$ $Bi_q = 5$

x	Fo	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$
0	0.1	0.028105	0.092544	-0.001715
	0.3	0.083798	0.355749	-0.006718
	0.5	0.119450	0.554855	-0.010535
	0.7	0.143779	0.705208	-0.013430
0.3	0.1	0.039318	0.160039	-0.037633
	0.3	0.0915427	0.406851	-0.042324
	0.5	0.124974	0.593557	-0.045904
	0.7	0.147787	0.734545	-0.0486190
0.6	0.1	0.073177	0.361796	-0.106236
	0.3	0.114996	0.559431	-0.10993
	0.5	0.141766	0.708935	-0.112858
	0.7	0.160034	0.821831	-0.115033
0.9	0.1	0.130014	0.696724	-0.148798
	0.3	0.154490	0.812395	-0.150996
	0.5	0.170158	0.899896	-0.152674
	0.7	0.180850	0.965972	-0.153946

TABLE 2

$Ko = 3$ $Lu_m = 0.5$ $Pn = 0.25$ $Lu_p = 100$ $x = 0.2$
$Bu = 0.05$ $Bi_m = 5$ $Bi_q = 10$

$\in$	Fo	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$
0.3	0.1	0.424967	0.578308	-0.170122
	0.3	0.461996	0.644172	-0.177473
	0.5	0.497282	0.696152	-0.183076
	0.7	0.529012	0.737471	-0.187408
0.5	0.1	0.528843	0.643156	-0.414194
	0.3	0.544720	0.711600	-0.433995
	0.5	0.579193	0.761216	-0.446593
	0.7	0.618502	0.798571	-0.455027
0.7	0.1	0.612309	0.656464	-1.085452
	0.3	0.653275	0.802714	-1.125559
	0.5	0.898788	0.907122	-1.156976
	0.7	1.07578	0.981544	-1.178971

TABLE 3

$\epsilon = 0.1$ $Lu_m = 0.7$ $Pn = 0.25$ $Bu = 0.05$ $Bi_m = 10$
$Bi_q = 5$ $x = 0.1$ $Lu_p = 100$

Ko	Fo	θ_1	θ_1	θ_1
3	0.1	0.85590	-0.444837	0.12319
	0.3	0.77823	-0.42246	0.12121
	0.7	0.68641	-0.39301	0.11873
	0.9	0.66006	-0.38373	0.11798
6	0.1	- 1.6430129	1.221579	0.1976622
	0.3	- 8.689804	1.685510	-0.1409345
	0.7	-11.360450	1.03628	-0.4117915
	0.9	-11.613551	0.853743	-0.400615
9	0.1	- 9.328636	4.290214	-0.338757
	0.3	-23.497460	7.510771	-1.968782
	0.7	-65.155449	9.093752	-3.683034
	0.9	-78.280935	8.57662	-4.079404

TABLE 4

$\epsilon = 0.1$ $Lu_m = 0.3$ $Ko = 3$ $Lu_p = 100$ $x = 0.2$
$Bu = 0.05$ $Bi_m = 5$ $Bi_q = 5$

Pn	Fo	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$
0.5	0.1	0.008156	-0.050856	-0.015727
	0.3	0.083086	0.323680	-0.022489
	0.5	0.128992	0.600992	-0.027593
	0.7	0.159563	0.805499	-0.031390
0.75	0.1	- 0.024278	-0.298944	-0.111912
	0.3	0.077005	0.235568	-0.0203869
	0.5	0.137085	0.625413	-0.027304
	0.7	0.176522	0.907945	-0.032379
1.0	0.1	- 0.065569	-0.638710	-0.005319
	0.3	0.0677707	0.098678	-0.017429
	0.5	0.145576	0.634602	-0.026618
	0.7	0.196509	1.020766	-0.033344

TABLE 5

$\in = 0.1$ Ko = 3 $Lu_p = 100$ x = 0.2 Bu = 0.05
$Bi_m = 5$ $Bi_q = 5$ Pn = 0.5

Lu_m	Fo	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$	$\theta_1(x, Fo)$
0.3	0.1	0.008156	−0.050856	−0.015727
	0.3	0.083086	0.323680	−0.022489
	0.5	0.128992	0.600992	−0.027593
	0.7	0.159563	0.805499	−0.031390
0.5	0.1	− 0.212896	−0.009036	−0.031529
	0.3	− 0.526132	0.592568	−0.057043
	0.5	0.577846	0.885353	−0.0677508
	0.7	− 0.571623	1.040666	0.072859

TABLE 6

$\in = 0.1$ $Lu_m = 0.7$ Ko = 3.0 Pn = 0.25 Bu = 0.0u
$Bi_m = 10$ $Bi_q = 5$ x = 0.1

Lu_p	Fo	$\theta_1(x, FO)$	$\theta_1(x, Fo)$	$\theta_1(x, Fd)$
300	0.1	0.85586	−0.44264	0.04066
	0.3	0.77836	−0.42008	0.04001
	0.7	0.68668	−0.39045	0.03919
	0.9	0.66037	−0.38113	0.03894
100	0.1	0.95590	−0.444837	0.12319
	0.3	0.77823	−0.42246	0.12121
	0.7	0.68641	−0.39301	0.11873
	0.9	0.66006	−0.38373	0.11798
500	0.1	0.855856	−0.44221	0.02435
	0.3	0.77838	−0.41962	0.02396
	0.7	0.68674	0.38995	0.02347
	0.9	0.66043	−0.38061	0.02332
700	0.1	0.855854	−0.44203	0.017382
	0.3	0.778399	−0.419417	0.01710
	0.7	0.686769	−0.3897307	0.01675
	0.9	0.66045	−0.38039	0.016644
900	0.1	0.855852	−0.441930	0.013513
	0.3	0.778406	−0.419305	0.013295
	0.7	0.686782	−0.38961	0.013022
	0.9	0.66047	−0.380272	0.012939
1100	0.1	0.855852	−0.441865	0.011053
	0.3	0.778410	−0.419235	0.010874
	0.7	0.686791	−0.389534	0.010651
	0.9	0.660483	−0.380195	0.010584
1300	0.1	0.855851	−0.441821	0.009350
	0.3	0.778412	−0.419186	0.009199
	0.7	0.686797	−0.389482	0.0090108
	0.9	0.660490	−0.380142	0.008953

TABLE 7

$\in = 0.3$ $Lu_m = 0.3$ $Ka = 3$ $Pn = 0.25$ $Lu_p = 100$
$x = 0.2$ $Bi_m = 5$ $Bi_q = 10$

Bu	Fo	$\theta_1(x, Fo)$	$\theta_1(x, Fa)$	$\theta_1(x, Fo)$
0.1	0.1	0.626775	0.564644	−0.048207
	0.3	0.690739	0.643888	−0.050656
	0.5	0.742755	0.710658	−0.052738
	0.7	0.785149	0.766894	−0.054506
0.05	0.1	0.626775	0.564644	−0.096414
	0.3	0.690739	0.643888	−0.101312
	0.5	0.742755	0.710658	−0.105476
	0.7	0.785149	0.766894	−0.109012
0.01	0.1	0.626775	0.564644	−0.482074
	0.3	0.690739	0.643888	−0.506562
	0.5	0.742755	0.710658	−0.527384
	0.7	0.785149	0.766894	−0.545061
0.005	0.1	0.626775	0.564644	−0.964148
	0.3	0.690739	0.643888	−1.013124
	0.5	0.742755	0.710658	−1.054768
	0.7	0.785149	0.766894	−1.090123

TABLE 8

$\in = 0.1$ $Lu_m = 0.7$ $Ko = 3.0$ $Pn = 0.25$ $Bu = 0.05$
$Bi_q = 5$ $x = 0.1$ $Ku_p = 100$

Bi_m	Fo	θ_1	θ_1	θ_1
10	0.1	0.85590	−0.444837	0.12319
	0.3	0.77823	−0.42246	0.12121
	0.7	0.68641	−0.39301	0.11873
	0.9	0.66006	−0.38373	0.11798
15	0.1	1.30712	−0.85381	0.165704
	0.3	1.17444	−0.82918	0.162964
	0.7	1.013528	−0.80205	0.159770
	0.9	0.96612	−0.79500	0.158870
20	0.1	1.74262	−1.28918	0.203208
	0.3	1.57579	−1.22702	0.19829
	0.7	1.37190	−1.15803	0.19262
	0.9	1.311058	−1.140315	0.191057

TABLE 9

$\epsilon = 0.3$ $Lu_m = 0.3$ $Ko = 3$ $Pn = 0.25$ $Lu_p = 100$
$x = 0.2$ $Bu = 0.05$ $Bi_m = 5$

Bi_q	Fo	$\theta_1(x, Fo)$	θ_1 (x, FO)	θ_1 (x, Fo)
10	0.1	0.626775	0.564644	-0.096414
	0.3	0.690739	0.643888	-0.1013124
	0.5	0.742754	0.710658	-0.105476
	0.7	0.785149	0.766894	-0.109012
15	0.1	0.947609	0.488525	-0.083936
	0.3	1.112832	0.717164	-0.098449
	0.5	1.263309	0.892625	-0.109122
	0.7	1.408986	1.025539	-0.116581
20	0.1	1.188155	0.522796	-0.081621
	0.3	1.513611	0.823257	-0.098558
	0.5	1.889364	1.036251	-0.107706
	0.7	2.377155	1.170930	-0.108558

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