

## Control Based on Feedback Linearization of a Mobile Manipulator Robot for Trajectory Tracking

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**Abstract:** The use of robotic systems has now become almost necessary in various fields. Of which, the realization of any hard or dangerous place requiring an implication of manipulation and locomotion, is necessarily entrusted to a mobile manipulator. In this work, we present a control approach that ensures the stability of the system, based on feedback linearisation. The mobile platform is controlled in such a way that it always brings the manipulator's terminal organ to the desired position. The control of the platform depends on the information of the measured joint positions of the manipulator to ensure the planning of its own movement. It is shown that the considered strategy solves the problem ensuring the closed-loop stability of the system, thus allowing the convergence of the tracking errors. The performance of the approach is validated by simulation tests, showing an acceptable performance.

**Key words:** Mobile manipulator, Control stability, Great Precision, feedback linearization control.

### Yörünge Takibi için bir Mobil Manipülâtör Robotunun Geri Besleme Doğrusallaştırmasına Dayalı Kontrol

**Öz:** Robotik sistemlerin kullanımı artık çeşitli alanlarda neredeyse gerekli hale gelmiştir. Bunlardan, manipülasyon ve hareketin bir imasını gerektiren herhangi bir sert veya tehlikeli yerin gerçekleştirilmesi, zorunlu olarak bir mobil manipülâtöre emanet edilmiştir. Bu çalışmada, geri beslemeli doğrusallaştırmaya dayalı olarak sistemin kararlılığını sağlayan bir kontrol yaklaşımı sunuyoruz. Mobil platform, manipülâtörün terminal organını her zaman istenen konuma getirecek şekilde kontrol edilir. Platformun kontrolü, kendi hareketinin planlanmasını sağlamak için manipülâtörün ölçülen eklem pozisyonlarının bilgisine bağlıdır. Ele alınan stratejinin sistemin kapalı döngü kararlılığını sağlayarak sorunu çözdüğü ve böylece izleme hatalarının yakınsamasına izin verdiği gösterilmiştir. Yaklaşımın performansı, kabul edilebilir bir performans gösteren simülasyon testleri ile doğrulanır.

**Anahtar kelimeler:** Mobil manipülâtör, Kontrol kararlılığı, Büyük Hassasiyet, geri besleme doğrusallaştırma kontrolü.

## 1. Introduction

Advances in technology have shaped modern robotics to the design of more complex system for handling tasks can be extremely complicated. This complication that can manifest itself to a single robotic system that will be composed of many basic systems, such as it can manifest itself by a robotic system trained to perform tasks primarily related to handling [1, 2]. Nowadays the most common term is the mobile manipulator, which refers to robot systems formed from a robotic arm mounted on a mobile platform. These systems combine on the one hand the advantages of mobile platforms and robotic arms, and on the other hand ensure the reduction of their disadvantages. For example, the mobile platform offers the manipulator unlimited working space. The additional degrees of freedom of the mobile platform also provide the user with more choices, whereas an arm offers several operational features. Although it appeared very early in the history of robotics [3], this concept has been mainly studied for less than ten years [4]. Most of the publications inherit the problems related to robotic weapons and deal with the state of the art, such as control [4-6], trajectory optimization or operational trajectory tracking [7, 8]. Despite the wide variety of problems to be solved and the corresponding publications [9], very little effort has been made on modeling. When the mobile manipulator has a holonomic platform, the arm modeling can be directly applied [9, 10]. In the case of a wheeled mobile platform, the non-slip rolling of the wheels on the ground implies a different modeling. The mobile platform cannot move instantaneously in an arbitrary direction, because of this constraint [9-14]. It is then said to be non-holonomic and it is necessary to consider the specific properties of mobile platforms [15]. The prerequisites to implement a good total system control often involve the study of the

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kinematics and dynamics of the mobile manipulator. Our study was initially justified by the generation of trajectories for a mobile manipulator in order to perform a given task [16], for example painting a surface or simply sweeping it. We have based our work on a study and a comparative analysis between works already done in the field, which considered simple structures of mobile manipulators to deal with trajectory tracking problems [17, 18]. And, other works, which consider such a complex structure: the flexible manipulators [19] and the omnidirectional mobile robot [20], using the technique of the calculated torque was considered to solve the problems of regulation and tracking of trajectory.

The contribution of this work is manifested in the application of a simple method of torque control calculated in the task space is suggested for the tracking control of a mobile manipulator considered to a complex structure. And finally, the effectiveness of the proposed system is evaluated through simulation by the MATLAB software of mobile manipulator robot.

## 2. Description of the mobile robot manipulator

The mobile manipulators can be built according to Platform, which differs by the mechanism from training employed. Mobile platforms most usually available to use a differential training or a training similar to a car.

A robot of the type unicycle east actuates by two independent wheels, it has possibly insane wheels to ensure its stability. Its centre of rotation is located on the axis connecting the two driving wheels. It is a robot non-holonomic; indeed, it is impossible to move it in a direction perpendicular to the wheels of locomotion [13, 17]. Its order can be very simple; it is indeed rather easy to move it of a point to another by a succession of simple rotations and straight lines.

## 3. Modeling of a mobile manipulator

The system to be considered is a mobile manipulator supported by two independently driven wheels with a common fixed axis to the platform and two passive self-aligning wheels. The wheeled platform is modeled as a non-nolonomic system in which slip is neglected due to idling [7, 17]. Therefore, the wheeled platform consists of three degrees of freedom that is reduced to two degrees of freedom due to the non-slip condition. On these assumptions that the kinematic model of robot is developed, the details of the development are given in section 3.1.

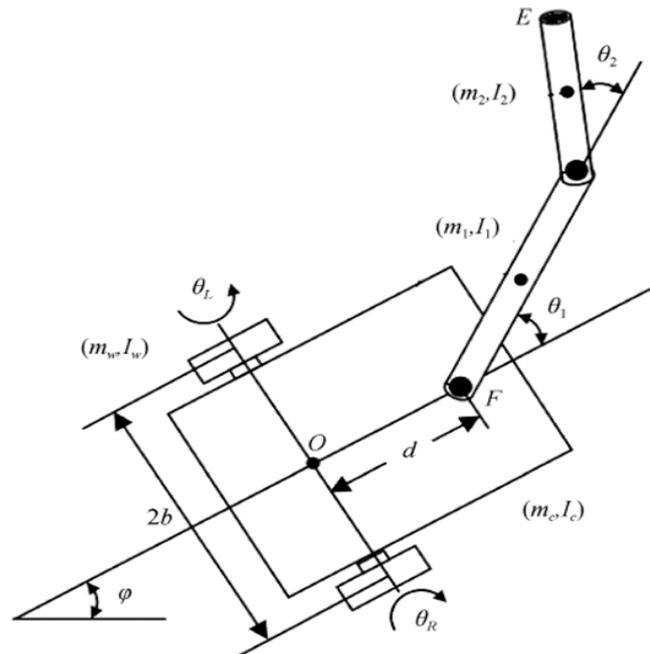


Figure 1. Mobile system of handling [14]

### 3.1. Kinematic modelling

Let us consider the mobile system of handling of Figure 1. For the mobile platform, the kinematic equation linear velocity at point F according to the speeds of wheel is given in the Equation 1 [14, 17]:

$$\begin{pmatrix} \dot{x}_F \\ \dot{y}_F \end{pmatrix} = \begin{pmatrix} r \\ 2b \end{pmatrix} \begin{pmatrix} (bC_0 + dS_0) & (bC_0 - dS_0) \\ (bC_0 - dS_0) & (bC_0 + dS_0) \end{pmatrix} \begin{pmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{pmatrix} \quad (1)$$

Where  $\theta_R$  and  $\theta_L$  are the angular velocities of the right-hand side and left, respectively. And  $S_0 = \sin(\varphi)$ ,  $C_0 = \cos(\varphi)$ . The linear velocity of the final effector is found while basing on the fact that its design speed is known and given by Equation 1. The speed of the final effector is written like:

$$\begin{pmatrix} \dot{x}_E \\ \dot{y}_E \end{pmatrix} = \begin{pmatrix} \dot{x}_F \\ \dot{y}_F \end{pmatrix} + \begin{pmatrix} C_0 & -S_0 \\ S_0 & C_0 \end{pmatrix} \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 + \dot{\varphi} \\ \dot{\theta}_2 \end{pmatrix} \quad (2)$$

Or  $J_{ij}$  ( $i, j=1, 2$ ) are elements of the fixed base Jacobien of the manipulator used, given by:

$$J_{11} = -L_1 S_1 - L_2 S_{12}, \quad J_{12} = -L_2 S_{12}, \quad J_{21} = L_1 C_1 - L_2 C_{12} \text{ and } J_{22} = L_2 C_{12}$$

$\theta_1, \theta_2$ : are the joint variables of the manipulator, with notations:  $S_i = \sin(\theta_i)$ ,  $C_i = \cos(\theta_i)$ ,

$$S_{ij} = \sin(\theta_i + \theta_j), \quad C_{ij} = \cos(\theta_i + \theta_j).$$

While combining Equation 1 and Equation 2, the differential kinematics of the mobile manipulator is obtained as follows:

$$\begin{pmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{x}_F \\ \dot{y}_F \end{pmatrix} = \begin{pmatrix} C_0 & -S_0 & 0 & 0 \\ S_0 & C_0 & 0 & 0 \\ 0 & 0 & C_0 & -S_0 \\ 0 & 0 & S_0 & C_0 \end{pmatrix} \begin{pmatrix} \left(\frac{r}{2}\right) - \left(\frac{r}{2b}\right)J_{11} & \left(\frac{r}{2}\right) + \left(\frac{r}{2b}\right)J_{11} & J_{11} & J_{12} \\ -\left(\frac{r}{2b}\right)(d + J_{21}) & \left(\frac{r}{2b}\right)(d + J_{21}) & J_{21} & J_{22} \\ \frac{r}{2} & \frac{r}{2} & 0 & 0 \\ -\left(\frac{r}{2b}\right)d & \left(\frac{r}{2b}\right)d & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta}_R \\ \dot{\theta}_L \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (3)$$

Who can be expressed in the following form:

$$\dot{x} = Jv \quad (4)$$

With:  $v(t) = [\dot{\theta}_r, \dot{\theta}_l, \dot{\theta}_1, \dot{\theta}_2]^T$ , by derivation Equation 4, we obtains:

$$\ddot{x} = \dot{J}v + J\dot{v} \quad (5)$$

This equation will be used in Equation 8, to make the link between the acceleration, the position and the motor torque of the robot.

#### 4. Dynamic modeling

The dynamic model of the system, is based on the determination of the relationship between the torque and the speed of the robot. In this section the details to obtain this relationship are given.

The dynamics of a mobile manipulator subjected to constraints non-holonomic can be obtained by using the Lagrangian one in the following form, [6], [17, 18]:

$$M(q)\ddot{q} + C(q, \dot{q}) = E(q)\tau - A^T(q)\gamma \quad (6)$$

The constraints non-holonomic written in the form of:  $A(q)\dot{q} = 0$ , with  $q = [x_c, y_c, \varphi, \theta_r, \theta_l, \theta_1, \theta_2]^T \in R^n$ , is the generalized coordinates.  $M(q) \in R^{n \times n}$  is the matrix of inertia of the system.  $C(q, \dot{q}) \in R^{n \times 1}$ ,  $A(q) \in R^{m \times n}$  are vectors centrifugal forces and Coriolis, respectively.  $A(q) \in R^{m \times n}$  is matrix of the constraints.

In order to eliminate the force from constraint  $\gamma$ ,  $S^T(q)A^T(q) = 0$ . We can find the vector of entry speed

$v(t) = [\dot{\theta}_r, \dot{\theta}_l, \dot{\theta}_1, \dot{\theta}_2]^T$ , for all  $\dot{q}$  given in the following equation:

$$\dot{q} = S(q)v(t) \quad (7)$$

$$\text{With: } s(q) = \begin{bmatrix} \frac{r(bc_0+ds_0)}{2b} & \frac{r(bc_0-ds_0)}{2b} & 0 & 0 \\ \frac{r(bs_0-dc_0)}{2b} & \frac{r(bs_0+dc_0)}{2b} & 0 & 0 \\ \frac{r}{2b} & \frac{-r}{2b} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Differentiating Equation 7, substituting the expression for  $\ddot{q}$  into Equation 6 and premultiplying by  $S^T$ , we get the Equation 8.

$$S^T(M(S\ddot{v}(t) + \dot{S}\dot{v}(t)) + C) = S^T\tau \quad (8)$$

Which can be reduced as follows:

$$\bar{M}\dot{v} + \bar{C} = \bar{\tau} \quad (9)$$

Where  $\bar{M} = S^TMS, \bar{C} = S^T(M\dot{S}\dot{v} + C), \bar{\tau} = S^T\tau$ ,

Since  $S$  is non-singular,  $\bar{M}$  is always symmetric and positive definite. Considering Equation 4, the Equation 9, can be expressed as:

$$\bar{M}\ddot{x} + \bar{C} = J^{-T}\tau \quad (10)$$

This model of Equation 10, will be used in the trajectory tracking, which is explained in section 4.1. The controller structure is shown in Figure 2.

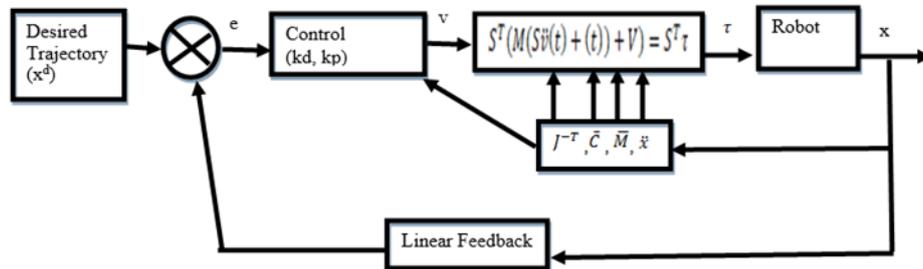


Figure 2. Modeling and control of a mobile manipulator.

#### 4.1 Path Tracking

The path tracking is based on a control that minimizes the error between the robot position and the desired position, details of this control are given below:

Suppose that the desired trajectory is described by  $\ddot{x}_d, \dot{x}_d$  and  $x_d$ , and since the general form of a mechanical system is given by Equation 10, its control is as follows:

$$J^T(\bar{M}\ddot{x} + \bar{C}) = \tau \quad (11)$$

Where auxiliary accelerations are given by:

$$\ddot{x} = \ddot{x}_d + k_d(\dot{x}_d - \dot{x}) + k_p(x_d - x) \quad (12)$$

Where  $e = x_d - x$  represents the tracking error of the system.  $k_p$  and  $k_d$  are respectively the proportional and derivatives gain matrices, which is also considered to be diagonal positive definite matrices. Substituting the dynamic model in the operational space of the mobile manipulator given by Equation 6, in the control law given by Equation 11. The result given in Equation 13.

$$\bar{M}(\ddot{e} + k_v \dot{e} + k_p e) = 0 \quad (13)$$

Where  $\bar{M} = S^T M S$  is invertible, the stability of the error (Equation 13), depends on the stability given in Equation 14:

$$\ddot{e} + k_v \dot{e} + k_p e = 0 \quad (14)$$

It is clear that for  $e = 0$  is an equilibrium point for the system illustrated in Equation 14 and let us consider the candidate Lyapunov function of the quadratic type of the following form,

$$V(q) = \frac{1}{2} \dot{e}^T \dot{e} + \frac{1}{2} e^T (k_p + \gamma k_d) e + \gamma \dot{e}^T e \quad (15)$$

In order to determine the stability of the closed loop system (Equation 14), let us now consider the time derivative of the function (Equation 15). The developments are illustrated in Equations (16) - (21).

$$\dot{V}(q) = \dot{e}^T \ddot{e} + \dot{e}^T (k_p + \gamma k_d) e + \gamma \dot{e}^T \dot{e} \quad (16)$$

$$\text{with } \ddot{e} = -k_p e - k_d \dot{e} \quad (17)$$

$$\dot{V}(q) = \dot{e}^T (-k_p e - k_d \dot{e}) + \dot{e}^T (k_p + \gamma k_d) e + \gamma \dot{e}^T (-k_p e - k_d \dot{e}) \quad (18)$$

$$= -\dot{e}^T k_p e - \dot{e}^T k_d \dot{e} + \dot{e}^T k_p e + \dot{e}^T \gamma k_d e - \gamma \dot{e}^T k_p e - \gamma \dot{e}^T k_d \dot{e} \quad (19)$$

$$= -\dot{e}^T k_d \dot{e} + \dot{e}^T \gamma k_d e - \gamma \dot{e}^T k_p e - \gamma \dot{e}^T k_d \dot{e} \quad (20)$$

$$\dot{V}(q) = -\dot{e}^T k_d (1 + \gamma) \dot{e} - \dot{e}^T \gamma (k_p - k_d) e \quad (21)$$

From Equations 21,  $\dot{V}(q)$ , is negative, we conclude that the closed-loop asymptotic stability indicate results for a sufficiently small epsilon( $\gamma$ ).

## 5. Results of simulation

To test the performance of the proposed study, the reference trajectory used is as follows:

$$x_{ef}^1(t) = 1 - \exp(-4t) \text{ (First simulation case) and } x_{ef}^2(t) = 3 \sin(t)^2 + 2 \cos(t)^2 \text{ (second simulation case)}$$

The main results of the simulation are given in the figures below.

The mobile manipulator robot moves from the start point to an end point (Figure 3), while following a desired trajectory of the end effector in the desired space, delimited by the small brown cycles. Nevertheless, it can be seen that there is some instability in the tracking, due to the non-holonomic constraints, which require the calculation of the steering angle at all times, whose manipulability index is slightly lower than the desired (maximum) value which should be equal to 1; in our case, it is stable around 0.748 (Figure 4 and 10 ) The linear velocity of the robot is given in figure 9 and 10 , the speed starts with an acceleration phase and then remains at a constant value, then in the approach phase by a deceleration until the arrival, this speed control is directly linked to the calculation of the motor torque of the robot that we have developed. Figures 7 and 8 illustrate that the moving platform brings the end-effector of the manipulator to the desired position. Note that the movement of the platform is not planned.

The control of the platform depends on the measured end-effector position information. The negative value of the velocities indicates that the moving platform moved backwards for a short period of time at the very beginning in order to reach the required heading angle. Therefore, the exposed backwards movement is not explicitly planned, which shows the performance of the developed control strategy.

- **First simulation case**

Desired trajectory is of form:  $x_{ef}^1(t) = 1 - \exp(-4t)$

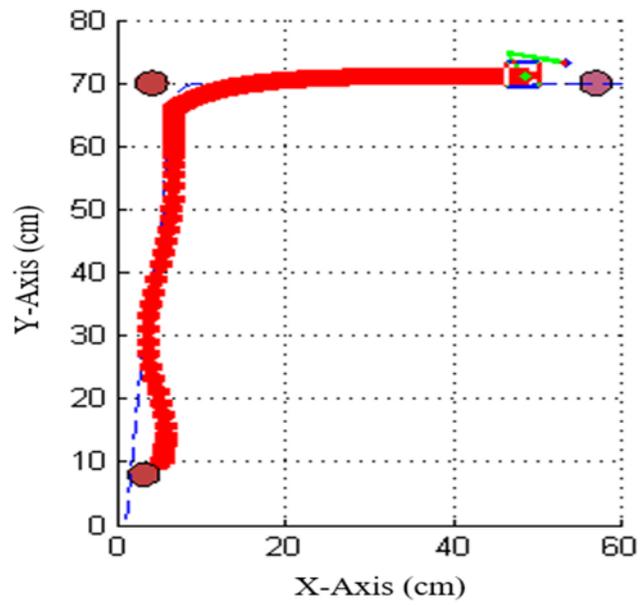


Figure 3. Robot trajectory tracking by the robot.

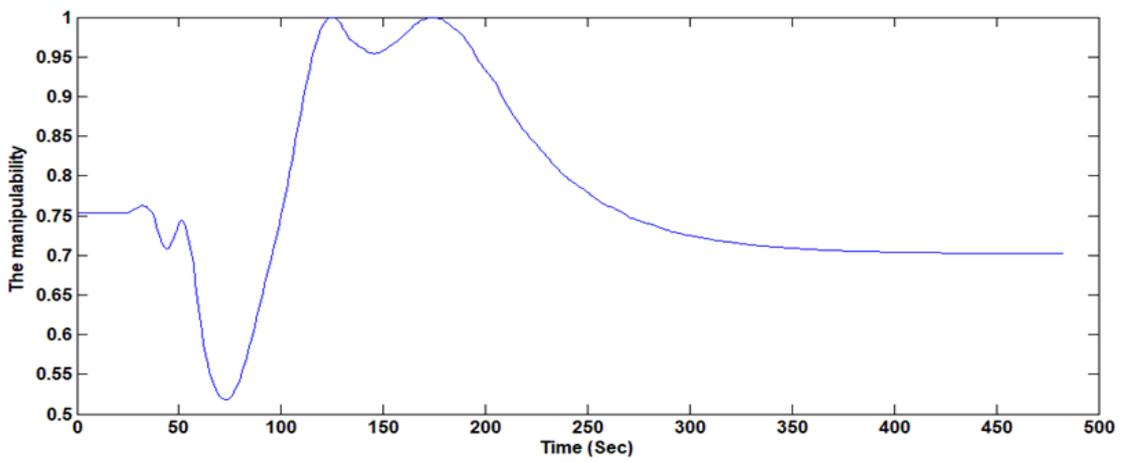


Figure 4. The manipulability.

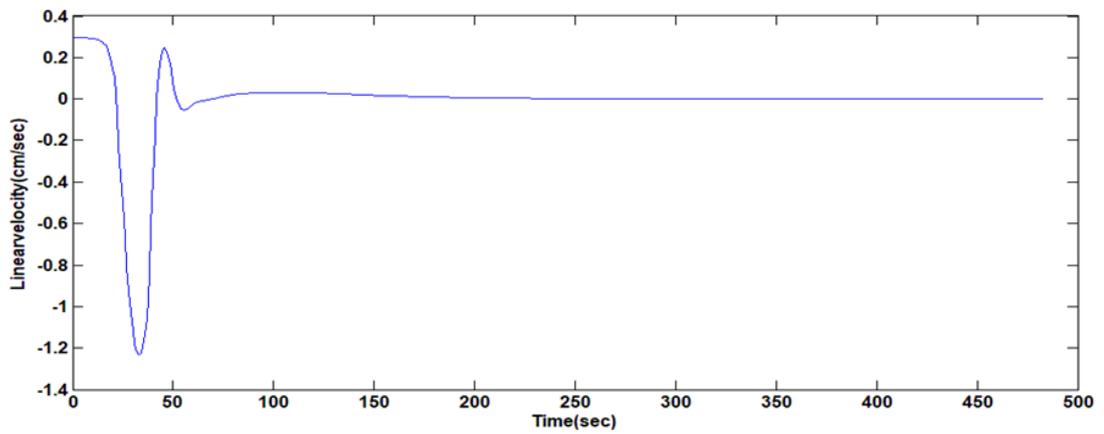


Figure 5. The linear velocity.

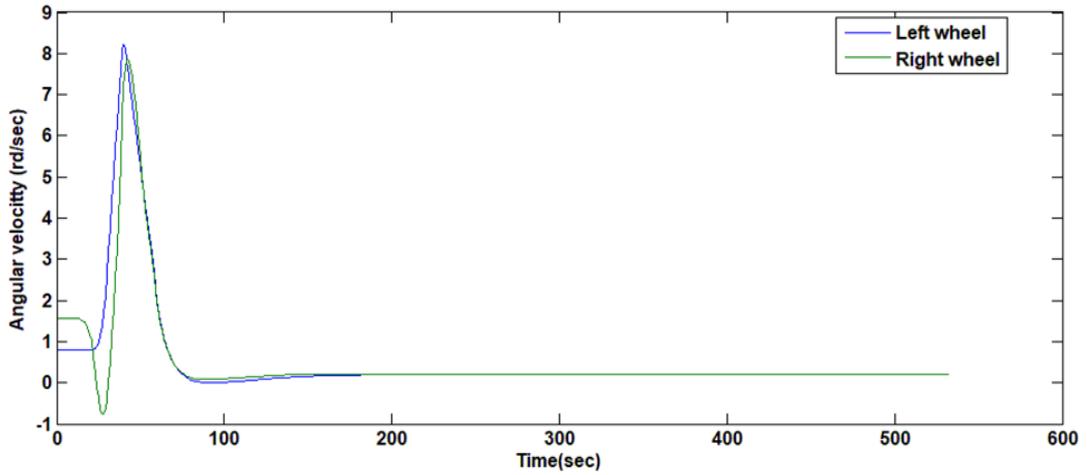


Figure 6. The angular velocity of wheels.

- **Second simulation case**  
Desired trajectory is of form:  $x_{ef}^2(t) = 3*\sin(t)^2 + 2*\cos(t)^2$

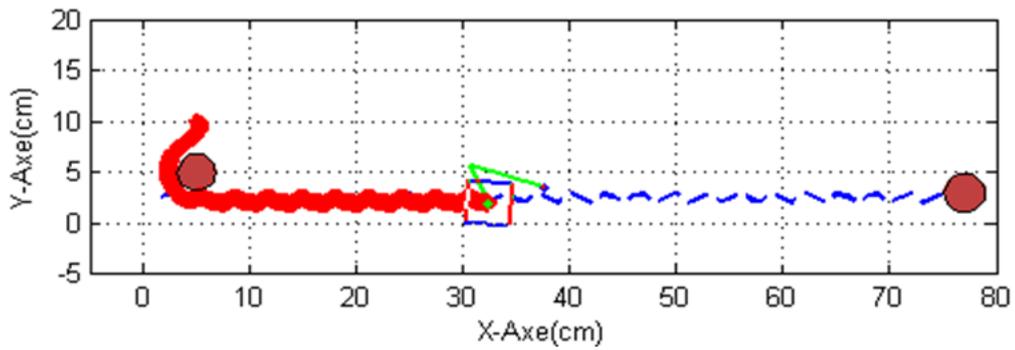


Figure 7. Robot trajectory tracking by the robot.

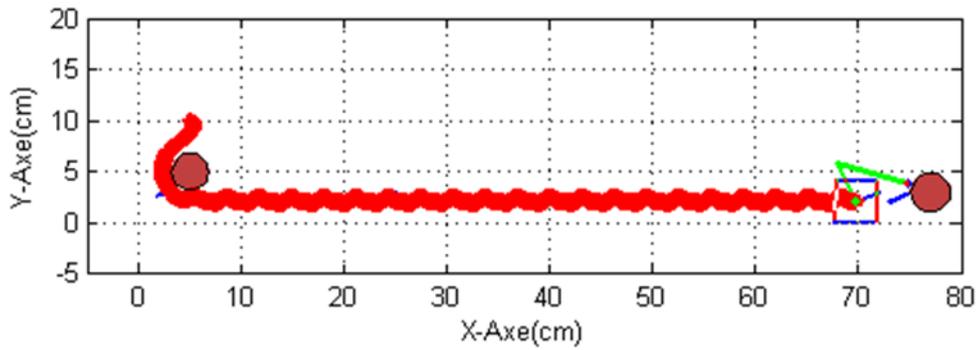


Figure 8. The final position of the robot.

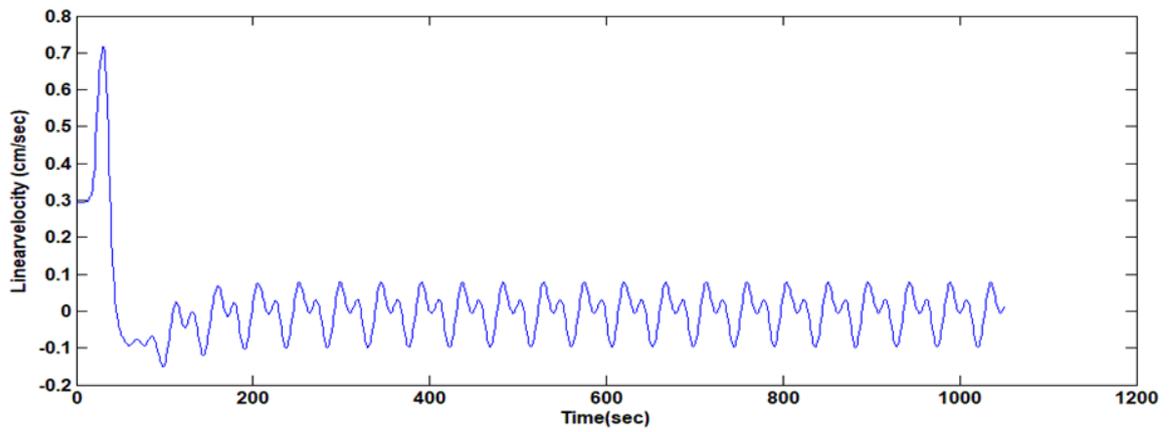


Figure 9 . Linearvolocity.

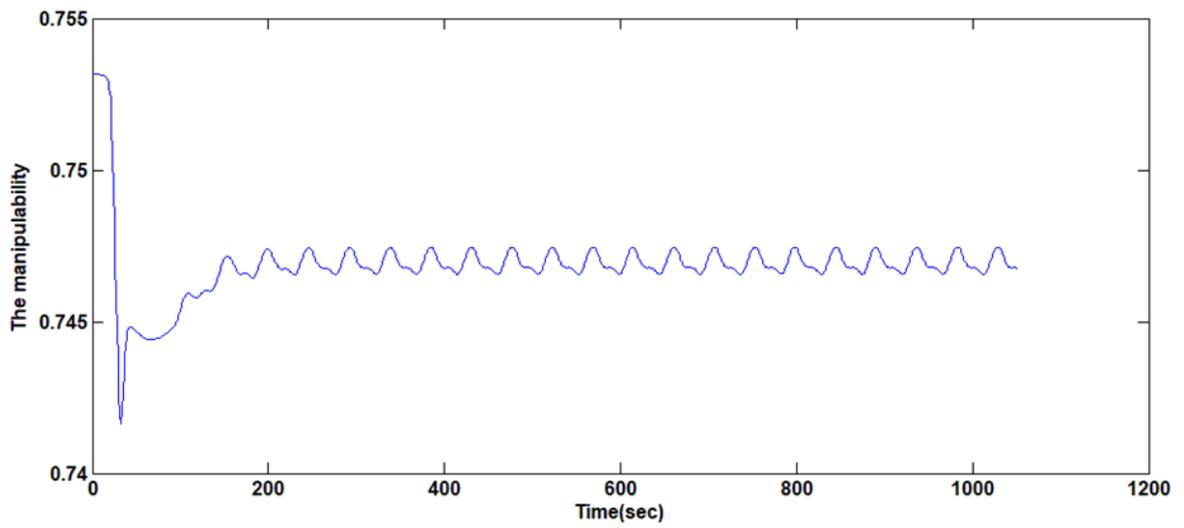


Figure 10. The manipulability.

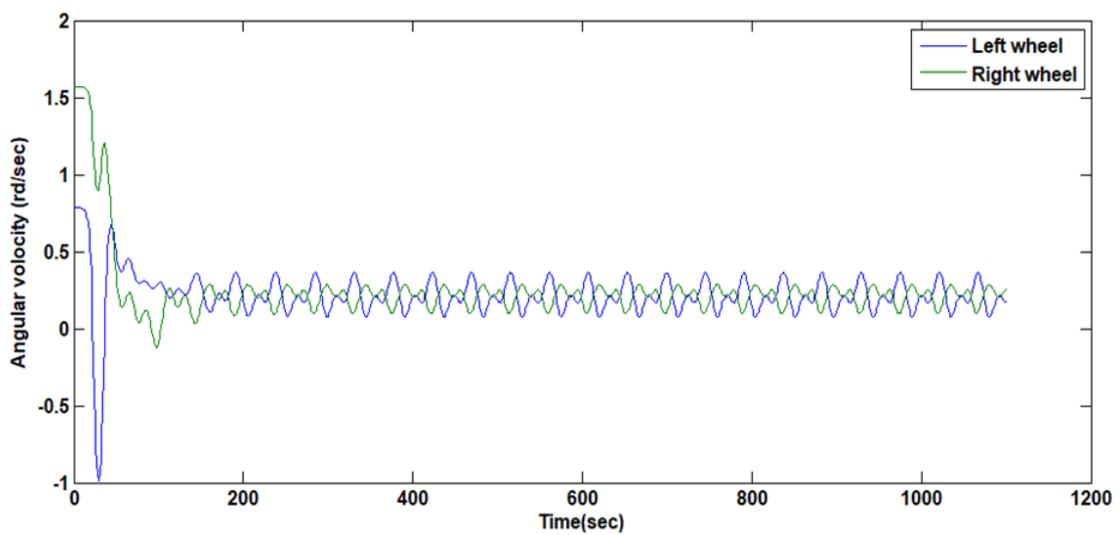


Figure 11 . Angular velocity of de wheels.

## 7. Conclusion

This work, focused on path following and control of the mobile manipulator, using a differential drive system as an example. The system platform was equipped with two link manipulators. In order to solve the problem of following the trajectory of the system subjected to non-holonomic constraints, we considered the control based on a dynamic model of a mobile manipulator. To solve the problem in the task space, the well-known computer torque control strategy, commonly used in the field of manipulator robots, has been considered. The stability of the entire closed loop system has been proven. The simulation results show that mobile manipulators can follow the reference path with great proximity, demonstrating the efficiency of the proposed calculated torque controller. Nevertheless, need to use the more robust PD-flou controller, Neuronal, passive, in order to better improve the performance of any structure (robot and its control).

## References

- [1] David DeVon et Timothy Bretl, "Kinematic and Dynamic Control of a Wheeled Mobile Robot", Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems San Diego, CA, USA, Oct 29 - Nov 2, 2007.
- [2] F. Pin, and J.C. Culioli, "Optimal Positioning of Combined Mobile Platform-Manipulator Systems for Material Handling Tasks", *Journal of Intelligent & Robotic Systems*, Vol. 6, 1992, pp. 165-182.
- [3] Dong Hun Shin and Kyung Hoon Park, "Velocity kinematic modeling for wheeled mobile robot", IEEE International Conference on Robotics and Automation, Proceedings 2001 ICRA Volume 4; 21-26 May 2001.
- [4] W. F. Carriker, P. K. Khosla, and B.H. Krogh, "Path Planning for Mobile Manipulators for Multiple Tasks Execution", *IEEE Transactions on Robotics and Automation*, Vol. 7, No. 3, 1991, pp. 403-408.
- [5] B. Bayle, J.-Y. Fourquet, et M. Renaud. « Génération des mouvements des manipulateurs mobiles : état de l'Art et perspectives ». *JESA*. Vol. 35. No. 6, pp. 809-845, 2001.
- [6] Yoshio Yarnamoto. Coordinated Control of a Mobile Manipulator. GRASP Laboratory Department of Mechanical Engineering and Applied Mechanics School of Engineering and Applied Science University of Pennsylvania, march 3, 1994.
- [7] N. Hootsmans, S. Dubowsky. The motion control of manipulators on mobile vehicles. In Proceedings of IEEE International Conference on Robotics and Automatics, IEEE, Sacramento, USA, vol. 3, pp. 2336{2341, 1991.
- [8] E. Padopoulos, J. Poulakakis. Planning and model-based control for mobile manipulators. In Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE, Takamatsu, J
- [9] Sarkar N, Yun X. and Kumar V., "Control of Mechanical Systems with Rolling Constraints: Application to Dynamic Control of Mobile Robots," *Int. J. Robotics Research*, Vol. 13, No. 1, February 1994, pp. 55-69.apan, 2000.
- [10] G. White. Simultaneous Motion and Interaction Force Control of a Nonholonomic Mobile Manipulator, Ph. D. dissertation, University of New York at Bu@alo, USA, 2006.
- [11] Y. Guo, P. Y. Woo. Adaptive fuzzy sliding mode control for robotic manipulators. In Proceedings of the 42nd IEEE Conference on Decision and Control, IEEE, Maui, USA, vol. 3, pp. 2174–2179, 2003.
- [12] A. Petitti, A. Franchi, D. Di Paola, and A. Rizzo, "Decentralized motion control for cooperative manipulation with a team of networked mobile manipulators," in 2016 IEEE International Conference on Robotics and Automation (ICRA). IEEE, May 2016, pp. 441–446.
- [13] T. Lee, "Robust adaptive attitude tracking on SO (3) with an application to a quadrotor uav," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 5, pp. 1924–1930, 2013.
- [14] Y.-H. Liu and S. Arimoto, "Decentralized Adaptive and Non-adaptive Position/Force Controllers for Redundant Manipulators in Cooperation," *The International Journal of Robotics Research*, vol. 17, no. 3, pp. 232–247, Mar 1998.
- [15] G Dai, Y Liu (2016) Coordination distribuée et contrôle de coopération pour les manipulateurs mobiles en réseau. *Transactions IEEE sur l'électronique industrielle* 64: 5065-5074.
- [16] N Chen, F Song, G Li, X Sun, C Ai (2013) Un contrôle de recul en mode glissant adaptif pour manipulateur mobile avec des contraintes non holonomiques. *Commun Nonlinear Sci Numer Simulat* 18: 2885-2899.
- [17] J Wen, D Bayard (1998) Nouvelle classe de lois de contrôle pour les manipulateurs robotiques. partie 1. cas non adaptatif. *Int J Control* 47: 1361-1385.
- [18] Asama, H., Sato, M., Bongoni, L., Kaetsu, H., Matsumoto, A., and Endo, I.: 1995, Development of an omnidirectional mobile robot with 3 DOF decoupling drive mechanism, in: Proc. of 1995 IEEE Internat. Conf. on Robotics and Automation, Vol. 2, pp. 1925–1930.
- [19] P. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Transactions on Automatic Control*, vol. 31, no. 4, pp.
- [20] C. Ott, A. Albu-Schaffer, A. Kugi, S. Stramigioli, and G. Hirzinger, "A Passivity Based Cartesian Impedance Controller for Flexible Joint Robots - Part I: Torque Feedback and Gravity Compensation," in Proc. of the 2004 IEEE International Conference on Robotics and Automation, April 2004, pp. 2659–2665.