Commun. Fac. Sci. Univ. Ank. Series A2-A3
V. 46. pp. 23-31 (2001)

AN ESTIMATION APPROACH FOR GAUSSIAN DISTRIBUTION ON DEGRADED IMAGES

Z. TELATAR

Department of Electronic Eng., Fac. of Engineering, Ankara University, Ankara, TURKEY

(Received March 01, 2001; Accepted March 28, 2001)

ABSTRACT

This paper introduces a novel approach to estimate the distribution of degradation effect on images. The basic noise distribution approach considered in restoration of degraded images uses a Gaussian structure to cover a large class of possible distributions. This is a general hypothesis in image processing. But, a problem encountered in image restoration, unless the disribution of blur effect is known, the restoration algorithm specially working on real time might have a large restoration error with unexpected results. Therefore, in order to obtain a high quality image from degraded one, an information about the distribution of noise might be an essential step through the forceful restoration.This paper presents a hypotesis test algorithm called chi-square goodness of fit test for estimating the noise distribution on degraded image. We handle mathematical analysis with examples and simulations by reformulating the algorithm in two dimension. According to the algorithm, estimating the distribution of degradation effect whether Gaussian or not on unknown image is related to a comparision between actual image having unknown distribution and reference test image. Successful experiments illustrate the performance of the resulting class of the algorithm and show significant benefits in obtaining the distribution of degradation effect on images.

KEYWORDS

Image processing, 2-D hypothesis test, Gaussian hypothesis, Chi-square.

1. INTRODUCTION

Generally, real world images are degraded by the effect of unfocused imaging systems or environmental factors such as atmospheric turbulence etc.. In addition to optical solutions which make a partial contribution to improvement image quality, such degraded images must be handled by using image processing techniques to obtain higher quality images than blurred one.

There are two general approaches to handle the restoration problem. In the first approach, blur function model parameters or previous information of each image region are known and degradation effect is easily removed from the image by using basic restoration techniques such as inverse filtering [6], blind deconvolution

[4]. In the other approach, degradation model parameters can not be known and they can adaptively or iteratively be estimated from the actual degraded image. But the basic concern arising with these restoration algorithms is, which type degradation effect has affected the images. On the other hand, what is the distribution of the degradation effect: Gaussian, Poisson, exponential, rectangular, etc..?

Many of the recently published restoration algorithms are based on estimation of degradation parameters which are supposed a Gaussian model hypothesis as a priori information. Telatar and Tuzunalp [6] have proposed an estimation based on edge information and restoration algorithm to remove the degradation effect on image with Gaussian distribution hypothesis. Molina, Katsaggelos and Mateos [4] have suggested a hyperparameter estimation method based on bayesian and regularization methods to restore degraded images by supposing a Gaussian noise. Elad and Feuer [2] have supposed a hypothesis of random noise having a Gaussian distribution to establish an adaptive superresolution restoration filter. According to these papers, in the condition of Gaussian degradation, restoration filter parameters have successfully been estimated by using Gaussian hypothesis with considerable results.

But, the possible problem arising restoration process mentioned above is that the distribution of degredation function is Gaussian or not. So, we have to test correctness of hypothesis to obtain a reliable restoration result. A number of hypothesis tests of distribution may be used to see whether it is normal or not [3,1,7,5]. We shall not discuss any of these special tests here, however, but rather handle the Chi-square goodness of fit test in two dimension. This is a novel approach to examine data to see if they fit some hypothesis. The general procedure involves the use of a statistic with an approximate distribution as a measure of discrepancy between an observed and theoretical data. A hypothesis of equivalence is then tested by studying the sampling distribution of this statistic. Chi-square goodness of fit test theory and our algorithm to estimate the distribution have been explained next sections.

The aim of this paper is to try to test of correctness of Gaussian hypotesis on degraded images. Section 2 explains the degradation model and the chi-square algorithm theory in detail. Section 3 explains our algorithm and section 4 presents some experimental results to test the algorithm and discussions. Finally, section 5 presents some concluding remaks.

2. Problem formulation and solution :

A Gaussian distribution has been modelled in previous studies as,

$$h(n_1,n_2)=\frac{1}{2\pi\sigma^2}e^{-\frac{(n_1^2+n_2^2)}{2\sigma^2}} \tag{1}$$

where n_1 and n_2 are dimensions of blur kernel, σ^2 is the variance of blur function and h is blur function in two dimension. Then, degraded image has been modelled as,

$$y(n_1,n_2)=g(n_1,n_2)*h(n_1,n_2)+v(n_1,n_2) \tag{2}$$

where, y, g, h and v are output of imaging system, original undegraded image blur function and additive noise based on imaging system respectively. According to assumption of Gaussian hypothesis, many researchers have realized different algorithms to solve the restoration problem and to obtain a clear image that close the original one. Each algorithm has also made a considerable improvement on images. But, the problem encountered in restoration is how we will model the degradation effect under different degradation scenerios. On the other hand, obtaining an information about distribution of blur function makes an important contribution to choose the correct filter model. The restoration of a degraded image is not issued in this paper. The chi-square goodness of fit test handled in the paper might only be a part of and efficient restoration process.

Let us introduce a system of notation for the above concepts that we will use throughout the rest of the paper. Let s be the number of cells or classes, n_j the observed frequency of the j-th cell, m the total number of observations and p_j the theoretical probability associated with the j-th cell .

$$m=\sum_{j=1}^{s} n_j \qquad \sum_{j=1}^{s} p_j = 1 \tag{3}$$

If we make the sum of the theoretical frequencies equal the sum of the observed frequencies then mp_j is the theoretical frequency of the j-th cell. Then chi-square calculations are made as in equation (4),

$$\chi^2=\sum_{j=1}^{s}\frac{(n_j-mp_j)^2}{mp_j} \tag{4}$$

Generally, the number of degree of freedom s' for the above test is calculated as,

$$s' = s - 1 \tag{5}$$

where s is observation number. We have already calculated for a distribution of both test and observed data with the same observation number m, mean value and standard deviation. So, we have three for degree of freedom for our example as,

$$s' = s - 3 \tag{6}$$

It is clear that the more "unusual" the observed distribution gets, the higher is chi-square; whereas if the observed is exactly like the expected, then chi-square equals zero.

As we make many more tosses and calculate their chi-square, the values of chi-square will fall between infinity and zero. Some will be bigger than our original calculated value and some will be smaller. If hardly any of the other values of chi-square are larger than our observed value, we might gather that our sample is a bit unusual. If however, there is a good proportion of the others with a chi-square just as large as calculated value, we should think our sample rather ordinary.

3. ALGORITHM

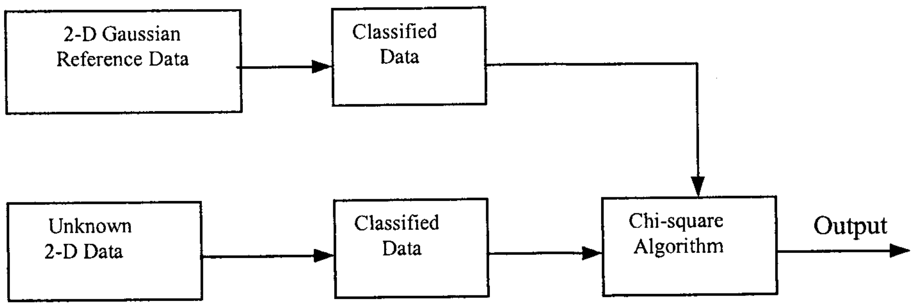

Figure-1. Chi-square goodness of fit test block diagram

According to the block scheme given in Figure-1, chi-square goodness of fit test algorithm has been expressed as below,

1- Establish a reference two dimensional filter having a Gaussian distribution,
2- Read the image which its distribution is unknown,
3- Classify both unknown image and reference two dimensional data (filter in step 1)),
4- Apply the chi square algorithm according to Equation (4),

Both actual image and test image have been classified in 20 groups. Consequently, degree of freedom has been calculated as 17 from the Equation (6) considering the parameters of Gaussian distribution such as standard deviation, mean and kernel. After four steps given above, algorithm produces a result and puts a star on the chi-square plot including four different curves. The curves in Figure 2 have been obtained from the standart tables in literature. In this figure, vertical and horizontal axis show chi-square and degree of freedom values respectively. If the star falls between the curves of probability P=0.1 and 0.9 with the calculated degree of freedom, hypothesis fits Gaussian distribution and we can say that "no suspect to hypothesis" for the test. If the star is above the curves, it can be said that result is "hypothesis precarious or disproved". If the star is under the curves, result is "data suspiciously good or too good".

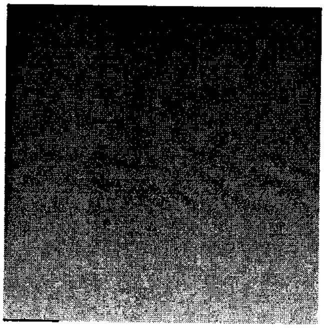

Figure-2. 256 gray level image for rows

Table-1.

	Distribution of degradation effect	Distribution of test image	Chi-square test Result (χ^2)	Conclusion
256 gray level image for rows	Non	Gaussian	50.28	Not Gaussian
	Degraded with Gaussian	Gaussian	14.66	Gaussian
	Degraded with Poisson	Gaussian	10.98	Gaussian
	Degraded with exponantial	Gaussian	52.40	Not Gaussian
	Degraded with rectangle	Gaussian	14.28	Gaussian
256 gray level image for colums	Non	Gaussian	50.28	Not Gaussian
	Degraded with Gaussian	Gaussian	14.66	Gaussian
	Degraded with Poisson	Gaussian	52.40	Not Gaussian
	Degraded with exponantial	Gaussian	10.98	Gaussian
	Degraded with rectangle	Gaussian	14.28	Gaussian
Child image	Non	Gaussian	196.65	Not Gaussian
	Degraded with Gaussian	Gaussian	14.66	Gaussian
	Degraded with Poisson	Gaussian	47.68	Not Gaussian
	Degraded with exponantial	Gaussian	145.74	Not Gaussian
	Degraded with rectangle	Gaussian	14.68	Gaussian

4. RESULTS and DISCUSSION

The performance of the proposed hypothesis test algorithm has been investigated with two 200 x 200 pixels artificial images, a 200 x 200 pixels photographic child image and a test image having Gaussian distribution. Some of the results are discussed in this section. Figure-3,4,5 and 6 show 256 gray level image with rows, 256 gray level image with colums, child image and Gaussian test data to be compared with the images respectively.

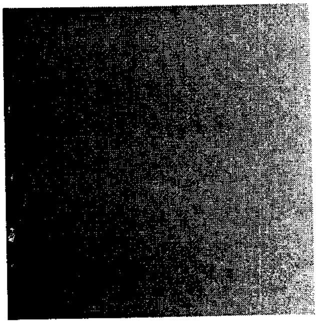

Figure-3. 256 gray level image for colums

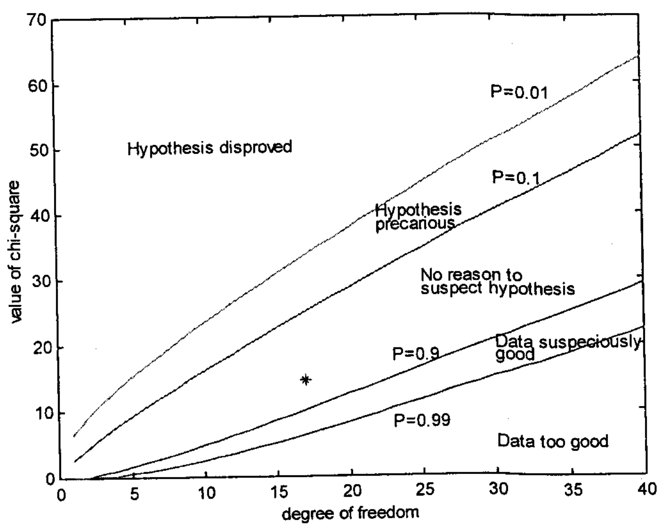

Figure-4. Chi-squre goodness of fit test evaluation regions.

Figure-5 Degraded child image

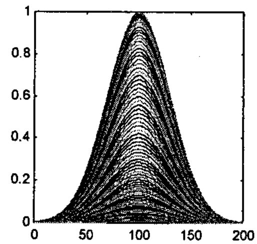

Figure-6 2-D Gaussian test data

Figure-7 Restored image with using Gaussian hypthesis

First, the image, which has no Gaussian distribution, shown in Figure-3 has been tested by the algorithm with degree of freedom of 17. Results in Figure-8 and Table-1 confirm that the image has no Gaussian distribution, as the result is outside the acceptance region with $\chi^2 = 50.28$ (hypothesis disproved). Second, the same image has been degraded by a Gaussian filter having a standard deviation with 3 and also tested by the algorithm. The test result is very considerable to correct the Gaussian hypothesis because the star is inside the acceptance region for Gaussian distribution (no reason to suspect hypothesis) as shown in Figure-9 and Table-1 with $\chi^2 = 14.66$. Then, the same image has been degraded poisson distribution. Result is very close to Gaussian distribution as the poisson distribution is similar to Gaussian distribution and χ^2 has been computed as 10.98 as shown in Table-1. Finally, image has been degraded by the exponential distribution and tested by the algorithm. Result has been computed as 52.40 in which the star is outside the acceptance region (hypothesis disproved).

The process for hypothesis test mentioned above has been applied to the other images shown in Figure-4 and real world photographic image in Figure-5. Results for Figure-4 look like as good as the results for Figure-3 shown in Figure-10 and in Table-1, as the images in Figure 2 and 3 have similar characteristics. Child image in Figure-4 also tested by the algorithm and results shown in Figure-11 and in Table-1 improve our hypothesis about Gaussian distribution. Child image has also restored using the Gaussian hypothesis shown in Figure-6. This considerable result confirms our algorithm.

The image also has been applied to our test algorithm after degraded by a rectangle distribution which has similar characteristics with the Gaussian distribution. Consequently, the results for Figure 2 and 3 are in the region of "no suspect to hypothesis" with $\chi^2 = 14.28$ and $\chi^2 = 14.68$.

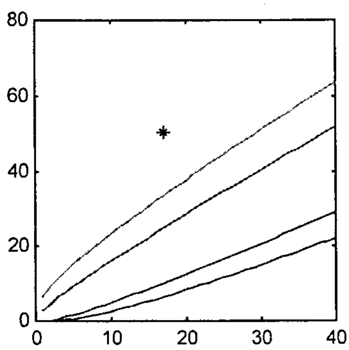

Figure-8. Chi-square test result for original undegraded image of figure-3

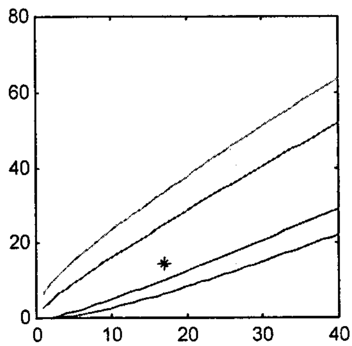

Figure-9. Chi square test result for Gaussian degraded image of figure-3

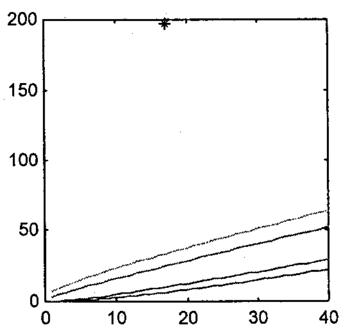

Figure-10. Chi-square test result for original undegraded child image

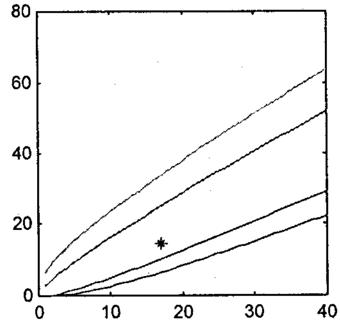

Figure-11.Chi-square test result for Gaussian degraded child image

5. CONCLUSION

This paper develops a hypothesis test algorithm for detecting the distribution of degradation effect having a Gaussian structure on image. Chi-square algorithm gives a result similar to Gaussian distribution for the distributions which are close to Gaussian such as rectangle, because the Gaussian distribution includes many of the other distribution types such as poisson, rectangle, etc.. But, the other distributions such as exponential etc.,which are far close to Gaussian structure.

If the degradation effect on image have a noise having Gaussian distribution, the chi-square test result is inside the region of "no reason to suspect to hypothesis". If the result is outside the region, we have to look for other distributions for efficient restoration.

The algorithm can be succesfully applied to noisy image problems on a basis of obtaining an information about distribution of degradation in two dimension. Our hypothesis test algorithm can also detect other distributions with a little change in algorithm.

REFERENCES

[1] Cox D.R., Miller H.D., (1994). "The Theory of Stochastic Process", Chapman&Hall,

[2] Elad M., Feuer A., (1999), "Superresolution restoration of an image sequence: Adaptive filtering approach", IEEE Trans. Image Proc., vol.8, n.3, pp.387-395.

[3] Metcalfe A. V., (1994), "Statistics in Engineering", Chapman&Hall,

[4] Molina R., Katsaggelos A.K., Mateos J., (1999), "Bayesian and regularization methods for hyperparameter estimation in image restoration", IEEE Trans. Image Proc.,vol.8, n.2, pp.231-246.

[5] Noonan, J.P., Polchlopek, H.M., (1993), A hypothesis testing technique for the wavelet transform in the presence of noise. Digital Signal Processing. Vol.3, pp.89-96.

[6] Telatar Z., Tüzünalp Ö., (1998), "Edge estimation and restoration of Gaussian degraded images", JIST Journal of Imaging Science and Technology, vol.42, n.4, pp.370-374.

[7] Zoubir A.M., Iskander D.R., (1999), "Bootstrapping bispectra: an application to testing for departure from Gaussianity of stationary signals",IEEE Trans. on Sig. Proc.,vol.47, n.3, pp.880-884.