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AN OVERVIEW OF TURBO DECODING ON FADING CHANNELS

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ABSTRACT

A review of turbo coding and decoding has been presented in the literature [1]. In that paper, turbo coding and decoding on AWGN (Additive White Gaussian Noise) channels has been elaborated. In wireless communications, a phenomennon called multipath fading is frequently encountered. Therefore, investigation of efficient techniques to tackle with the destructive effects of fading is essential. Turbo coding has been proven as an efficient channel coding technique for AWGN channels. Some of the work found in the literature shows that use of turbo codes on fading channels also yields promising results. In this paper, we present an overview of turbo decoding on fading channels.

KEYWORDS: turbo codes, turbo coding and decoding, fading channels

1. INTRODUCTION

Turbo codes opened an era in the field of error correcting codes and communications. Along with the increasing demands in mobile communications in todays world, effective techniques have become essential in order to tackle with the negative effects of the frequently encountered phenomenon, so called multi-path fading. It is well known that turbo coding and decoding have closed the gap between the so far achieved coding gains and the Shannon's theoretical channel capacity limit as little as a few tenths of a dB in AWGN (Additive White Gaussian Noise) channels. Turbo codes are also promising to yield good results in terms of BER (Bit Error Rate) performances on fading channels.

In this paper, we present an overview of turbo decoding on fading channels. Before presenting the work found in the related literature, we give the necessary modifications to a turbo decoder to be used on fading channels.

2. TURBO DECODING ON FADING CHANNELS

In wireless communications, there is a phenomenon called *"multipath fading"*, or shortly *"fading"*, which occurs due to the multiple reflective paths in the transmission media. Received signal amplitude and phase fluctuate due to the superposition of several reflected versions of the transmitted signal. Therefore, besides the noise, fading must be considered as another factor affecting the receiver performance.

In this paper, we present the necessary modifications to a turbo decoder so that, it can be used in fading channels. Then, we give a few examples of the work found in literature on "turbo decoding with adaptive channel estimation on fading channels".

2.1 Modifications To Turbo Decoder To Be Used on Fading Channels

To facilitate the following detailed explanations, we first present a rate ½ turbo encoder and turbo decoder in Figures 1 and 2, respectively. For more details about the turbo encoding and decoding scheme, interested reader is referred to the literature [1]. Note that only difference here than the scheme in [1] is that the turbo encoder is yielding a rate ½, i.e. a punctured code. This is achieved by deleting one of the parity bits (Y_{1k} and Y_{2k}) in each signaling interval respectively. That means parity bit is taken from either of the component codes [RSC (Recursive Systematic Convolutional) code C1 or RSC code C2] in a pre-determined order.

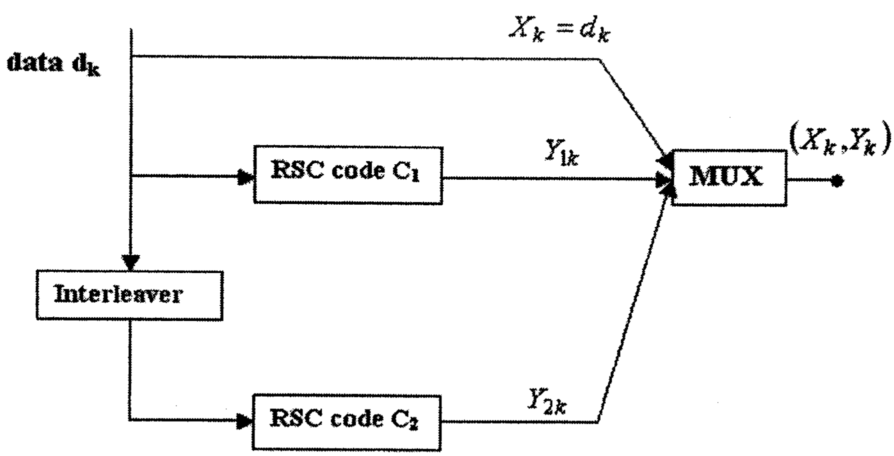

Figure 1. Rate 1/2 (punctured) turbo encoder

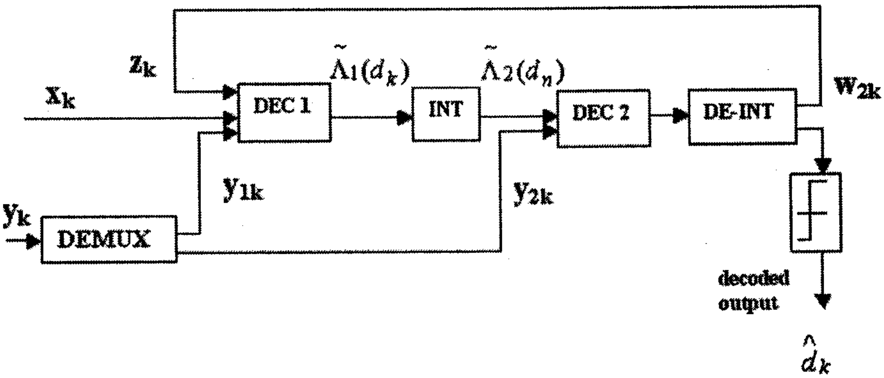

Figure 2. Turbo decoder

Let us consider a scenario as follows. An information bit sequence $\{d_k\}$ consists of N independent bits d_k is encoded by a rate 1/2 punctured turbo encoder with two identical rate 1/2 RSC encoders as the component encoders. Therefore, encoder output sequence would be $C_1^N = \{C_1 \ldots\ldots C_k \ldots\ldots C_N\}$ where $C_k = (X_k, Y_k)$. Note that Y_k, the parity bit, corresponds to y_{1k} (parity bit produced by the first component encoder) and y_{2k} (parity bit produced by the second component encoder) respectively in each signaling interval since a punctured code is considered. Also suppose that BPSK (Binary Phase Shift Keying) modulation is used, i.e. bit one is mapped to +1, and bit zero is mapped to −1. Note that here we assume that the average transmitted signal energy per information bit is normalized to one, i.e. $E_b = 1$.

Remember that when AWGN discrete memoryless channels are considered, the received sequence $R_1^N = \{R_1 \ldots\ldots R_k \ldots\ldots R_N\}$ where $R_k = (x_k, y_k)$ was defined by

$$x_k = X_k + i_k$$
$$y_k = Y_k + q_k$$

where i_k and q_k are two independent noises with the same variance σ^2, and zero mean value. However, for fading channels in the presence of AWGN, received bits can be modeled as given in Equation (1).

$$x_k = a_k^s X_k + i_k$$
$$y_k = a_k^p Y_k + q_k \qquad (1)$$

where a_k is the fading coefficient which can be modeled as a random variable. Note that a_k^s and a_k^p corresponds to the fading coefficients of systematic bit and parity bit respectively. In practice, most fading channels can be modeled either with Rayleigh or Rician probability distribution. In this chapter, we consider a Rayleigh fading channel with sufficient channel interleaving. Probability density function (pdf) of a Rayleigh random variable is given by Equation (2):

$$p_A(a_k) = 2a_k \exp(-a_k^2) \qquad a_k > 0 \qquad (2)$$

At this point, we must note that channel interleaving is different than the interleaving involved in turbo encoding and decoding. Encoded data are interleaved before transmission through the fading channel. The main reason for using a channel interleaver can be attributed to the fact that fading channels have memory, and therefore by using a channel interleaver errors would be distributed at the output of the channel. If sufficient channel interleaving were performed (fully interleaved channel), fading coefficients a_k's could be made independent. This allows us to use the codes designed for memoryless channels for fading channels. It has been noted in [2] that while channel interleaver does not change the capacity of the channel, assumption of a channel with memory as memoryless limits the achievable performance of the decoder.

When a turbo decoder is used in fading channels, it must be modified to incorporate the fading coefficients. To accomplish that, the transition metric in the BCJR (Bahl, Cocke, Jelinek and Raviv) algorithm $\gamma_i(R_k, m', m)$ must be reformulated. For fully interleaved channel and known fading amplitudes, transition metric $\gamma_i(R_k, m', m)$, as given in the literature [3], can be expressed as:

$$\begin{aligned}\gamma_i(x_k, y_k, a_k^s, a_k^p, S_{k-1} = m', S_k = m) = p(x_k \mid d_k = i, a_k^s) \\ .p(y_k \mid d_k = i, a_k^p, S_{k-1} = m', S_k = m) \\ .q(d_k = i \mid S_{k-1} = m', S_k = m) \\ .\pi(S_k = m \mid S_{k-1} = m')\end{aligned} \qquad (3)$$

where p(.|.) is the transition probability of the discrete Gaussian memoryless channel. Since the convolutional encoder is a deterministic machine, $q(d_k = i \mid S_{k-1} = m', S_k = m)$ is either 0 or 1, and can be determined from the encoder trellis. $\pi(S_k = m \mid S_{k-1} = m')$ are the state transition probabilities of the trellis, and are defined by the encoder input statistics. Generally, it is assumed that both input symbols are equally likely, i.e. $\Pr\{d_k=1\}= \Pr\{d_k=-1\}=1/2$ and as a result of this assumption $\pi(S_k = m \mid S_{k-1} = m') = 1/2$ since there are two possible transitions from each state. However, in one of the two approaches, $\pi(S_k = m \mid S_{k-1} = m')$ are considered as *"a priori probabilities"* and updated by each decoder using the *"extrinsic information"* provided by the other decoder in an iterative decoding scheme [4].

The probability $p(x_k \mid d_k = i, a_k^s)$ is conditionally Gaussian with mean value $a_k^s d_k$ and variance σ^2:

$$p(x_k \mid d_k = i, a_k^s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{\frac{-1}{2\sigma^2}[x_k - (a_k^s i)]^2\} \quad where\, i = -1, +1 \tag{4}$$

Similarly,

$$p(x_k \mid d_k = i, a_k^p, S_{k-1} = m', S_k = m) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{\frac{-1}{2\sigma^2}[y_k - (a_k^p Y_k)]^2\} \tag{5}$$

where Y_k is the parity bit generated by the encoder when $d_k = i, S_{k-1} = m', S_k = m$, and it is known from the encoder trellis, being either -1 or $+1$.

By replacing Equations (4) and (5) with their counterparts in the expression for $\gamma_i(R_k, m', m)$ given for AWGN channels, we finally obtain:

$$\gamma_i(R_k,m',m)=\frac{1}{2\pi\sigma^2}\frac{\exp(z_k/2)}{1+\exp(z_k)}\exp(z_k/2)$$
$$.q(d_k=i\mid S_k=m,S_{k-1}=m') \tag{6}$$
$$.\exp\{\frac{-1}{2\sigma^2}[x_k-(a_k^s i)]^2\}\exp\left\{\frac{-1}{2\sigma^2}\left[\{x_k-(a_k^s i)\}^2+\{y_k-(a_k^p Y_k)\}^2\right]\right\}$$

As we explained above, fading amplitudes appeared in the transition metric of the decoding algorithm. Therefore, it is now apparent that knowledge of the fading amplitudes has a direct effect on the decoder performance. In the following sections, we give some examples on adaptive channel estimation in conjunction with turbo decoding.

2.2 Turbo Decoding with Adaptive Channel Estimation for Rayleigh Flat Fading Channels

Authors presented different approaches found in the literature on "adaptive iterative detection for turbo codes on flat fading channels" in the literature [5]. There are mainly three approaches as given in Figure 3.

The simplest approach, shown in Figure 3. (a), consists of a channel estimator followed by the turbo decoder. In this approach, received information and parity bits from the fading channel is passed on to the channel estimator before the turbo decoder starts its iterative operation. Then fading amplitude estimates are passed on to the turbo decoder for an improved decoding performance.

Second approach, shown in Figure 3. (b) is quite similar to the first one. Major difference is that in the second approach channel estimator receives information from the turbo decoder as the turbo decoder iteratively produces soft information. In other words, there is a "feedback" from the turbo decoder to the channel estimator. With this approach, channel estimator is expected to produce better estimates of the fading amplitudes as the iterations of the turbo decoder continues, and consequently these "refined" estimates may improve the BER performance of the turbo decoder. In the literature, either the soft information, i.e. LLR of information and parity bits $\Lambda(d_k),\Lambda(Y_k)$, or the hard information, i.e. estimated information and parity bits $\hat{d}_k,\hat{y}_k$, are transmitted to the channel estimator. Depending on this, it is called either SDF (Soft Decision Feedback) or HDF (Hard Decision Feedback). Due to the information exchange between the decoder and the channel estimator, this approach is obviously better than the first approach.

Third approach can be described as "iterative joint data and parameter estimation". In this approach, channel estimation is jointly performed with turbo decoding. Some of the work related to this approach is found in [2], [6] and [7]. Detailed study of this subject is beyond the scope of this paper.

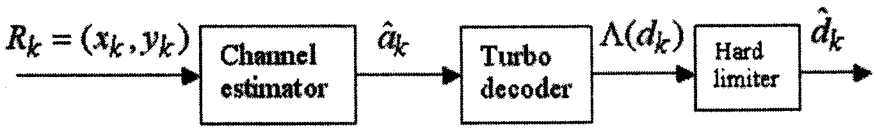

Figure 3. (a) A decoupled channel estimator without feedback from the turbo decoder

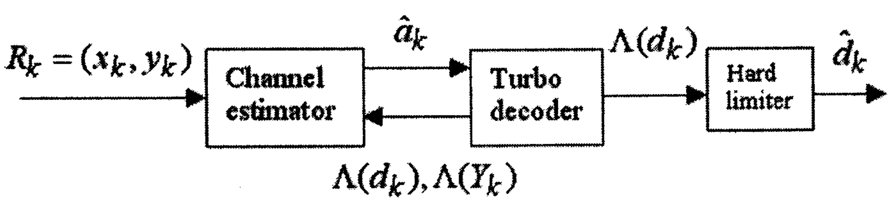

Figure 3. (b) A decoupled channel estimator with feedback from the turbo decoder

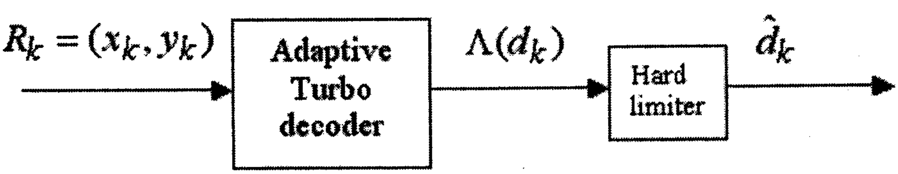

Figure 3. (c) Iterative joint data and parameter estimation

An example of the second approach given in Figure 3. (b) is found in the literatures [8] and [9]. Their approach is depicted in Figure 4. Received signal model was given earlier given in Equation (1):

$$x_k = a_k^s X_k + i_k$$
$$y_k = a_k^p Y_k + q_k$$

For ease of exposition we can write it as:

$$R_k = a_k C_k + n_k \tag{7}$$

where $R_k = (x_k, y_k)$, $C_k = (X_k, Y_k)$, $a_k = (a_k^s, a_k^p)$ and n_k is an i.i.d. (independent and identically distributed) AWGN with variance σ^2, and zero mean value. Operation of this system is as follows.

Received bits $R_k = (x_k, y_k)$ are first buffered before channel estimation starts. In the first estimation iteration ($q = 1$), switch is in position 1 and therefore the absolute value of $R_k = (x_k, y_k)$ is passed on to the channel estimator. We must emphasize that the q here is the estimation iteration, which is different than the iterations of the turbo decoder. After channel deinterleaving, estimated fading amplitudes $a_k^{(q)}$ are passed on to the turbo decoder. Note that the turbo decoder must be modified to yield the LLR (Logarithm of the Likelihood Ratio) of the parity bits ($\Lambda^{(q)}(Y_k)$) in addition to the LLR of the information bits ($\Lambda^{(q)}(d_k)$). In Figure 4 hard decision feedback is used, i.e. LLR's are passed through a hard limiter with a zero threshold to obtain two levels, either –1 or +1. When turbo decoder operates a few iterations, channel estimator uses these updated estimates of the information and parity bits to better estimate the fading amplitudes. Note that for the following iterations of the channel estimator ($q \geq 2$) the switch is at position 2. This procedure is repeated after every couple of iterations of the turbo decoder to provide better estimates of the information and parity bits to the channel estimator, and channel estimator, in turn, yields better estimates of the fading amplitudes to the turbo decoder.

There are several approaches that have been used for the channel estimator. The LMS algorithm and GOBA (General Optimum Block Adaptive) algorithm are suggested in [8] and [9]. Besides these, Wiener filter, average Wiener filter, and Kalman filter are suggested in [2]. Channel estimation, in its entirety, is another broad topic of communications, and will not be covered in this paper.

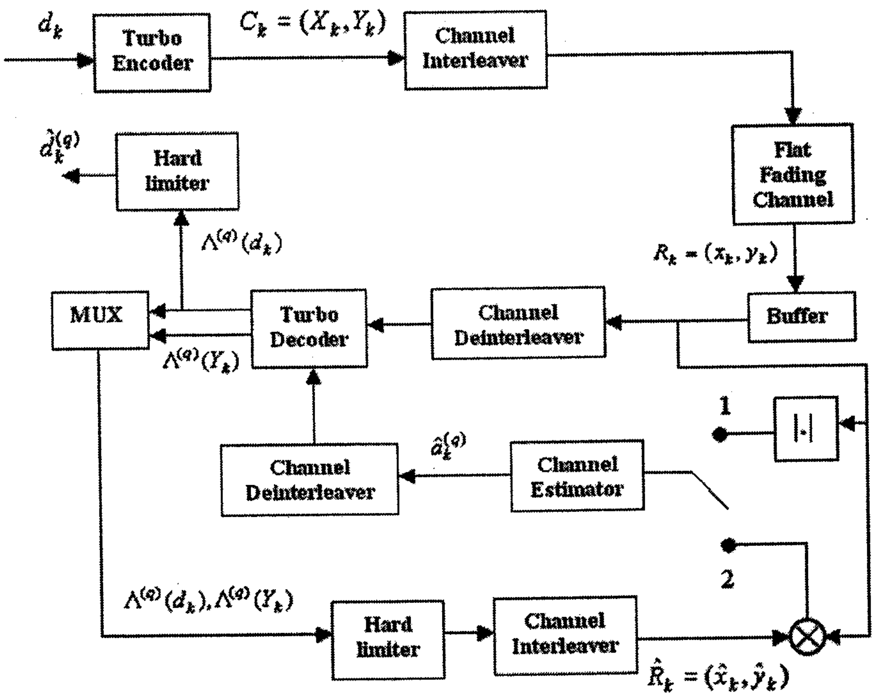

Figure 4. Turbo coding and decoding with decision directed channel re-estimation

3. CONCLUSIONS

In this paper, we presented an overview of turbo decoding on fading channels. Before presenting the work found in the related literature, we gave the necessary modifications to a turbo decoder to be used on fading channels.

ÖZET

Turbo kodlama ve kod çözmenin bir incelemesi literatür [1]'de verilmiştir. O makalede, AWGN (Toplamsal Beyaz Gaussian Gürültü) kanallarında turbo kodlama ve kod çözme konuları ayrıntılarıyla verilmiştir. Kablosuz haberleşmede, çok yollu bayılma etkisi olarak anılan bir olayla sıklıkla karşılaşılmaktadır. Bundan dolayı, bayılmanın bozucu etkileriyle mücadele için etkili yöntemlerin araştırılması elzemdir. Turbo kodlama AWGN kanallar için etkili bir kanal kodlama tekniği olarak ispatlanmıştır. Literatürde bulunan bazı çalışmalar turbo kodların bayılma kanallarında da ümit verici sonuçlar verdiğini göstermektedir. Bu makalede, bayılma kanallarında turbo kod çözme konusunun genel bir tanıtımı yapılmaktadır.

ANAHTAR KELİMELER: Turbo kodlar, turbo kodlama ve kod çözme, bayılma kanalları

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