REFRACTION IN THE CASE OF VELOCITY INCREASING LINEARLY WITH DEPTH AND DIPPING REFRACTOR

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ABSTRACT

In this paper the time-distance relation for head waves in "dipping refractor" is calculated assuming that the real seismic media may be expressed by the velocity fonction $V = V_0 (1 + KZ)$, depending on depth. A numerical example is also given.

INTRODUCTION

Horizontal refractor case, with velocity as a fonction of depth, has been studied by various authors. For the case of dipping refractor. Laski (1972) has developed an algorithm for an approximate solution of time-distance relation with the velocity function taken as $V = V_1$

$$\left[\frac{L + qz}{L} \right]^{\frac{1}{q}}$$

where

q = dimensionless constant

L = a constant with the dimension of length

Z = depth below datum.

The greatest difficulty encountered in solving the problem is to find the coordinates of the point at which the seismic ray reaches the refractor under the critical angle.

In this paper the equation giving the coordinates of the above mentioned point has been derived taking the velocity function to be $V = V_0 (1+KZ)$ and thus, the problem is solved.

Calculation of the coordinates

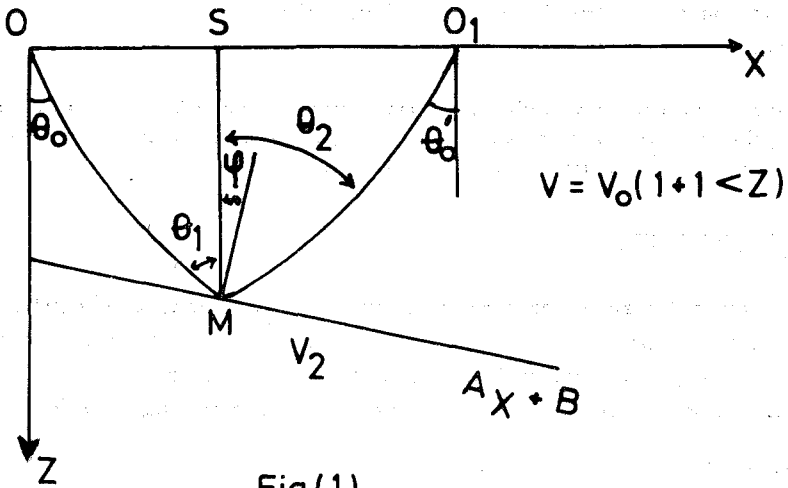 Notation (Fig–1)

  V_0 = velocity at the surface

  V = velocity at Z

  Z = depth

  K = constant

$Z = AX + B$ = Equation of the line expressing the refractor

  V_2 = Velocity of the refracting layer.

Fig (1)

Equation of the coordinates of the point of incidence at critical angle on the positive x direction

$\mathrm{Sin}\,(\theta_1+\varphi) = V_0\,(1+KZ)/V_2$ gives

$\mathrm{Sin}\theta_1\,\mathrm{Cos}\varphi + \mathrm{Cos}\theta_1\,\mathrm{Sin}\varphi = V_0\,(1+KZ)/V_2$ \hfill (1)

using $X = \dfrac{1}{K\,\mathrm{Sin}\varphi_0}\,(\mathrm{Cos}\theta_0 - \mathrm{Cos}\theta_1)$ and

$Z = \dfrac{1}{K\,\mathrm{Sin}\theta_0}\,(\mathrm{Sin}\theta_1 - \mathrm{Sin}\theta_0)$. Favre (1958).

It can be written

$$\sin \theta_1 = (ZK + 1) \sin \theta_0 \tag{2}$$

$$\cos \theta_1 = \cos \theta_0 - K X \sin \theta_0 \tag{3}$$

by combination of (2), (3) and (1)

we find $[(KZ + 1) \cos \varphi - KX \sin\varphi] \sin \theta_0 + \sin \varphi \cos\theta_0$

$$- V_0 (1 + KZ)/V_2 = 0 \tag{4}$$

on the other hand when θ_1 is eliminated from

$$X = \frac{1}{K \sin\theta_0} (\cos \theta_0 - \cos\theta_1) \text{ and } Z = \frac{1}{K \sin\theta_0} (\sin\theta_1 - \sin\theta_0) \tag{5}$$

and the result is divided by $K \sin\theta_0$, the following equation is reached

$$(X^2 + Z^2) K \sin\theta_0 - 2X \cos\theta_0 + 2Z \sin\theta_0 = 0 \tag{6}$$

solving θ_0 from equation (4) and (6) an equation dependent on X and Z can be obtained.

If we multiply both sides of the equations (4) and (6) by 2X and $\sin\varphi$ respectively and summing the two equations we obtain: $K \sin \theta_0 \sin\varphi$

$(X^2 + Z^2) + 2X \sin\theta_0 [(KZ + 1) \cos\varphi - KX \sin\varphi - KX \sin\varphi]$

$$- \frac{2X V_0 (1 + KZ)}{V_2} = 0 \tag{7}$$

Solving for $\sin\theta_0$ yields,

$$\sin\theta_0 = \frac{2 \times d}{V_2 (e+a)} \tag{8}$$

where

$$d = V_0 (1 + K Z) \tag{9}$$

$$a = 2x [(KZ + 1) \cos\varphi - KX \sin\varphi] \tag{10}$$

$$e = K \sin\varphi (X^2 + Z^2) + 2 Z \sin\varphi \tag{11}$$

If the value of $\sin\theta_0$ is placed in the equation (4) it results:

$$\frac{ad}{V_2 (e + a)} + \sin \varphi \cos \theta_0 - \frac{d}{V_2} = 0 \tag{12}$$

According to (12)

$$\cos \theta_0 = -\frac{ad}{\sin \varphi\, V_2\, (e+a)} + \frac{d}{\sin \varphi\, V_2} \tag{13}$$

If $\cos \theta_0$ is expressed in term of $\sin\theta_0$ (12) becomes

$$\frac{d^2}{V_2 \sin\varphi}\; \frac{e}{(e+a)} = \sqrt{1-\left(\frac{2xd}{V_2\,(e+a)}\right)^2} \tag{14}$$

squaring both sides of (14)

$$\frac{d^2}{V_2{}^2\, \sin^2\varphi}\left[\frac{e}{e+a}\right]^2 = 1 - \frac{(2xd)^2}{V_2{}^2\,(e+a)^2} \qquad \text{and} \tag{15}$$

$$\frac{d^2 e^2 - \sin^2\varphi\,[V_2{}^2\,(e+a)^2 - (2xd)^2\,]}{V_2{}^2\,(e+a)^2\, \sin^2\varphi} = 0 \tag{16}$$

is obtained.

In this last equation e, a and d are dependent on X and Z. The refractor in the X; Z plane is expressed as $Z = AX + B$. When Z is replaced by $AX + B$ in the equation (12) an equation dependent only on X is obtained.

Abscissa of the point at which the ray reaches the refractor under critical angle is one of the roots of equation (16). To solve the equation (13) it should be arranged with resprct to X.

From (9) it can be written

$$d^2 = d_2 X^2 + d_1 X + d_0$$

where

$$d_2 = (V_0\, KA)^2$$

$$d_1 = 2\, V_0{}^2\, KA\, (KB + 1)$$

$$d_0 = V_0{}^2\, (K^2 B^2 + 2\, KB + 1) \tag{17}$$

Similarly, using (10) and (11)

$$a = a^1{}_2 x^2 + a^1{}_1\, x$$

where

$$a^1_2 = 2 \text{ K (A Cos } \varphi - \text{Sin } \varphi)$$

$$a^1_1 = 2 \text{ Cos } \varphi \text{ (BK + 1)} \tag{18}$$

since $A = \tan \varphi$; $a^1_2 = 0$

$$a^2 = (a^1_2)^2$$

$$e = e^1_2 \, x^2 + e^1_1 \, x + e^1_0$$

where

$$e^1_2 = \text{K Sin } \varphi \, (1 + A^2)$$

$$e^1_0 = 2 \text{ A Sin } \varphi \, (1 + \text{KB}) \tag{19}$$

$$e^1_0 = \text{B Sin } \varphi \, (2 + \text{KB})$$

In this stage if the terms d^2e^2 and $(e + a)^2$ will be arranged with respect to the powers of X, equation (16) should also be written in the powers of X.

$$e^2 = (e^1_2 \, x^2 + e^1_1 \, x + e^1_0)^2 = e_4 x^4 + e_3 x^3 + 3e_2 x^2 + e_1 x^2 + e_0 .$$ When the similar terms are equated:

$$e_4 = (e^1_2)^2$$

$$e_3 = 2 \, e_2^1 \, e^1_1$$

$$e_2 = (e^1_1)^2 + 2 \, e^1_2 \, e^1_0 \tag{20}$$

$$e_1 = 2 \, e_1^1 \, e^1_0$$

$$e_0 = (e^1_0)^2$$

Because d^2e^2 is the sixth power of X; $d^2e^2 = e_4 d_2 x^6 + (e_4 d_1 + e_3 d_2)x^5 + (e_4 \, d_0 + e_3 d_1 + e_2 d_2) \, x^4 + (e_3 d_0 + e_2 d_1 + e_1 d_2) \, x^3 + (e_2 d_0 + e_1 d_1 + e_0 d_2) \, x^2 + (e_1 d_0 + e_0 d_1) \, x + e_0 d_0$. On the other hand, when $a^2_2 = 0$ is taken into account the following equation can be obtained by using equations (18), (20)

$$(a + e)^2 = e_4 x^4 + (e_3 + 2 \, a^1_1 \, e^1_2) \, x^3 + (a_2 + e_2 + 2 \, a^1_1 e^1_1) \, x^2 +$$

$$(e_1 + 2 \, a^1_1 \, e^1_0) \, x + e_0$$

Accordingly when a polynominal in power six is written as:

$$\sum_{n=0}^{6} P_n X^n \tag{21}$$

coefficients of (21) can be as follows using equations (12)–(20)

$$P_6 = e_4\, d_2$$

$$P_5 = e_4\, d_1 + e_3 d_2$$

With $f_1 = e_4 d_0 + e_3 d_1 + e_2 d_2$; $f_2 = e_3 d_0 + e_2 d_1 + e_1 d_2$

$$P_4 = f_1 - \text{Sin}^2\, \varphi\, V^2_2\, e_4 + 4\, \text{Sin}^2\, \varphi\, d_2$$

$$P_3 = f_2 - \text{Sin}^2\, \varphi\, V^2_2\, (e_3 + 2\, a^1_1\, e^1_2) + 4\, \text{Sin}^2\, \varphi\, d_1 \qquad (22)$$

With $g_1 = e_2 d_0 + e_1 d_1 + e_0 d_2$; $g_2 = a_2 + e_2 + 2\, a^1_1\, e^1_1$

$$P_2 = g_1 - \text{Sin}^1\, \varphi\, V^2_2\, g_2 + 4\, d_0\, \text{Sin}^2\, \varphi$$

$$P_1 = e_1 d_0 + e_0 d_1 - V^2_2\, (e_1 + a^1_1\, e^1_0)\, \text{Sin}^2\, \varphi$$

$$P_0 = e_0 d_0 - V^2_2\, e_0\, \text{Sin}^2\, \varphi$$

When p values found in equations (22) are replaced in the equation (21) the smallest root of (21) will provide the abscissa value of the point of incidence. The other roots form the trivial solutions.

It goes without saying that it is tedius and not practical to solve the problem through the equation (21). But we can try an approximate solution, sufficently accurate, which replace favourably the equation (21). **Approximate Solution:** Under the common conditions the V_0; V_2 and B parameters are more than 1 and $\text{Sin}\varphi$, K parameters are less than 1 in the polinominal (21). Therefore coefficients P_2; P_1 and P_0 are great and other coefficients are negligible.

Then, putting $\alpha = P_2$; $\beta = P_1$ and $\gamma = P_0$

$$X = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad \text{is the approximate solution} \qquad (23)$$

Calculation of Time: Fig (1)

Replacing the values $\theta_1 = -\varphi + \arcsin\left(\dfrac{V_0\,(1 + KZ)}{V_2}\right)$ \qquad (24)

and $\theta_0 = \arcsin\left[\dfrac{\text{Sin}\theta_1}{1 + KZ}\right]$ in the equation $t = \dfrac{1}{KV_0}\left[\log\left(\tan\dfrac{\theta}{2}\right)\right]^{\theta_1}_{\theta_0}$

$$(25)$$

time value t can be calculated easily between 0 and M

Computation of the Coordinates on the Negative x Direction:

Coordinates may be calculated from the coefficients P, obtained from (22) by keeping V_0; B; K values unchanged and putting $-\varphi$ for φ X value found should be multiplied by -1.

Numerical Computation:

The input values are:

$V_0 = 1000$ m/s

$V_2 = 2000$ m/s

$B = 400$ m

$\varphi = 5^0$

$K = 0.0008333$

From (22) the coefficients of the sixth order polynominal:

$P_6 = 0,2847.10^{-10}$

$P_5 = 0,1057014.10^{-5}$

$P_4 = 0,010164$

$P_3 = 1,705931$

$P_2 = -14482,8$

$P_1 = -6064908$

$P_0 = 1156219.10^4$

give as smallest root x = 266,28 m.

From (Z = AX + B), Z is found 423,29 m.

Using the formula (24) the value obtained for

Sin $(\theta_1 + \varphi)$ is 0,676364

At the point of incidence $V_0 (1 + KZ)/V_2 = 0,676366$

Thus, it is proved that Sin $(\theta_1 + \varphi)$ is equal to

$V_0 (1 + KZ)/V_2$ with an error of 0,0001

Approximate solution:
The formula (23) gives:

X = 265,24 m.

Z = 423,20 m.

As we can see approximation is perfect.

Time Calculation: Can be easly performed by using (25).

Between (0;0) and (266.28; 423,29) Fig (1)

$t_{OM} = 0,427$ Sec.

Using $\operatorname{Sin} \theta^1_0 = \dfrac{\operatorname{Sin} \theta_2}{1 + KZ}$ and

$SO_1 = \dfrac{1}{K \operatorname{Sın} \theta^1_0} (\operatorname{Cos} \theta^1_0 - \operatorname{Cos} \theta_2) = 359,1$ m

$t_{MO_1} = 0,474$ Sec. is obtained.

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