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Knockdown factors for cylindrical shells caused by torsional Mode-I type geometric imperfections under axial compression

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ABSTRACT

Geometrical imperfection, which is generally a result of manufacturing process and service conditions, plays a crucial role in load-bearing capacity of shell structures. This study presents a numerical study on knockdown factors of cylindrical shells as a result of torsional Mode-I type of geometric imperfections under compressive loads. The deformation patterns obtained from liner bifurcation analysis (LBA) for torsional Mode-I shape are used as a source of geometric imperfection. Then, geometrically nonlinear buckling analysis with imperfect model (GNIA) is incorporated with LBA in ANSYS Workbench to obtain limit loads of imperfect structures. A parametric study is thus performed to investigate the influence of imperfection depth on the load-bearing capacity considering a wide range of cylindrical shell configurations. Local and global buckling characteristics of the imperfect shells are examined and knockdown factors are characterized by three distinct regions as a basis of normalized imperfection depth. For a large number of shell configurations, a scattering of knockdown factors against normalized imperfection depth is given with mathematical expressions evolving lower and upper bounds. These expressions provide the minimum and maximum values of knockdown factors for a given imperfection depth, which can be treated as a design tool to ensure safety of the shell structure.

1. Introduction

Cylindrical shells are major structural elements that are widely used in most industries such as, storage tanks, silos, launch-vehicle systems, pressure hulls and other engineering applications. This kind of structures is mostly subjected to compressive loadings due to weight of the structural components in which they are connected. Cylindrical shell structures especially undergo local or global buckling failure as a result of axial compression. For this reason, prediction of buckling load plays an important role in the design stage of cylindrical shells. If it occurs, this type of failure mode tends to rapid and complete destruction of the shell structure. For a perfect cylindrical shell structure (without any imperfection), the critical buckling stress is expressed by Equation1 [1].

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\mu^2)}}\frac{t}{R} \qquad (1)$$

where E is Young's modulus, μ is Poisson's ratio, t is wall thickness and R is cylinder radius. Load-bearing capacity of a shell structures is quite sensitive to geometric imperfections even though they are very small. A geometric imperfection can be described as the deviations of geometric features from the ideal cylindrical shape. Generally, the pattern of a geometric imperfection in the shell is a result of utilized manufacturing process and interactions of the components in the construction [2]. These imperfections cause drastic variation of the actual buckling load in comparison with that of the perfect (ideal) shell structure [3]. Donnell [4] and Flügge [5] are the first studies to develop non-linear formulations taking into account large-scale initial geometric imperfections to evaluate actual buckling loads of imperfect structures. Later, Koiter [6] proposed an analytical method to obtain scatterings of the experimental geometric imperfection trends. However, the buckling loads are very sensitive to the form and type of geometric imperfections. There are various forms of geometric imperfections of shell structures, for example, out of roundness, local dimple, eccentricity, sinusoidal wave-type deviations, etc. Each has a characteristic influence on the buckling load of

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cylindrical shell structures. Many forms of geometric imperfections on the shell buckling have been studied in references [7-9].

The knockdown factor referred to as KDF, the ratio of the buckling load of an imperfect cylindrical shell to the predicted buckling load of an ideal cylinder obtained from Equation 1 ($F_{cr}=2\pi Rt\sigma_{cr}$) is commonly used to characterize influence of the geometric imperfections on the load-bearing capacity of the shell structure. According to a series of test results, an empirical design guideline was proposed by NASA [10] which provides lower-bound KDFs for the design of cylindrical shells as shown in Equation 2. This guideline is very conservative and resulting cylindrical shells would be so redundant and inefficient, thereby affecting the payloads for these structures [10].

$$KDF = 1 - 0.92\left(1 - e^{\frac{-\sqrt{R/t}}{16}}\right) \quad (2)$$

For axially compressed cylindrical shell with initial geometric imperfection, Evkin and Lykhachova [11] implemented an energy barrier method to estimate buckling load and knockdown factors considering elasto-plastic buckling case. They conducted a parametric analysis of the structure and derived a formula for design buckling load which splits up the zone of the high sensitivity of the shell to imperfections [11]. Initial geometric imperfections caused by different mode shapes under axial compression is studied by Kim [12] to develop practical design equations and charts predicting buckling strength of cylindrical shells and tanks. Among the shell structures, cylindrical shells are a standard structure (membrane stress dominant by nature of the structure form) which are quite sensitive to imperfections. For this reason, more robust KDF's for the design of shells are a primary factor for efficient construction. Several important studies in the literature, concerning the initial geometric imperfection for axially compressed cylindrical shells, are: (i) robust KDF's for the design of cylindrical shells [13], (ii) experimental and numerical campaign on low KDF's [14], (iii) improved KDF's for composite cylindrical shells with geometric imperfections [15], and (iv) buckling of quasi-perfect cylindrical shell under axial compression [16].

Type of loading is a factor that affects the geometric imperfection of the shell structure as well as the geometrical characteristics of imperfections. Combined loadings lead to more complex buckling responses in terms of geometrically imperfect structures. To increase the load-bearing capacity and avoid the stress-failure before reaching the critical load, Mahdy et al. [17] investigated buckling and stress-competitive failure analysis of composite cylindrical shells under axial compression and torsion. Similarly, the buckling behaviour of imperfect cylindrical shells subjected to torsion examined by Zhang and Han [18] using a singular perturbation technique. However, most of the studies in the available literature focus on a limited range of geometric imperfection patterns. The influence of the geometric imperfections caused by the torsional interactions on the buckling load of cylindrical shells under axial compression remains still unclear.

This study presents lower and upper bound of knockdown factors for the geometrically imperfect cylindrical shells under axial compression. Torsional Mode-I type deformation patterns are considered as a geometric imperfection. The influence of deformation patterns and imperfection depths is examined at various cylindrical shell configurations. In the available literature, no study concerning torsional Mode-I type of geometric imperfections has been found for axially compressed cylindrical shells. In this way, this study is expected to be have a contribution to fill this gap in the literature.

2. Material and Method

2.1 Shell Geometry

The geometry of a cylindrical shell structure is schematically illustrated in Figure 1. There are three independent geometric parameters in the construction of a cylindrical shell which are denoted as shell length L, wall thickness t and cylinder radius R. A reference cylindrical coordinate system is presented in Figure 1 at which r, θ and z denote radial, circumferential and vertical directions, respectively. Schematic illustration of a torsional Mode-I type of geometric imperfection is shown in Figure 1b, where Δw denotes the level of imperfection depth. Since the maximum value of the imperfection depth is critical in buckling behaviour, Δw is selected as an imperfection parameter.

For general knockdown factor (KDF) evaluations, it is useful to consider normalized shell parameters. Therefore, dimensionless parameters, such as radius-to-thickness ratio R/t, ratio of the imperfection depth to the wall thickness $\Delta w/t$, and length-to-radius ratio L/R are used for KDF evaluation. To reveal influence of each normalized parameter on knockdown factors (KDF's), a parametric study is performed in two stages as can be seen in Table 1.

Table 1. Shell configurations for the parametric study.

Stage I			Stage II		
R/t	$\Delta w/t$	L/R	R/t	$\Delta w/t$	L/R
100	0.1	2	100	0.1	2
200	0.2			0.2	3
400	⋮			⋮	4
800	1			1	5
1600	1.2			1.2	6
	1.5			1.5	
	2			2	
	3			3	

*Factorial design methodology is considered and total number of tested configurations is 5x14x1+1x14x5=140.

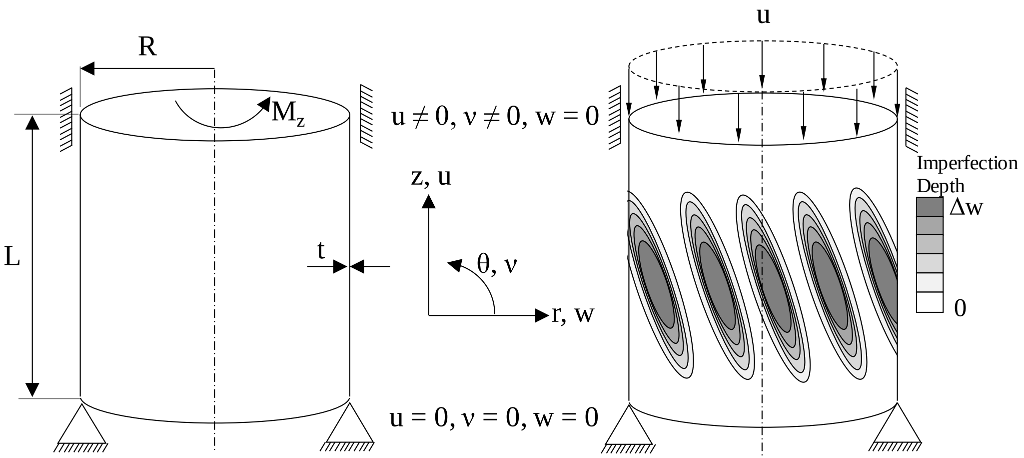

Figure 1. Cylindrical shell geometry with boundary conditions and dimensions a) perfect shell structure under the action of a torsional moment M_z, b) axially compressed imperfect structure with the torsional Mode-I type of geometric imperfection.

In Stage I, the influence of *R/t* and *Δw/t* on KDF is systematically investigated at *L/R*=2 taking *R*=100 mm. In the second stage, a similar procedure is applied at a constant *R/t*=100 to understand the influence of *L/R* on the imperfection sensitivity.

2.2 Numerical Analysis

Details of the finite element model, boundary conditions and numerical analysis are presented in this section. A numerical model is constructed in ANSYS Workbench package program. Figure 1 displays the boundary conditions employed in the numerical model. Bottom end of the shell model is clamped and the top of the shell is free to move in *u* and *v* direction, and displacement *u* is applied progressively until the buckling occurrence as shown in Figure 1. The numerical analysis takes nonlinear geometry and imperfection referred to GNIA (geometrically nonlinear elastic analysis with imperfection included) into account in this study. An elastic material model is considered (*E*= 200GPa and μ= 0.3 where E and μ are Young's modulus and Poisson's ratio, respectively) since the elastic buckling behaviour of the shells is investigated. Shell181, four-node quadrilateral shell element with large displacement capability, is selected for the numerical analysis. For the analysis of shell structures, the ideal element size is suggested to be $0.5\sqrt{Rt}$ [13] for the numerical analysis. For this reason, a variable element size according to the above formula is considered for each shell configuration.

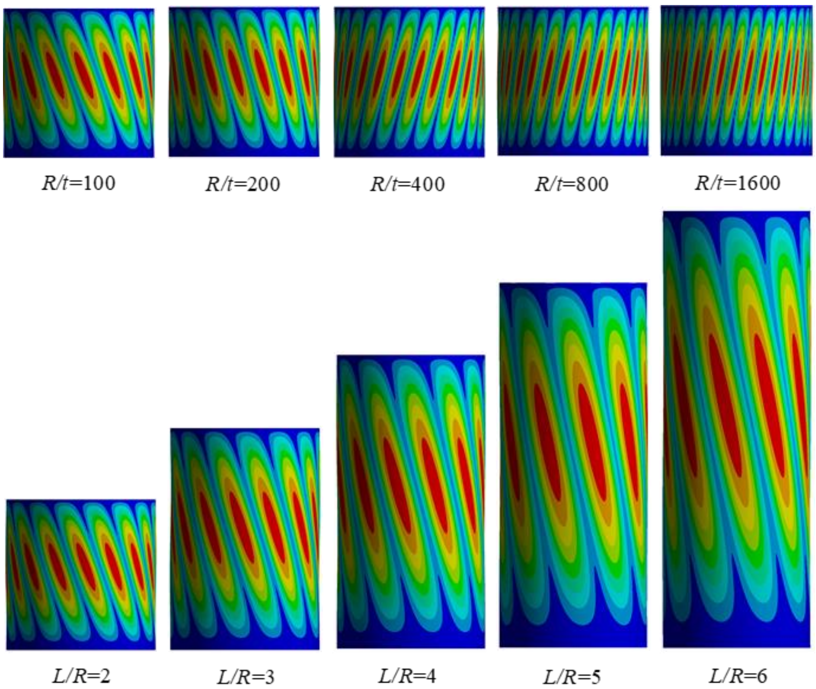

Figure 2. Torsional Mode-I deformation patterns as a source of geometric imperfection for the cylindrical shells at different geometry configurations.

2.3 Characteristics of geometric imperfection

There are different types of initial geometric imperfections as a result of service conditions and manufacturing processes of the cylindrical shells. One of them is the localized geometric imperfection with a certain orientation which is circumferentially distributed over the cylindrical shell surface, as shown in Figure 1 as a result of torsional pre-loadings. If a cylindrical shell is subjected to a torsional moment of M_z due to service conditions, a typical Mode-I deformation pattern forms. This fairly produces a geometric imperfection in the shell structure. It may cause a loss in the load-bearing capacity of the cylindrical shell under axial compression in service conditions. To investigate this phenomena, deformation patterns of the shells should be retrieved after applying a torsional moment of M_z to create an imperfect structure to be tested for axial compression. The deformation pattern of a particular mode shape is obtained by performing a linear elastic bifurcation analysis, LBA, (eigenvalue analysis). This kind of analysis is carried out on a perfect model without taking into account the imperfections. To extract and use the torsional Mode-I deformation pattern of each cylinder configuration as a geometric imperfection, linear buckling analysis is conducted and obtained results are presented in Figure 2. As can be seen, the mode shapes vary depending on the shell parameters R/t and L/R. In the linear buckling analysis, the maximum depth of imperfection Δw is adjusted to 1 mm. Therefore, a scaling factor on the deformed shell is used to obtained desired $\Delta w/t$ values. After this step, the deformed cylindrical shells (imperfect shells) are subjected to axial compression until the buckling occurrence. A flowchart for determining the buckling load of an imperfect shell configuration is described in Figure 3. KDF of each configuration is calculated considering the ratio of the buckling load of imperfect cylindrical shells to the perfect (ideal) case ($F_{cr}=2\pi Rt\sigma_{cr}$).

3. Results and Discussion

The results of the numerical analysis of the cylindrical shell configurations are presented to investigate knockdown factors (KDF's) of geometrically imperfect structures. Equilibrium path (load-axial displacement curve) is an important tool to indicate the general buckling behaviour of an imperfect shell structure. For this reason, the load-axial deformation curve of a selected configuration, where R/t=100, L/R=2 and $\Delta w/t$=0.9, is plotted in Figure 4 for the assessment of general buckling behaviour. As can be seen, there are several critical points (A, B, C, D and E) on the equilibrium curve which represents local buckling, non-linear collapse and post-buckling stages of the imperfect cylindrical shell. Additionally, formation of buckles and corresponded deformation patterns at each critical points are shown in Figure 4. The results show that local buckling occurs as a first failure mode (Point A) and shortly afterwards second local buckling mode observed (Point B), then it results in a non-linear collapse (Point C) with a buckling load equals to about 310 kN. At the first local buckling instant, torsional deformation strips are still visible and shortly afterwards the mode shape turns into a different shape, which has circumferentially localized dimples over the shell surface, as seen in Figure 4 (picture for point B). After this point, the mode shape does not radically change during the non-linear collapse and post-buckling stages (see points D and E). However, the depth of the dimples increases and the number of dimples is prone to decrease during the loading history. It is noteworthy that the initial torsional deformation pattern disappears in axial compression, which is replaced by localized dimples.

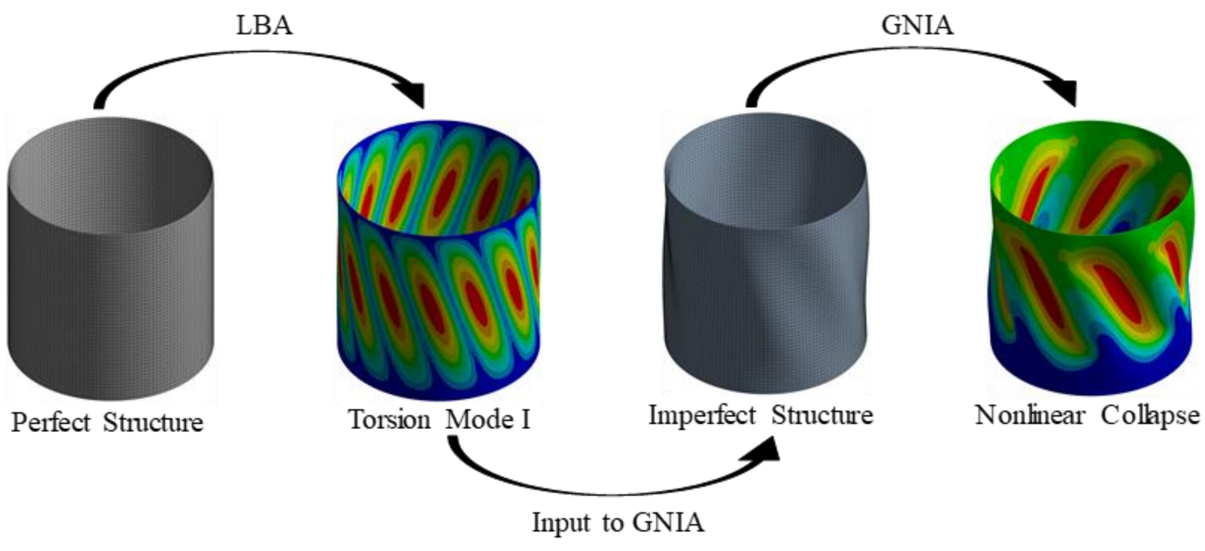

Figure 3. Flowchart of the non-linear buckling analysis of an imperfect cylindrical shell structure under axial compression. At the first step, LBA is applied to obtain torsional Mode-I type geometric imperfection. Secondly, the imperfect structure (deformed shape) exported to GNIA buckling analysis using a scale factor for the arrangement of imperfection depth Δw. At the last step, axial compression is applied to the imperfect shells until the global buckling occurrence.

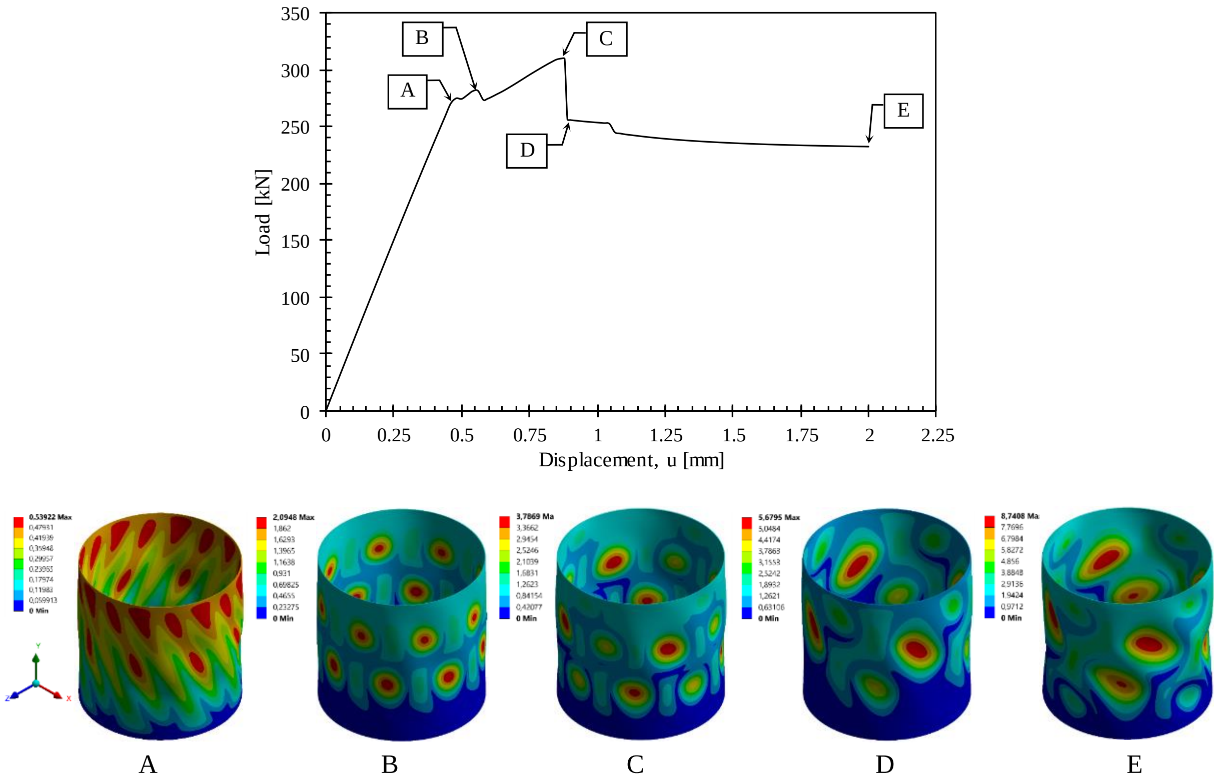

Figure 4. Load-displacement diagram of the imperfect shell with the critical buckling stages.

Change of KDF concerning *Δw/t* is plotted in Figure 5 to evaluate the role of *Δw/t* on the load-bearing capacity of geometrically imperfect cylindrical shells. To perform this task, a shell configuration for which *R/t* and *L/R* equal to 100 and 4 is selected, respectively. It is more convenient to divide KDF curve into three distinct regions (Region I, II and III). In Region I, it is seen that even very small *Δw/t* values cause a reduction of about 40% in load-bearing capacity (KDF≈0.6) of the cylinder. This is because cylindrical shells are a standard structure (membrane stress dominant) and are quite sensitive to imperfections. However, no additional change in KDF is observed until the next region since the imperfection depth does not reach a threshold value to induce extra bending stress in the shell structure.

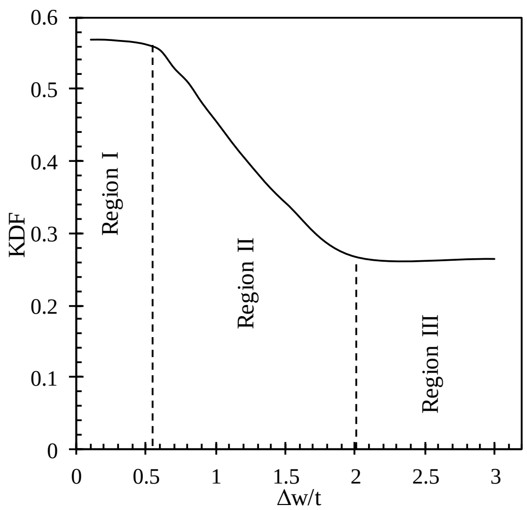

Figure 5. KDF vs *Δw/t* diagram

On the other hand, KDF drops drastically up to Region III due to additional bending effects of the *Δw/t* values. For this reason, Region II at which *Δw/t* varies between 0.6 and 2 may be called a critical interval in terms of torsional Mode-I type imperfection sensitivity (see Figure 5). Beyond this region, KDF reaches a stability region in which no further changes are observed.

Knockdown factor is a multivariate function of whole set of shell parameters such as, *R/t*, *L/R* and *Δw/t*. However, it is a quite challenging task to produce exact solutions of KDF as a function of the aforementioned shell parameters due to stochastic nature of geometric imperfections. For this reason, it is more practical to demonstrate a scattering of the KDF values for the whole set of shell families, as depicted in Figure 6. In this way, the variation of KDF at a certain shell configuration can be represented depending on the *Δw/t* values. In terms of design perspective, it is more useful to give lower and upper bound equations to evaluate the maximum and minimum KDF values at a particular *Δw/t*. Furthermore, it provides an interval for the KDF values to make sure that the shell structure with torsional Mode-I type of geometric imperfection is safe under the action of axial compression. The lower bound may be expressed with a preliminary exponential function, which is found using a trial error approach considering various form of mathematical functions, as the following:

$$(KDF)_{LB} = Ae^{-B\sqrt{\Delta w/t}} \tag{3}$$

Similarly, the upper bound may be expressed with a preliminary power function:

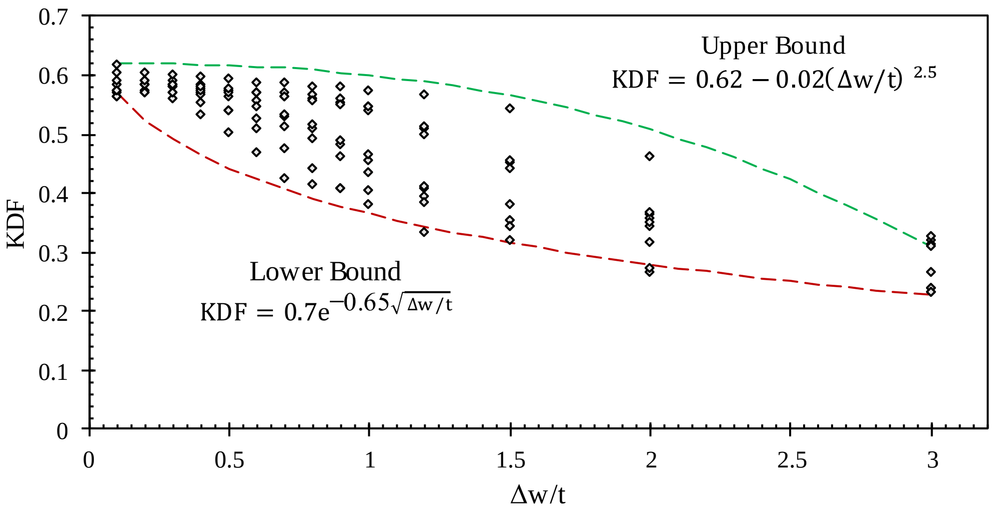

Figure 6. Scattering of the KDF values for the whole set of shell configurations

$$(KDF)_{UB} = C - D(\Delta w/t)^n \tag{4}$$

where *A*, *B*, *C*, *D* and *n* are positive real numbers. In this case, proposed expressions give more accurate trend of lower and upper bounds than polynomial or linear functions. Eqs. 3 and 4 predict the limit values of KDF's at a random cylindrical shell configuration regardless of R/t and L/R. The coefficients A, B, C, D and n are estimated using Least Square Method as 0.7, 0.65, 0.62, 0.02 and 2.5, respectively.

4. Conclusions

The current study investigates the influence of torsional Mode-I type of geometric imperfections on the load-bearing capacity of cylindrical shells under axial compression. A parametric study is performed covering a wide range of cylindrical shell configurations. Knockdown factors (KDF's) caused by the geometric imperfections are examined and relevant deformation patterns are illustrated. The results obtained from the current study are highlighted as the followings:

- It is concluded that even an inconsiderable amount of geometric imperfection depth may cause nearly 40% reduction in the load-bearing capacity of the cylindrical shells having a torsional Mode-I type of geometric imperfection.
- It is seen that KDF slightly increases with the increasing values of *R/t* and *L/R*. This is an indication of lower imperfection sensitivity to higher values of both *R/t* and *L/R*.
- The influence of *Δw/t* on the knockdown factor (KDF) can be characterized by three distinct regions: The first region is accepted to be a plateau indicating the threshold value of *Δw/t* for additional decrement of KDF. The second region is the critical range of the *Δw/t* values. No considerable change in KDF is achieved in the last region.
- The KDF values emerge a stochastic distribution (no visible correlation with the shell parameters) and it is difficult to establish an efficient relationship. However, Eqs. 3 and 4 are proposed to estimate a local minima and maxima of KDF's stochastic distribution as a function of *Δw/t.*

Declaration

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The authors also declared that this article is original, was prepared in accordance with international publication and research ethics, and ethical committee permission or any special permission is not required.

Author Contributions

İ. Kocabaş and H. Yılmaz conceived the study together. İ. Kocabaş supervised the research, analysed the data and drafted / finalized the paper. H. Yılmaz conducted the numerical analysis, contributed to the data analysis, proposals of equations and revised the paper.

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