On Deferred Statistical Convergence of Sequences of Sets in Metric Space

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Abstract.

In this paper mainly, Wijsman deferred statistical convergence of sequence of sets in an arbitrary metric space is defined and some basic theorems are given. Besides new results, some results in this paper are the generalization of the results given in [3], [15] and [18].

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1. INTRODUCTION

The concept of statistical convergence for real valued sequences has been introduced by Fast and Steinhaus independently in the same year 1951. Since then, several authors (Some of them, Fridy [6], Fridy-Miller [7], Fridy-Orhan [8], Kaya, et.al. [9], Kolk [10] and Mursaleen [14], e.t.c.) investigated this subject and it is applied different areas of mathematics, such as summability theory, analytic number theory, Fourier series, e.t.c.

Let K be a subset of $\mathbb{N}$ and $K(n) = \{k \leq n \ : \ k \in \mathbb{N}\}$. Then the asymptotic density of K is defined as follows if the limit exists

$$\delta(K) := \lim_{n \to \infty} \frac{|K(n)|}{n}$$

where the vertical bars denotes the cardinality of the inside set.

On the contrary to the convergence of the real valued sequences, in literature, there are only well known three type convergence method for sequence of sets: Wijsman, Hausdorff and Kuratowski (see [2], [11], [19]).

Beer [3] interested Kuratowski convergence of sequence of sets and obtained the Arzela-Ascoli Theorem from the compactness theorem for sequence of sets.

Nuray and Rhoades [15] defined Wijsman statistical convergence of sequence of sets. Later, in the paper [18] by using lacunary sequence, this concept is generalized to the lacunary statistical convergence and some parallel results in [15] is given.

Let us start with fundamental definitions.

Let (X, ρ) be a metric space. For any nonempty closed subsets $A, A_k \subseteq X$, we say that the sequence (A_k) is Wijsman convergent to A if,

$$\lim_{k \to \infty} d_x(A_k) = d_x(A) \tag{1.1}$$

for each $x \in X$.

In (1.1) the symbol $d_x(B)$ denotes the distance of the point x to the set B and defined by

$$d_x(B) := \inf\{\rho(x, a) \ : \ a \in B\}$$

Wijsman convergency of the sequence (A_n) to A is denoted by $W - \lim A_n = A$.

In [1], Agnew defined the deferred Cesàro mean as a generalization of Cesàro mean for real or complex valued sequences $x = (x_n)$ by

$$\frac{1}{q(n) - p(n)} \sum_{k=p(n)+1}^{q(n)} x_k$$

where $p = (p(n))$ and $q = (q(n))$ are the sequences of non-negative integers satisfying

$$p(n) < q(n) \text{ and } \lim_{n \to \infty} q(n) = \infty. \tag{1.2}$$

Throught the paper, we will use p and q instead of $p(n)$ and $q(n)$ only for simplicity.

The sequence (A_k) is said to be Wijsman strongly deferred Cesàro summable to the set A if for each $x \in X$,

$$\lim_{n \to \infty} \frac{1}{q - p} \sum_{k=p+1}^{q} |d_x(A_k) - d_x(A)| = 0 \tag{1.3}$$

hold. In this case, we write $WD - \lim_{n \to \infty} A_k = A$.

Deferred density of $K \subset \mathbb{N}$ is defined as if the limit exists:

$$\delta_D(K) := \lim_{n \to \infty} \frac{1}{q - p} |\{p < k \le q \ : \ k \in K\}|.$$

The sequence (A_k) is said to be Wijsman deferred statistically convergent to A if for every $\varepsilon > 0$ and $x \in X$,

$$\lim_{n \to \infty} \frac{1}{q - p} |\{p < k \le q \ : |d_x(A_k) - d_x(A)| \ge \varepsilon\}| = 0 \tag{1.4}$$

hold. In this case, we write $WDS - \lim_{k \to \infty} A_k = A$.

If we consider the cases, (i) $p = 0$, $q = n$ and (ii) $p = k_{n-1}$, $q = k_n$ for a lacunary sequence $\theta = (k_n)$ in (1.4) and (1.3), these two concept coincide with Wijsman statistical convergence-Wijsman strongly Cesàro summability and Wijsman lacunary statistical convergence-Wijsman lacunary strongly summability of sequence of sets (see [15],[18]) respectively.

Theorem 1.1. *Let (X, ρ) be a metric space. If $(A_n)_{n \in \mathbb{N}}$, $(B_n)_{n \in \mathbb{N}}$ and $(C_n)_{n \in \mathbb{N}}$ are sequences of sets such that the inclusion $A_n \subset B_n \subset C_n$ hold for all $n \in \mathbb{N}$. Then, the following statements are hold.*
(i) If $W - \lim A_n = W - \lim C_n = T$, then $W - \lim B_n = T$,
(ii) If $WDS - \lim A_n = WDS - \lim C_n = T$, then $WDS - \lim B_n = T$.

Proof. The case (i) can be proof easily by using standart methods of analysis. So, it is omitted here.

(ii) Let $x \in X$ be an arbitrary fixed point and consider the sequences $(d_x(A_n))_{n \in \mathbb{N}}$, $(d_x(B_n))_{n \in \mathbb{N}}$ and $(d_x(C_n))_{n \in \mathbb{N}}$. It is clear from the inclusion $A_n \subset B_n \subset C_n$ that the inequality

$$(1.5) \qquad d_x(C_n) \leq d_x(B_n) \leq d_x(A_n)$$

holds for all $n \in \mathbb{N}$. From this inequality, we have

$$\begin{aligned}
\{ \quad & p < k \leq q \ : \ |d_x(B_k) - d_x(T)| \geq \varepsilon \} = \\
= \quad & \{p < k \leq q \ : \ d_x(B_k) \geq d_x(T) + \varepsilon\} \cup \\
& \cup \{p < k \leq q \ : \ d_x(B_k) \leq d_x(T) - \varepsilon\} \\
\subset \quad & \{p < k \leq q \ : \ d_x(A_k) \geq d_x(T) + \varepsilon\} \cup \\
& \cup \{p < k \leq q \ : \ d_x(C_k) \leq d_x(T) - \varepsilon\}
\end{aligned}$$

for $\varepsilon > 0$. It is also clear that the following inclusion

$$\begin{aligned}
\{p < k \leq q \quad : \quad & |d_x(A_k) - d_x(T)| \geq \varepsilon\} \\
\supset \quad & \{p < k \leq q \ : \ d_x(A_k) \geq d_x(T) + \varepsilon\},
\end{aligned}$$

and

$$\begin{aligned}
\{p < k \leq q \quad : \quad & |d_x(C_k) - d_x(T)| \geq \varepsilon\} \\
\supset \quad & \{p < k \leq q \ : \ d_x(C_k) \leq d_x(T) - \varepsilon\}
\end{aligned}$$

are true and we have

$$\delta_D(\{p < k \leq q \ : \ d_x(A_k) \geq d_x(T) + \varepsilon\}) = 0,$$
$$\delta_D(\{p < k \leq q \ : \ d_x(C_k) \leq d_x(T) - \varepsilon\}) = 0.$$

Therefore,

$$\delta_D(\{p < k \leq q \ : \ |d_x(B_k) - d_x(T)| \geq \varepsilon\}) = 0.$$

This gives the desired proof.

$\square$

Definition 1.2. Let $(A_n)_{n \in \mathbb{N}}$ and $(B_n)_{n \in \mathbb{N}}$ be sequences of sets. If deferred density of $\{n \in \mathbb{N} \ : \ A_n \neq B_n\}$ is zero, then we say that the sequence $(A_n)_{n \in \mathbb{N}}$ is deferred almost all eqiual to the sequence $(B_n)_{n \in \mathbb{N}}$ and it is denoted by $(A_n) \equiv (B_n)$ $(D - a.a.e.)$.

Theorem 1.3. *Let (X, ρ) be a metric space, $(A_n)_{n \in \mathbb{N}}$ and $(B_n)_{n \in \mathbb{N}}$ be sequences of sets such that $(A_n) \equiv (B_n)$, $(D - a.a.e.)$. Then, Wijsman deferred statistical convergency of the sequence $(A_n)_{n \in \mathbb{N}}$ implies Wijsman deferred statistical convergency of the sequence $(B_n)_{n \in \mathbb{N}}$, vice versa.*

Proof. Assume that $WDS - \lim_{n \to \infty} A_n = A$, i.e.

$$(1.6) \qquad \lim_{n \to \infty} \frac{1}{q - p} |\{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}| = 0$$

holds for $x \in X$.

Since $A_k \neq B_k$ $(D - a.a.e.)$, then we have

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(1.7)
$$\lim_{n\to\infty} \frac{1}{q-p}|\{p < k \leq q \ : \ A_k \neq B_k\}| = 0.$$

Also, the set
$$\{p < k \leq q \ : \ |d_x(B_k) - d_x(A)| \geq \varepsilon\},$$

can be represent as

(1.8)
$$\{p < k \leq q \ : \ A_k = B_k\} \cup \{p < k \leq q \ : \ A_k \neq B_k\}$$

for the k when $|d_x(B_k) - d_x(A)| \geq \varepsilon$.

From (1.6), (1.7) and (1.8) we have

$$\lim_{n\to\infty} \frac{1}{q-p}|\{p < k \leq q \ : \ |d_x(B_k) - d_x(A)| \geq \varepsilon\}| = 0$$

and this gives the proof. The converse case can be prove suitable changes. $\qquad\square$

Corollary 1.4. *Let (X,ρ) be a metric space, $(A_n)_{n\in\mathbb{N}}, (B_n)_{n\in\mathbb{N}}$ and $(C_n)_{n\in\mathbb{N}}$ be a sequences of sets such that $A_n \subset B_n \subset C_n$ $(D-a.a.e.)$.*

If $WDS - \lim_{n\to\infty} A_n = WDS - \lim_{n\to\infty} C_n = T$, then $WDS - \lim_{n\to\infty} B_n = T$.

Definition 1.5. A sequence $(A_n)_{n\in\mathbb{N}}$ of sets is said to be bounded if for each $x \in X$ there exists positive M_x such that $d_x(A_n) < M_x$ for all $n \in \mathbb{N}$. The set of all bounded sequence of sets is denoted by L_∞, i.e.,

$$L_\infty := \{(A_n)_{n\in\mathbb{N}} \ : \ \sup_{n\in\mathbb{N}} d_x(A_n) < \infty \text{ for each } x \in X\}.$$

Theorem 1.6. *Let (X,ρ) be a metric space, $(A_n)_{n\in\mathbb{N}}$ be a sequence of sets and A be a nonempty closed subset of X. Then, the following statements are true:*

(i) If $WD - \lim_{n\to\infty} A_n = A$, then $WDS - \lim_{n\to\infty} A_n = A$.

(ii) The converse of (i) is not true in general.

(iii) $WDS \cap L_\infty = WD \cap L_\infty$.

Proof. (i) Since $WD - \lim_{n\to\infty} A_n = A$, then we have

$$\lim_{n\to\infty} \frac{1}{q-p} \sum_{k=p+1}^{q} |d_x(A_k) - d_x(A)| = 0,$$

for $x \in X$. Therefore, the following inequality

$$\frac{1}{q-p} \sum_{k=p+1}^{q} |d_x(A_k) - d_x(A)| \geq \frac{1}{q-p} \sum_{\substack{k=p+1 \\ |d_x(A_k)-d_x(A)|\geq\varepsilon}}^{q} |d_x(A_k) - d_x(A)|$$

$$\geq \frac{1}{q-p}|\{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}|.\varepsilon$$

hold for every $\varepsilon > 0$. Letting limit for $n \to \infty$ we obtain desired proof.

(ii) Let us consider the sequence $(A_k)_{k\in\mathbb{N}}$ as follows for an arbitrary p and q satisfying (1.1):

$$A_k = \begin{cases} \{k\}, & p < k \leq p + [\![\sqrt{q-p}]\!], \ n \in \mathbb{N}, \\ \{0\}, & otherwise. \end{cases}$$

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It is clear that

$$\lim_{n\to\infty} \frac{1}{q-p}|\{p < k \le q \quad : \quad |d_x(A_k) - d_x(\{0\})| \ge \varepsilon\}| =$$
$$= \lim_{n\to\infty} \frac{[\![\sqrt{q-p}]\!]}{q-p} = 0,$$

but

$$\lim_{n\to\infty} \frac{1}{q-p} \sum_{k=p+1}^{q} |d_x(A_k) - d_x(\{0\})| \ge \lim_{n\to\infty} \frac{[\![\sqrt{q-p}]\!]([\![\sqrt{q-p}]\!] + 1)}{2(q-p)}$$
$$= \frac{1}{2} \ne 0$$

This shows that the converse of (i) is not true.

(iii) Let $(A_k) \in WDS \cap L_\infty$ be an arbitrary sequence of sets. From the boundedness of $(A_k)_{k\in\mathbb{N}}$ for each $x \in X$ there exists a positive number M_x such that $d_x(A_k) \le M_x$ for all $k \in \mathbb{N}$. Therefore,

$$|d_x(A_k) - d_x(A)| \le d_x(A_k) + d_x(A) \le M_x + d_x(A)$$

holds for all $x \in X$. So, for any $\varepsilon > 0$, we have

$$\frac{1}{q-p} \sum_{k=p+1}^{q} |d_x(A_k) - d_x(A)| =$$

$$= \frac{1}{q-p} \left(\sum_{\substack{k=p+1 \\ |d_x(A_k)-d_x(A)|\ge\varepsilon}}^{q} + \sum_{\substack{k=p+1 \\ |d_x(A_k)-d_x(A)|<\varepsilon}}^{q} \right) |d_x(A_k) - d_x(A)|$$

$$\le \frac{2M_x'}{q-p}|\{p < k \le q \ : \ |d_x(A_k) - d_x(A)| \ge \varepsilon\}| + \varepsilon$$

where $M_x' = M_x + d_x(A)$.

After taking limit for $n \to \infty$, the proof is obtained. $\qquad\square$

Definition 1.7. A method $D[p,q]$ is called properly deferred, if the sequence $\left(\frac{p}{q-p}\right)$ is bounded in addition to $p = (p_n)_{n\in\mathbb{N}}$ and $q = (q_n)_{n\in\mathbb{N}}$ are satisfying (1.2).

In the following theorem the method WS and WDS are compared under some restriction on $p(n)$ and $q(n)$.

Theorem 1.8. $WS \subset WDS$ if and only if D is proper.

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Proof. Let $(A_n)_{n\in\mathbb{N}}$ be a sequence of closed subsets of (X,d) and assume that $WS-\lim_{n\to\infty} A_n = A$. Then, WDS transformation of the sequence $(A_n)_{n\in\mathbb{N}}$ is

$$\frac{1}{q-p}|\{p < k \le q \quad : \quad |d_x(A_k) - d_x(A)| \ge \varepsilon\}| =$$

$$= \frac{1}{q-p}|\{1 \le k \le q \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}|$$

$$-\frac{1}{q-p}|\{1 \le k \le p \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}|$$

$$= (\frac{q}{q-p})\frac{1}{q}|\{1 \le k \le q \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}|$$

$$-(\frac{p}{q-p})\frac{1}{p}|\{1 \le k \le p \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}|.$$

Regarding to the above equality we can say that WDS transformation of the sequence $(d_x(A_n))$ is the linear combination of the WS transformation of the sequence $(d_x(A_n))$.

It is clear from the Silverman-Toeplitz Theorem [13] that this transformation is regular if and only if $\left[\frac{p}{q-p} + \frac{q}{q-p}\right]$ is bounded. That is, the sequence $\left(\frac{p}{q-p}\right)$ must be bounded. $\square$

Theorem 1.9. $WS \supset WDS[p,q]$ *when* $q(n) = n$ *for all* $n \in \mathbb{N}$.

Proof. Let $(A_n)_{n\in\mathbb{N}}$ be a sequence of sets which is $WDS-$convergent to a set $A \subset X$ for $x \in X$. We shall show that $A_n \to A(WS)$. Let for each $n \in \mathbb{N}$

$$p(n) = n^{(1)} > p(n^{(1)}) = n^{(2)} > p(n^{(2)}) = n^{(3)} > \cdots,$$

From this partition we have

$$\{k \le n \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\} = \{k \le n^{(1)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}$$
$$\cup \; \{n^{(1)} < k \le n \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}$$

and

$$\{k \le n^{(1)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\} = \{k \le n^{(2)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}$$
$$\cup \; \{n^{(2)} < k \le n^{(1)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}.$$

If we continue this process consecutively we obtain

$$\{k \le n^{(h-1)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\} = \{k \le n^{(h)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}$$
$$\cup \; \{n^{(h)} < k \le n^{(h-1)} \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}$$

where h is a certain positive integer such that $n^{(h)} \ge 1$ and $n^{(h+1)} = 0$.

From the above discussion, we have

$$\frac{1}{n}|\{k \le n \; : \; |d_x(A_k) - d_x(A)| \ge \varepsilon\}| = \sum_{m=0}^{h} \left(\frac{n^{(m)} - n^{(m+1)}}{n}\right) L_m$$

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for every n, where L_m is a sequence as follows

$$(L_m) := \left(\frac{1}{n^{(m)} - n^{(m+1)}} |\{ n^{(m+1)} < k \le n^{(m)} \ : \ |d_x(A_k) - d_x(A)| \ge \varepsilon \}| \right).$$

Let us consider the matrix $T = (t_{n,m})$ as

$$t_{n,m} := \begin{cases} \frac{n^{(m)} - n^{(m+1)}}{n}, & m = 0, 1, 2, \cdots, h \\ 0 \quad, & otherwise. \end{cases}$$

where $n^{(0)} = n$.

It is clear that Wijsman statistical transformation of the sequence $(A_n)_{n \in \mathbb{N}}$ is the T transformation of the sequence (L_m).

The matrix $T = (t_{n,m})$ is satisfied regularity conditions (see in [13],[16]). Therefore, the sequence $(A_n)_{n \in \mathbb{N}}$ Wijsman statistical convergence to A at the point $x \in X$.
□

From Theorem 1.8 and Theorem 1.9, we have following result.

Corollary 1.10. $WS = WDS[p, n]$ *if and only if*

$$\left(\frac{p}{n - p} \right)$$

is bounded.

Theorem 1.11. *Let $q = (q(n))_{n \in \mathbb{N}}$ be a sequence which contains almost all natural numbers. Then,*
$WDS - \lim_{n \to \infty} A_n = A$ *implies* $WS - \lim_{n \to \infty} A_n = A$.

Proof. Let us choose sufficiently large natural numbers m such that the set $\{q(n) \ : \ n \in \mathbb{N}\}$ contains all natural numbers which is greater than m. Then, we may set a sequence of natural numbers such that

$$k_1 = k_2 = \cdots = k_m = 1$$

and $q_{k_n} = n$ for each $n > m$.

Since $WDS - \lim_{n \to \infty} A_n = A$ for p and q, then we have

$$WDS - \lim_{n \to \infty} A_n = A$$

for p_{k_n} and q_{k_n}. Therefore, theorem follows from Theorem 1.9.
□

From the above Theorems it is clear that any proper WDS method and WS method are mutually consistent. Unfortunately, this is not true for any two proper WDS method. For to see this, let us consider $WDS[2n, 4n]$ and $WDS[2n-1, 4n-1]$, and sequence of sets as

$$A_n = \begin{cases} \{ \frac{-n}{2} \}, & n \ is \ even, \\ \\ \{ \frac{n+1}{2} \}, & n \ is \ old. \end{cases}$$

It is clear that

$$WDS[2n, 4n] - \lim_{n \to \infty} A_n = \{0\},$$

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and

$$WDS[2n - 1, 4n - 1] - \lim_{n \to \infty} A_n = \left\{ \frac{1}{2} \right\}.$$

Let us consider the sequences $p = (p(n))$, $q = (q(n))$, $p' = (p'(n))$ and $q' = (q'(n))$ such that

(1.9)
$$p \leq p' < q' \leq q$$

for all $n \in \mathbb{N}$.

In the following theorems by considering (1.9), the methods $WDS[p', q']$ and $WDS[p, q]$ are compared.

Theorem 1.12. *If the sets $\{k \ : \ p < k \leq p'\}$ and $\{k \ : \ q' < k \leq q\}$ are finite for all $n \in \mathbb{N}$, then*

$$WDS - \lim_{n \to \infty} A_n = A \ w.r.t \ (p' \ and \ q')$$

implies

$$WDS - \lim_{n \to \infty} A_n = A \ w.r.t \ (p \ and \ q).$$

Proof. Let us assume that for each $x \in X$ the sequence of sets $(A_n)_{n \in \mathbb{N}}$ is Wijsman deferred statistical convergent to A with respect to p' and q'. For an arbitrary $\varepsilon > 0$, we have

$$\{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\} = \{p < k \leq p' \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\} \cup$$
$$\cup \{p' < k \leq q' \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\} \cup \{q' < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}.$$

It is also clear that following inequality

$$\frac{1}{q - p}|\{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}| \leq$$

$$\leq \quad \frac{1}{q' - p'}|\{p < k \leq p' \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}| +$$

$$+ \quad \frac{1}{q' - p'}|\{p' < k \leq q' \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}| +$$

$$+ \quad \frac{1}{q' - p'}|\{q' < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}|$$

holds. On taking limit for $n \to \infty$, it is obtained that

$$\lim_{n \to \infty} \frac{1}{q - p}|\{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\}| = 0.$$

$\square$

Theorem 1.13. *Under the condition (1.9), if $\left(\frac{q-p}{q'-p'} \right)$ is bounded then $WDS[p, q] \subset WDS[p', q']$.*

Proof. Since the inclusion

$$\{p' < k \leq q' \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\} \subset \{p < k \leq q \ : \ |d_x(A_k) - d_x(A)| \geq \varepsilon\},$$

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is hold, then we have

$$\frac{1}{q'-p'}|\{p' < k \le q' \quad : \quad |d_x(A_k) - d_x(A)| \ge \varepsilon\}| \le$$

$$\le \quad \frac{q-p}{q'-p'}\frac{1}{q-p}|\{p < k \le q \ : \ |d_x(A_k) - d_x(A)| \ge \varepsilon\}|.$$

If we take limit for $n \to \infty$, the proof is completed. $\qquad\qquad\square$

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