Turkish **J**ournal of **M**athematics and **C**omputer **S**cience
TJMCS

http://tjmcs.matder.org.tr/

@TJMCS
© MatDer
http://www.matder.org.tr/

Location and Simultaneous Correction of Burst/Solid Burst Errors

Pankaj Kumar Das

Abstract. This paper proposes a class of error locating codes that locates burst/solid bursts of length s or less as well as corrects burst/solid bursts of length $b(< s)$ or less. Lower and upper bounds on parity check digits for such codes are obtained. Examples of such codes are also provided. Further, comparisons between the bounds of these codes with other types of codes:
(a) codes that correct all bursts/solid bursts of length s or less, and
(b) codes that detect solid bursts of length s or less as well as correct solid bursts of length $b(< s)$ or less,
are also provided.

2010 AMS Classification: 94B05, 94B20, 94B65

Keywords: Syndromes, Parity check digits, Burst, Solid bursts, Error locating codes.

1. Introduction

Wolf and Elspas [18] introduced the concept of error location coding which is the midway between error detection and error correction. The codes using such concept are known as *error locating codes* and they are found to be efficient in feedback communication systems. In such systems, the code length is divided into some finite number of mutually exclusive sub-blocks and the receiver tests each sub-block of received digits for the presence of errors. If the code detects an error, the code has the capacity of locating the corrupted sub-block. Then the system can request the retransmission of the corrupted sub-block instead of the whole block and this process may be repeated for each incoming corrupted sub-block. The length of the sub-blocks can be chosen relatively small which requires less number of parity checks and this improves the information rate of the system.

During transmission of data from one place to another, data may be encountered with errors due to noise in the channel. The nature of errors depends upon the behaviour of the communication channel. In channels, like radio channels, telephone line, errors follow a particular pattern. They occur in a clustered way, not independently. This type of errors is known as burst errors. A burst of length b, due to Fire [9], may be defined as follows:

Definition 1.1. *A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which is nonzero.*

Further, in some communication channels viz. semiconductor memory data [11], supercomputer storage system [1], errors are not only in clustered way, but also all the clustered components are in error. Such errors are known as solid burst and may be defined as follows:

Definition 1.2. *A solid burst of length b is a vector whose all the b consecutive components are nonzero and rest are zero.*

For more study on solid bursts, one may refer to [2, 3, 4, 5, 6, 14, 15, 16, 17].

Fujiwara and Kitakami [10] proposed a class of error locating codes that is suitable to computer memory systems organized with b-bit byte-organized semiconductors memory systems. They have further proposed another class of error locating codes [12] that corrects all single-bit errors and indicate a location of the erroneous byte which includes e-bit errors. This type of codes is very suitable for an application to memory systems. This motivates us to study a class of codes that corrects any burst/solid burst of length b or less and simultaneously locates any burst/solid burst of length $s(> b)$ or less . In [2, 3, 13, 7], codes that correct and locate bursts/solid bursts of length s or less separately are studied. For simultaneous correction and location of burst/solid burst errors, the codes studied in these papers will not be efficient. In view of this, it is important to study codes that are capable of not only locating such errors, but also correcting some of them.

The paper is organized as follows. Section 1 i.e., the Introduction gives brief view of the importance of the study of the paper, basic definition and some related works to our study. In Section 2, we obtain gives lower and upper bounds on the number of parity check digits required for a linear code that locates any solid burst of length s or less and simultaneously corrects any solid burst of length $b(< s)$ or less. This is followed by an example. In Section 3, similar bounds are obtained for burst error, followed by an example. Further, Section 4 provides comparisons between the necessary (lower bound) and sufficient numbers (upper bound) of parity check digits of these codes with other types of codes mentioned in abstract. Similar comparisons are given for bounds obtained in Section 3 and codes correcting burst errors. Section 5 gives the conclusion.

For a (n, k) linear code over $GF(q)$ capable of locating any (solid) burst of length s or less within a sub-block and simultaneously correcting any (solid) burst of length $b(< s)$ or less, the following conditions are required to be satisfied [8]:
(i) The syndrome resulting from the occurrence of a (solid) burst error of length b or less must be non-zero and distinct from the syndromes resulting from any other (solid) burst errors of length b or less.
(ii) The syndrome resulting from the occurrence of a (solid) burst of length s or less

within any one sub-block must be distinct from the all zero-syndrome.
(*iii*) The syndrome resulting from the occurrence of any (solid) burst of length s or less within a single sub-block must be distinct from the syndrome resulting likewise from (solid) burst of length s or less within any other sub-block.
(*iv*) The syndrome resulting from the occurrence of any (solid) burst of length s or less within any single sub-block must be distinct from the syndrome resulting likewise from any (solid) burst of length b or less.

Kindly note that for the (solid) burst of length b or less, the conditions (*ii*) to (*iv*) are not required to be considered, the condition (*i*) is sufficient. In the paper, by a linear code we mean to be a subspace of n-tuples over $GF(q)$. The length n of the code that consists of k information digits and $n-k$ parity check digits, is divided into m mutually exclusive sub-blocks. The length of each sub-block is $t = \frac{n}{m}$.

2. Lower and Upper Bounds for Solid Burst Error

In this section, we study linear codes over $GF(q)$ that are capable of locating all solid burst of length s or less within a sub-block and simultaneously correcting any solid burst of length $b(< s)$ or less. First, we provide a lower bound on the number of parity check digits required for such a code. The proof is based on the technique used in Theorem 4.16, Peterson and Weldon [13].

Theorem 2.1. *The number of parity check digits in an (n, k) linear code over $GF(q)$ subdivided into m sub-blocks of length t each, that locates any solid burst of length s or less within a sub-block and simultaneously corrects any solid burst of length $b(< s)$ or less is bounded from below by*

$$n-k \geq \log_q \left\{ 1 + \sum_{i=1}^{b} (n-i+1)(q-1)^i + m \sum_{i=b+1}^{s} (q-1)^i \right\}. \tag{2.1}$$

Proof. The theorem is proved by counting the number of syndromes according to the conditions $(i)-(iv)$ and then setting this number less than or equal to q^{n-k} , the number of maximum possible syndromes.
For correcting solid bursts of length b or less, according to the condition (*i*), syndromes produced by such errors must be nonzero and distinct. The number of such syndromes is

$$\sum_{i=1}^{b} (n-i+1)(q-1)^i. \quad \text{(refer [2])}$$

We know that the conditions (*ii*) to (*iv*) are taken care of by condition (*i*) for solid burst of length b or less. Therefore for locating solid burst of length $s(> b)$ or less, we need to count the syndromes produced by solid burst of length i only, where $b+1 \leq i \leq s$. The number of such syndromes is given by

$$\sum_{i=b+1}^{s} (q-1)^i. \quad \text{(refer [3])}$$

Thus, the total number of such syndromes including the vector of all zeros is atleast

$$1+\sum_{i=1}^{b}(n-i+1)(q-1)^i+m\sum_{i=b+1}^{s}(q-1)^i.$$

Therefore, we must have

$$q^{n-k}\geq 1+\sum_{i=1}^{b}(n-i+1)(q-1)^i+m\sum_{i=b+1}^{s}(q-1)^i.$$

or,

$$n-k\geq \log_q\left\{1+\sum_{i=1}^{b}(n-i+1)(q-1)^i+m\sum_{i=b+1}^{s}(q-1)^i\right\}.$$

□

Now we provide a theorem that gives an upper bound on the number of check digits required for the construction of a linear code considered in Theorem 2.1. This bound assures the existence of such a linear code. The proof is based on the technique used in Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [14], also Theorem 4.7 Peterson and Weldon [13]).

Theorem 2.2. *There shall always exist an (n,k) linear code over GF(q) subdivided into m sub-blocks of length t each, that locates any solid burst of length s or less within a sub-block and simultaneously corrects any solid burst of length $b(<s)$ or less provided that*

$$\begin{aligned} q^{n-k} \quad > \quad & 1+\sum_{i=1}^{b}\sum_{l=1}^{b}(n-l-i+1)(q-1)^{i+l-1}+\sum_{i=b}^{s-1}(q-1)^i \\ & +(m-1)\left\{\sum_{i=b+1}^{s}(t-i+1)(q-1)^i+\sum_{i=1}^{s}\sum_{l=1}^{s-1}(t-i+1)(q-1)^{i+l}\right. \\ & \left.-\sum_{i=1}^{b}\sum_{l=1}^{b-1}(t-i+1)(q-1)^{i+l}\right\}. \end{aligned}$$

Proof. By constructing an $(n-k)\times n$ parity check matrix H suitably as below for the desired code, we can prove the existence of such a code.

Suppose that the columns of the first $m-1$ sub-blocks of H and the first $j-1$ columns $h_1,h_2,\ldots,h_{j-1}$ of the m^{th} sub-block have been added appropriately. We lay down the condition to add j^{th} column of the m^{th} sub-block as follows:
According to condition (i), h_j should not be a linear sum of immediately preceding up to $b-1$ consecutive columns $h_{j-1},h_{j-2},\ldots,h_{j-b+1}$, together with any b or fewer consecutive columns from amongst the first $j-b$ columns $h_1,h_2,\ldots,h_{j-b}$, i.e.,

$$\begin{aligned} h_j \quad \neq \quad & (u_{j-1}h_{j-1}+u_{j-2}h_{j-2}+\cdots+u_{j-\alpha}h_{j-\alpha}) \\ & +(v_ih_i+v_{i+1}h_{i+1}+\cdots+v_{i+\beta-1}h_{i+\beta-1}), \end{aligned}$$

where $u_i, v_i \in GF(q)$ are nonzero coefficients; $\alpha \leq b-1$, $\beta \leq b$ and the columns h_i's in the second bracket are any b or less consecutive columns among the first $(j-1-\alpha)$ columns.

This condition ensures that there shall not be a code vector which can be expressed as sum (difference) of two solid bursts of length b or less each. The number of choices of these coefficients is given by (refer [2])

$$\sum_{i=1}^{b}\sum_{l=1}^{b}(n-l-i+1)(q-1)^{i+l-1}. \tag{2.2}$$

Now according to condition (ii) and (iii), the syndrome of any solid burst error of length $s(>b)$ or less within a sub-block must be nonzero and distinct from those syndromes resulting from such errors within any other sub-block.
In view of this, h_j can be added provided that

$$h_j \neq u_{j-1}h_{j-1} + u_{j-2}h_{j-2} + \cdots + u_{j-l}h_{j-l},$$
where $l < s$ and the coefficients u_i's are nonzero.

Also,

$$\begin{aligned} h_j \quad &\neq \quad (u_{j-1}h_{j-1} + u_{j-2}h_{j-2} + \cdots + u_{j-\alpha+1}h_{j-\alpha+1}) \\ &\quad\quad +(v_i h_{l+i} + v_{i+1}h_{l+i+1} + \cdots + v_{l+i+\beta-1}h_{l+i+\beta-1}), \end{aligned}$$

where $\alpha, \beta \leq s$; $u_i, v_i \in GF(q)$ are nonzero and h_{l+i}'s are any s (or less) consecutive columns corresponding to any l^{th} sub-block among the $m-1$ sub-blocks.

The number of such linear combinations is given by (refer [3])

$$\left\{1+\sum_{i=1}^{s-1}(q-1)^i\right\}\left\{1+(m-1)\sum_{i=1}^{s}(t-i+1)(q-1)^i\right\}. \tag{2.3}$$

Again according to condition (iv), the syndrome resulting from the occurrence of any solid burst of length s or less within any single sub-block must be distinct from the syndromes resulting from solid bursts of length b or less. But, the number of syndromes of solid bursts of length b or less is already computed in *expr.*(2.2). Therefore, the number of syndromes computed in *expr.*(2.3) and distinct from *expr.*(2.2), is given by

$$\left\{1+\sum_{i=1}^{s-1}(q-1)^i\right\}\left\{1+(m-1)\sum_{i=1}^{s}(t-i+1)(q-1)^i\right\}$$
$$-\left\{1+\sum_{i=1}^{b-1}(q-1)^i\right\}\left\{1+(m-1)\sum_{i=1}^{b}(t-i+1)(q-1)^i\right\}$$
$$=\sum_{i=b}^{s-1}(q-1)^i+(m-1)\left\{\sum_{i=b+1}^{s}(t-i+1)(q-1)^i\right.$$
$$\left.+\sum_{i=1}^{s}\sum_{l=1}^{s-1}(t-i+1)(q-1)^{i+l}-\sum_{i=1}^{b}\sum_{l=1}^{b-1}(t-i+1)(q-1)^{i+l}\right\}. \tag{2.4}$$

Thus, the total number of linear combinations to which h_j can not be equal is given by $expr.(2.2)+expr.(2.4)$.
At worst all these combinations might yield distinct sum. Therefore, h_j can be added to H provided that

$$q^{n-k}>1+expr.(2.2)+expr.(2.4)$$

or

$$q^{n-k} > 1+\sum_{i=1}^{b}\sum_{l=1}^{b}(j-l-i+1)(q-1)^{i+l-1}+\sum_{i=b}^{s-1}(q-1)^i$$
$$+(m-1)\left\{\sum_{i=b+1}^{s}(t-i+1)(q-1)^i+\sum_{i=1}^{s}\sum_{l=1}^{s-1}(t-i+1)(q-1)^{i+l}\right.$$
$$\left.-\sum_{i=1}^{b}\sum_{l=1}^{b-1}(t-i+1)(q-1)^{i+l}\right\}.$$

Replacing j by n gives the theorem. □

Example 2.1. For a (10, 4) linear code over $GF(2)$, we construct the following 6×10 parity check matrix H, according to the synthesis procedure given in the proof of Theorem 2.2 by taking $t=5$, $m=2$, $b=2$ and $s=3$.

$$H=\begin{bmatrix}1&0&0&0&0&&0&1&0&0&1\\0&1&0&0&0&&0&0&1&0&1\\0&0&1&0&0&&0&1&0&1&1\\0&0&0&1&0&&0&1&1&0&1\\0&0&0&0&1&&0&0&1&1&1\\0&0&0&0&0&&1&0&0&1&1\end{bmatrix}$$

It can be seen from Table 3.1 that the syndromes of any solid burst of length 2 or less are all nonzero and distinct. Further, the syndromes of any solid burst of length 3 are all nonzero and distinct in different sub-blocks and also distinct from those resulting from solid burst of length 2 or less. This shows that the code that is the null space of this matrix can locate any solid burst of length 3 or less within a sub-block and simultaneously correcting solid bursts of length 2 or less.

Table 3.1
Error pattern - syndromes Table

Error-patterns	Syndromes	Error-patterns	Syndromes
For correction		0001100000	000110
Solid burst of length 1		0000110000	000011
1000000000	100000	0000011000	101101
0100000000	010000	0000001100	111010
0010000000	001000	0000000110	011101
0001000000	000100	0000000011	110100
0000100000	000010	**For location**	
0000010000	000001	Solid burst of length 3	
0000001000	101100	1st sub-block	
0000000100	010110	1110000000	111000
0000000010	001011	0111000000	011100
0000000001	111111	0011100000	001110
1100000000	110000	0000011100	111011
0110000000	011000	0000001110	110001
0011000000	001100	0000000111	100010

Remark 2.1. *It can also be verified that syndromes of the solid bursts of length 3 are all nonzero and distinct among selves as well as distinct from syndromes of solid bursts of length 2 or less. Therefore, the code discussed above not only corrects all solid bursts of length 2 or less, but also corrects all solid bursts of length 3.*

3. Lower and Upper Bounds for Burst Error

This section extends the study of Section 2 to burst error, defined by Fire [9]. We obtain the lower and upper bounds on the number of parity-check digits required linear codes over $GF(q)$ that are capable of locating all bursts of length s or less within a sub-block and simultaneously correcting any burst of length $b(< s)$ or less.

Theorem 3.1. *The number of parity check digits in an (n, k) linear code over $GF(q)$ subdivided into m sub-blocks of length t each, that locates any burst of length s or less within a sub-block and simultaneously corrects any burst of length $b(< s)$ or less is bounded from below by*

$$n-k \geq b-1+\log_q\left\{(n-b+1)(q-1)+m(q^{s-b+1}-q)+1\right\}.$$

Proof. The theorem is also proved by the same method used in Theorem 2.1, i.e., by counting the number of syndromes according to the conditions $(i)-(iv)$ and then setting this number less than or equal to q^{n-k}.
For correcting bursts of length b or less, according to the condition (i), syndromes produced by such errors must be nonzero and distinct. The number of such syndromes, including the zero syndrome, is

$$q^{b-1}[(n-b+1)(q-1)+1]. \quad \text{(refer Theorem 4.16, [13])}$$

We know that the conditions (ii) to (iv) are taken care of by condition (i) for burst of length b or less. Therefore for locating burst of length $s(>b)$ or less, we need to count the syndromes produced by burst of length i only, where $b+1 \leq i \leq s$. The number of such syndromes is given by

$$q^s - q^b. \quad \text{(refer [7])}$$

Thus, the total number of such syndromes including the vector of all zeros is atleast

$$q^{b-1}[(n-b+1)(q-1)+1] + m(q^s - q^b).$$

Therefore, we must have

$$q^{n-k} \geq q^{b-1}[(n-b+1)(q-1) + m(q^{s-b+1} - q) + 1]$$

or

$$n-k \geq b-1+\log_q\left\{(n-b+1)(q-1) + m(q^{s-b+1} - q) + 1\right\}.$$

□

In the following, we obtain an upper bound on the number of check digits required for the construction of a linear code considered in Theorem 3.1. This bound makes sure about the existence of such a linear code. The proof follows the technique of Theorem 2.2.

Theorem 3.2. *There shall always exist an (n,k) linear code over GF(q) subdivided into m sub-blocks of length t each, that locates any solid burst of length s or less within a sub-block and simultaneously corrects any solid burst of length $b(<s)$ or less provided that*

$$\begin{aligned} q^{n-k} >&(q-1)^{2(b-1)}[(q-1)(n-2b+1)+1] + (2-m)(q^{s-1}-q^{b-1}) \\ &+ (m-1)\left\{q^{2(s-1)}[(q-1)(t-s+1)+1] - q^{2(b-1)}[(q-1)(t-b+1)+1]\right\}. \end{aligned}$$

Proof. The existence of such a code is also proved by constructing an $(n-k)\times n$ parity check matrix H for the desired code as follows:
Select any nonzero $(n-k)$-tuple as the first column h_1 of the matrix H. After having selected the first $n-1$ columns $h_1, h_2, \ldots, h_{n-1}$ appropriately, we lay down the condition to add n^{th} column as follows:
According to the condition (i), h_n should not be a linear combination of immediately preceding $b-1$ consecutive columns $h_{n-1}, h_{n-2}, \ldots, h_{n-b+1}$, together with any b or fewer consecutive columns from amongst the first $n-b$ columns $h_1, h_2, \ldots, h_{n-b}$ i.e.,

$$\begin{aligned} h_n \neq &(u_1 h_{n-1} + u_2 h_{n-2} + \cdots + u_{b-1} h_{n-b+1}) + (v_i h_i + v_{i+1} h_{i+1} \cdots + v_{b-1} h_{b-1}), \\ &\text{where } u_i, v_i \in GF(q) \text{ and } h_i\text{'s are any } b \text{ consecutive columns} \\ &\text{amongst the first } n-b \text{ columns.} \end{aligned}$$

This condition ensures that the code vector can correct all bursts of length b or less. The number of choices of the coefficients u_i and v_i, including the zero vector, is given by (refer Theorem 4.17, [13])

$$(q-1)^{2(b-1)}[(q-1)(n-2b+1)+1]. \tag{3.1}$$

Now for locating the corrupted sub-block, according to condition (ii) and (iii), the syndrome of any solid burst error of length $s(> b)$ or less within a sub-block must be nonzero and distinct from those syndromes resulting from such errors within any other sub-block. In view of this, h_n can be added to H such that h_n should not be a linear combination of the immediately preceding $s-1$ or less consecutive columns from the m^{th} sub-block, together with a linear combination of s or less consecutive columns from amongst the remaining $m-1$ sub-blocks. The number of such linear combinations is given by (refer [7])

$$q^{s-1} + q^{s-1}(m-1)\Big\{q^{s-1}[(q-1)(t-s+1)+1]-1\Big\}. \tag{3.2}$$

Again according to condition (iv), the syndrome resulting from the occurrence of any burst of length s or less within any single sub-block must be distinct from the syndromes resulting from bursts of length b or less. Therefore, the number of syndromes of any burst of length more than b but less than or equal to s within any single sub-block is given by

$$\begin{aligned}
&q^{s-1} + (m-1)\Big\{q^{2(s-1)}[(q-1)(t-s+1)+1] - q^{s-1}\Big\}\\
&- q^{b-1} - (m-1)\Big\{q^{2(b-1)}[(q-1)(t-b+1)+1] - q^{b-1}\Big\}\\
=&(2-m)(q^{s-1}-q^{b-1}) + (m-1)\Big\{q^{2(s-1)}[(q-1)(t-s+1)+1]\\
&- q^{2(b-1)}[(q-1)(t-b+1)+1]\Big\}.
\end{aligned} \tag{3.3}$$

Thus, the total number of linear combinations to which h_n can not be equal is given by $expr.(3.1) + expr.(3.3)$. At worst all these combinations might yield distinct sum. Therefore, h_n can be added to H provided that

$$q^{n-k} > expr.(3.1) + expr.(3.3)$$

or

$$\begin{aligned}
q^{n-k} >&(q-1)^{2(b-1)}[(q-1)(n-2b+1)+1] + (2-m)(q^{s-1}-q^{b-1})\\
&+ (m-1)\Big\{q^{2(s-1)}[(q-1)(t-s+1)+1] - q^{2(b-1)}[(q-1)(t-b+1)+1]\Big\}.
\end{aligned}$$

This proves the theorem. □

Example 3.1. Consider a (10, 3) binary code whose parity check matrix H given below, constructed according to the synthesis procedure given in the proof of Theorem 3.2 by taking $q = 2$, $t = 5$, $m = 2$, $b = 2$ and $s = 3$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

It can be verified from Table 3.1 that the syndromes of any burst of length 2 or less are all nonzero and distinct. Further, the syndromes of any burst of length 3 are all nonzero and distinct in different sub-blocks and also distinct from those resulting from burst of length 2 or less. Therefore, the code can locate any burst of length 3 or less within a sub-block and simultaneously correcting bursts of length 2 or less.

Table 3.1

Error pattern - syndromes Table

Error-patterns	Syndromes	Error-patterns	Syndromes
For correction		0000001100	1001001
Burst of length 1		0000000110	1101100
1000000000	1000000	0000000011	0110110
0100000000	0100000	**For location**	
0010000000	0010000	Burst of length 3	
0001000000	0001000	1st sub-block	
0000100000	0000100	1110000000	1110000
0000010000	0000010	1010000000	1010000
0000001000	0000001	0111000000	0111000
0000000100	1001000	0101000000	0101000
0000000010	0100100	0011100000	0011100
0000000001	0010010	0010100000	0010100
Burst of length 2		2nd sub-block	
1100000000	1100000	0000011100	1001011
0110000000	0110000	0000010100	1001010
0011000000	0011000	0000001110	1101101
0001100000	0001100	0000001010	0100101
0000110000	0000110	0000000111	1111110
0000011000	0000011	0000000101	1011010

Remark 3.1. *It can also be verified that syndromes of the bursts of length 3 are all nonzero and distinct among selves as well as distinct from syndromes of bursts of length 2 or less. Therefore, the above code corrects all bursts of length 3 or less.*

4. Comparisons Between Numbers of Parity Check Digits

Detection of error, location of error and correction of error are all important with respect to the requirement of a system or situation. Accordingly codes are constructed to deal with. The numbers of parity check digits required for the three types of codes are different. The less is the number of parity check digits, the more is the rate of information.

4.1. **Comparison of bounds for solid bursts.** In this subsection, we make comparisons between the necessary (lower bound) and sufficient number (upper bound) of parity check digits required for a code discussed in Section 2, with the following types of codes:
(a) codes that correct all solid bursts of length s or less, and
(b) codes that locate any solid burst of length s or less within a sub-block and simultaneously correct solid burst of length $b(< s)$ or less.

The necessary and sufficient number of parity check digits required for a code of type (a) are given as follows.

Theorem 4.1. [2] *The necessary number of parity check digits for a code which is able to corrects all solid bursts of length s or less is given by*

$$q^{n-k} \geq 1 + \sum_{i=1}^{s} (n-i+1)(q-1)^i.$$

Theorem 4.2. [2] *There shall always exist an (n, k) linear code over $GF(q)$ that corrects all solid bursts of length s or less $(n > 2s)$ provided that*

$$q^{n-k} > 1 + \sum_{i=1}^{s} \sum_{l=1}^{s} (n-l-i+1)(q-1)^{i+l-1}.$$

Further, the necessary and sufficient number of parity check digits required for a code of type (b) are given below.

Theorem 4.3. [5] *The necessary number of parity check digits for a (n, k) linear code over $GF(q)$ that corrects any solid burst of length b or less and simultaneously detects any solid burst of length $s(> b)$ or less is given by*

$$q^{n-k} \geq 2 + \sum_{i=1}^{b} (n-i+1)(q-1)^i.$$

Theorem 4.4. [5] *There shall always exist an (n, k) linear code over $GF(q)$ that corrects solid burst of length b or less and simultaneously detects any solid burst of length $s(> b)$ or less provided that*

$$q^{n-k} > 1 + \sum_{i=1}^{b} \sum_{l=1}^{b} (n-l-i+1)(q-1)^{i+l-1} + \sum_{i=b}^{s-1} (q-1)^i.$$

It is evident from Table 4.1 that the necessary number of parity check digits required for a code correcting any solid burst of length b or less and simultaneously detecting any solid burst of length $s(> b)$ or less (Theorem 4.3) is less than or equal to the necessary number of parity check digits required for a code locating any solid burst of length s or less within a sub-block and simultaneously correcting any solid burst of length $b(< s)$ or less (Theorem 2.1) which in turn is again less than or equal to the necessary number of parity check digits required for a code correcting all solid bursts of length s or less (Theorem 4.1). In other words, the numbers of parity checks are in increasing order.

Table 4.1

Comparison of necessary number of check digits for codes correcting & detecting, correcting & locating, and only correcting solid burst errors.

m	t	s	$b(< s)$	n	$n-k$ correction and detection	$n-k$ location and correction	$n-k$ only correction
2	5	5	2	10	5	5	6
3	5	5	2	15	5	6	7
4	5	5	2	20	6	6	7
5	5	5	2	25	6	7	7
6	5	5	2	30	6	7	8

A similar comparison between the sufficient number of parity check digits required for the existence of codes that correcting & detecting (as mentioned in Theorem 4.4), correcting & locating (Theorem 2.2), only correcting solid burst errors (Theorem 4.2) is also done. From Table 4.2, it is clear that the sufficient numbers of parity check digits required for codes correcting & detecting, correcting & locating, only correcting solid burst errors are in increasing order.

Table 4.2

Comparison of sufficient number of check digits for codes correcting & detecting, correcting & locating, and only correcting solid burst errors.

m	t	s	$b(< s)$	n	$n-k$ correction and detection	$n-k$ location and correction	$n-k$ only correction
5	5	5	2	25	7	9	9
10	5	5	2	50	8	10	11
15	5	5	2	75	9	11	11
20	5	5	2	100	9	11	12
25	5	5	2	125	9	12	12

4.2. **Comparison of bounds for bursts.** In this subsection, we make comparisons between the necessary (lower bound) and sufficient number (upper bound) of parity check digits required for a code discussed in Section 3, with the linear codes that correct all bursts of length s or less.

Theorem 4.5. [13] *The necessary number of parity check digits for a code which is able to corrects all bursts of length s or less is given by*

$$q^{n-k} \geq q^{s-1}[(q-1)(n-s+1)+1].$$

Theorem 4.6. [13] *There shall always exist an (n, k) linear code over $GF(q)$ that corrects all bursts of length s or less $(n > 2s)$ provided that*

$$q^{n-k} > q^{2(s-1)}[(q-1)(n-2s+1)+1].$$

It is clear from the following Table 4.3, the necessary number of parity check digits required for a code considered in Theorem 3.1 is less than or equal to the necessary number of parity check digits required for a code mentioned in Theorem 4.5. The numbers of parity checks are in increasing order.

Table 4.3

Comparison of necessary number of check digits for codes correcting & locating, and only correcting burst errors.

m	t	s	$b(< s)$	n	$n-k$ location and correction	$n-k$ only correction
2	5	5	2	10	7	7
3	5	5	2	15	7	8
4	5	5	2	20	8	9
5	5	5	2	25	8	9
6	5	5	2	30	8	9

A similar comparison between the sufficient number of parity check digits required for the existence of codes that correcting & locating (Theorem 3.2), only correcting burst errors (Theorem 4.6) is also done. From Table 4.4, it is also found that the sufficient numbers of parity check digits required for codes correcting & locating, only correcting burst errors are in increasing order.

Table 4.4

Comparison of sufficient number of check digits for codes correcting & locating, and only correcting burst errors.

m	t	s	$b(< s)$	n	$n-k$ location and correction	$n-k$ only correction
5	5	5	2	25	11	13
10	5	5	2	50	13	14
15	5	5	2	75	13	15
20	5	5	2	100	14	15
25	5	5	2	125	14	15

5. Conclusion

The bounds on the parity checks required for the existence of the codes discussed in this paper are obtained. Construction of such codes has also been dealt with. The author feels that there may be a systematic way of constructing such codes. Further, the study may be extended to other types of errors.

References

[1] J. Arlat and W.C. Carter, Implementation and Evaluation of a (b, k)-Adjacent Error-Correcting/Detecting Scheme for Supercomputer Systems, IBM J. Res. Develop., vol. 28, No. 2, 159-169 (1984).
[2] P.K. Das, Codes Detecting and Correcting Solid Burst Errors, Bulletin of Electrical Engineering and Informatics, vol. 1, No. 3, 225-232 (2012).
[3] P.K. Das, Codes Detecting and Locating Solid Burst Errors, Romanian Journal of Mathematics and Computer Science, vol. 2, No. 2, 1-10 (2012).
[4] P.K. Das, Blockwise Solid Burst Error Correcting Codes, International Journal on Information Theory, vol. 1, No. 1, 11-17 (2012).
[5] P.K. Das, Codes correcting and simultaneously detecting solid burst errors, Computer Engineering and Application Journal, vol. 2, No. 1, 143-150 (2013).
[6] P.K. Das, Difference Equations and SBEC Optimal Codes, Journal of Mathematics and Computer Science, vol. 7, No. 4, 258-265 (2013).
[7] B.K. Dass, Burst error locating linear codes. J. Inf. and Optimization Sciences, vol. 3, No. 1, 77-80 (1982).
[8] B.K. Dass and S. Madan, Repeated Burst Error Locating Linear Codes, Discrete Mathematics, Algorithms and Applications, vol. 2, No. 2, 181-188 (2010).
[9] P. Fire, A class of multiple-error-correcting binary codes for non-independent errors, Sylvania Report RSL-E-2, Sylvania Reconnaissance Systems Laboratory, Mountain View, Calif, (1959).
[10] E. Fujiwara and M. Kitakami, A Class of Error Locating Codes for Byte-Organized Memory System, IEEE Trans. Inform. Theory, IT-40, 1857-1865 (1994).
[11] D.W. Jensen, Block code to efficiently correct adjacent data and/or check bit errors, Patent number: US6604222 B1, Date of Patent Aug. 5, (www.google.com/patents/US6604222), (2003).
[12] M. Kitakami and E. Fujiwara, A Class of Error Locating Codes- SEC-Se/bEL Codes, IEICE Trans. Fundamentals, E78-A, No. 9, 1-6 (1995).
[13] W.W. Peterson and E.J. Weldon(Jr.), *Error-Correcting Codes*, 2nd edition, The MIT Press, Mass, 1972.
[14] G.E. Sacks, Multiple error correction by means of parity-checks, IRE Trans. Inform. Theory, IT-4, 145-147 (1958).
[15] A.G. Schillinger, A class of solid burst error correcting codes, Polytechnic Institute of Brooklyn, N.Y., Research Rept. PIBMRI, April, 1223-64 (1964).
[16] B.D. Sharma and B.K. Dass, Adjacent error correcting binary perfect codes, J. Cybernetics, vol. 7, 9-13 (1977).
[17] S.G.S. Shiva and C.L. Sheng, Multiple solid burst-error- correcting binary codes, IEEE Trans. Inform. Theory, IT-15, 188-189 (1969).
[18] J. Wolf and B. Elspas, Error-locating codes a new concept in error control, IEEE Transactions on Information Theory, vol. 9, No. 2, 113-117 (1963).

Pankaj Kumar Das (pankaj4thapril@yahoo.co.in)– Department of Mathematics, Shivaji College (University of Delhi), Raja Garden,Delhi-110027