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Stability Analysis of Nanobeams by Modified Finite Element Transfer Matrix Method

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Abstract: In this study, the modified finite element-transfer matrix method was adapted for nanobeam's stability analysis. The nanobeam's stability equation was first established with the help of Euler beam theory. Using differential equation, the finite element matrix of the element was first deduced, followed by the Ricatti transfer matrix. The suitability of the present method was demonstrated using an example reported in the literature. In our reported method, the matrix dimensions are significantly reduced compared to the classical finite element method, and therefore the solution time is shortened. This method can be used mainly for the solution of multi-span and variable cross-section nanobeams.

Keywords: Nanobeam, Finite Element, Transfer Matrix Method, Euler Beam

Nano Kirişlerin Değiştirilmiş Sonlu Elemanlar Taşıma Matrisi Yöntemi ile Stabilite Analizi

Öz: Bu çalşmada değiştirilmiş Sonlu elemanlar–taşıma matrisi yöntemi nano kirişlerin stabilite analizi için uyarlanmıştır. Çalışmada önce nano kirişin stabilite denklemi yerel olmayan Euler kiriş teorisi yardımıyla oluşturulmuştur. Diferansiyel denklemin çözümü ile önce eleman sonlu elamanlar matrisi elde edilmiş daha sonra yapılan dönüşümle Ricatti taşıma matrisi elde edilmiştir. Çalışmanın sonunda sunulan yöntemin uygunluğunu literatürden alınan bir örnek üzerinde gösterilmiştir. Sunulan yöntem ile matris boyutları klasik sonlu elemanlar yöntemine göre kayda değer bir şekilde azalmakta ve dolayısıyla çözüm süresi de kısalmaktadır. Sunulan yöntem özellikle çok açıklıklı ve değişken kesitli nano kirişlerin çözümünde kullanılabilir.

Anahtar Kelimeler: Nano kiriş, Sonlu Eleman, Taşıma Matrisi Yöntemi, Euler Kiriş

1. Introduction

New generation materials with enhanced properties and functionality are produced through nanotechnology. The beam is widely used in many applications and, therefore the mechanical properties of these nanostructures depicts an essential role in the design of nanostructures. Nanostructures have attracted the attention of many researchers because of their widespread use in engineering applications.

Glabisz et al. [1] developed an algorithm to analyze the stability of Euler–Bernoulli nanobeams. Exact solutions were used for the analysis of the prismatic nanobeam. Shariati et al. [2] investigated the vibrations and stability of functionally graded nanobeams by numerical and analytical methods.

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The effect of nonlocal parameters and different boundary conditions of nanobeams were studied by Kumar et al. [3], Zhang et al. [4], Mohammadi et al. [5], and Wang et al. [6]. Eringen [7] introduced the nonlocal elasticity theory and developed the basic field equations of nonlocal continuum field theories [8, 9]. This theory was used in the analysis of many nanostructures [10-20]. Karličić and Cajić [21] used the incremental harmonic balance method to analyze the dynamic stability of a nanobeam system with the help of Eringen's nonlocal elasticity theory.

Behdad et al. [22] employed the two-phase local/nonlocal elasticity to investigate the size-dependent stability and vibration of viscoelastic functionally graded porous Timoshenko nanobeams for the first time. Sourani et al. [23] used nonlocal strain gradient theory to investigate the Euler–Bernoulli nanobeam's dynamic stability under time-dependent axial loading. They assumed the cross-section of nanobeam as rectangular and boundary conditions as simply-supported. They derived strain–displacement relations using the Von Kármán equations.

Hamed et al. [24] derived equations for beam (local and nonlocal) to study the influence of perforation parameters on buckling loads and static bending of nanobeams by considering all the boundary conditions. Their model is helpful in modeling nanoresonators and nano actuators used in nanotechnology. Arda and Aydogdu [25] studied the stability of a nanobeam under a time-varying axial loading. They investigated the effect of a small-scale parameter on the dynamic displacement and critical dynamic buckling load of nanobeams. Eltaher et al. [26] used higher-order shear deformation beam theories to investigate the influence of thermal load and shear force on the buckling of nanobeams.

Several numerical methods are used in structural analysis, and the finite element method is the powerful and the most common numerical method [27-30]. The transfer matrix method is an effective method used in mechanics developed by Holzer [31] for torsion vibrations of rods. Dokanish [32] combined FEM and TMM to analyze the vibration of structures. Rong et al. [33, 34] used the finite element transfer matrix method for eigenvalue problems of structures. Ozturk et al. [35] implemented a modified finite element transfer matrix method for the structure static analysis. Bozdogan and Khosravi Maleki [36] applied a modified finite element transfer matrix method to the heat transfer problem.

This study has adopted the Modified Finite Element-Transfer Matrix (MFETM) method for the stability analysis of nanobeams. Nonlocal Euler beam theory is used for nanobeams in the study. In the classical finite element method, matrix system size increases with increase in the number of elements. In the MFETM method, the size of the system matrix is independent of the number of elements.

In the study, it has been accepted that the material has linear elastic behavior.

2. Material and Method

According to the nonlocal elasticity theory, the differential equation expressing the Euler beam's stability state is written as follows [37,38].

$$EI\frac{d^4v}{dz^4}-(e_0a)^2P\frac{d^4v}{dz^4}+P\frac{d^2v}{dz^2}=0 \tag{1}$$

Where υ is the deflection function, z is the beam axis, EI is the bending stiffness, e_0a is the scale coefficient that incorporates the small scale effect. P is the axial force.

The differential equation (3) is obtained by substituting the transformation in equation (2) to make the 4th order homogeneous ordinary differential equation to dimensionless.

$$\varepsilon = \frac{z}{L} \tag{2}$$

$$[EI - (e_0 a)^2 P]\frac{d^4 v}{d\varepsilon^4} + PL^2 \frac{d^2 v}{d\varepsilon^2} = 0 \tag{3}$$

By using equation (4), short version of equation (3) can be written as equation (5)..

$$\overline{EI} = [EI - (e_0 a)^2 P] \tag{4}$$

$$\overline{EI}\frac{d^4 v}{d\varepsilon^4} + PL^2 \frac{d^2 v}{d\varepsilon^2} = 0 \tag{5}$$

The differential equation (6) can be written by modifying equation (5),

$$\frac{d^4 v}{d\varepsilon^4} + \lambda^2 \frac{d^2 v}{d\varepsilon^2} = 0 \tag{6}$$

The λ in the differential equation (6) is defined as follows,

$$\lambda = \sqrt{\frac{PL^2}{\overline{EI}}} \tag{7}$$

The solution of the dimensionless differential equation (6) is,

$$v(\varepsilon) = c_1 + c_2\varepsilon + c_3\cos(\lambda\varepsilon) + c_4\sin(\lambda\varepsilon) \tag{8}$$

If the derivative of the differential equation (8) is taken, the rotation function is found as follows.

$$v'(\varepsilon) = c_2 - c_3\lambda\sin(\lambda\varepsilon) + c_4\lambda\cos(\lambda\varepsilon) \tag{9}$$

The deflection and rotation values at the ends of the beam are obtained using functions (8) and (9).

$$v(0) = c_1 + c_3 \tag{10}$$

$$v'(0) = c_2 + c_4\lambda \tag{11}$$

$$v(1) = c_1 + c_2 + c_3\cos(\lambda) + c_4\sin(\lambda) \tag{12}$$

$$v'(1) = c_2 - c_3\lambda\sin(\lambda) + c_4\lambda\cos(\lambda) \tag{13}$$

The equations (10), (11), (12), and (13) can be expressed in matrix form as follows.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \lambda \\ 1 & 1 & \cos(\lambda) & \sin(\lambda) \\ 0 & 1 & -\lambda\sin(\lambda) & \lambda\cos(\lambda) \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{bmatrix} v(0) \\ v'(0) \\ v(1) \\ v'(1) \end{bmatrix} \tag{14}$$

The matrix equation (14) can be expressed in abbreviated form as follows.

$$\begin{bmatrix} v(0) \\ v'(0) \\ v(1) \\ v'(1) \end{bmatrix} = A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{15}$$

Shear force and bending moment expressions at beam ends are written as below in dimensionless form.

$$V = \frac{\overline{EI}}{L^3}\frac{d^3 v}{d\varepsilon^3} + \frac{P}{L}\frac{dv}{d\varepsilon} \tag{16}$$

$$M = \frac{\overline{EI}}{L^2}\frac{d^2 v}{d\varepsilon^2} \tag{17}$$

By substituting the deflection equation (8) in the equations (16) and (17) and performing the necessary calculations, the equations (18) and (19) can be written as.

$$V(\varepsilon) = \frac{P}{L}c_2 \tag{18}$$

$$M(\varepsilon) = -\frac{\overline{EI}}{L^2}[c_3\lambda^2\cos(\lambda\varepsilon) + c_4\lambda^2\sin(\lambda\varepsilon)] \tag{19}$$

By the sign convention in the one-dimensional finite element method, the force expressions at the ends of beams can be expressed by the following equations,

$$F_1 = V(0) = \frac{P}{L}c_2 \tag{20}$$

$$F_2 = -M(0) = \frac{\overline{EI}}{L^2}\lambda^2 c_3 \tag{21}$$

$$F_3 = -V(1) = -\frac{P}{L}c_2 \tag{22}$$

$$F_4 = M(1) = -\frac{\overline{EI}}{L^2}[c_3\lambda^2\cos(\lambda) + c_4\lambda^2\sin(\lambda)] \tag{23}$$

The equations (20), (21), (22), and (23) can be written in matrix form as follows.

$$\begin{bmatrix} 0 & \frac{P}{L} & 0 & 0 \\ 0 & 0 & \frac{\overline{EI}}{L^2}\lambda^2 & 0 \\ 0 & -\frac{P}{L} & 0 & 0 \\ 0 & 0 & -\frac{\overline{EI}}{L^2}\lambda^2\cos(\lambda) & -\frac{\overline{EI}}{L^2}\lambda^2\sin(\lambda) \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{24}$$

The abbreviated representation of the matrix equation (24) can be written as follows.

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = B \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{25}$$

The matrix equation (26) is written by combining the matrix relations (15) and (25).

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = BA^{-1} \begin{bmatrix} v(0) \\ v'(0) \\ v(1) \\ v'(1) \end{bmatrix} \tag{26}$$

The matrix equation (26) is abbreviated as follows.

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = K \begin{bmatrix} v(0) \\ v'(0) \\ v(1) \\ v'(1) \end{bmatrix} \tag{27}$$

where K is the stability stiffness matrix of the Euler beam according to the nonlocal elasticity theory. The stiffness matrix (27) can be written as equation (28) in the form of sub-matrices.

$$K = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} \tag{28}$$

where k_1, k_2, k_3, and k_4 are the K matrix sub-matrices, and their dimensions are 2×2.

The matrix equation (27) can be expressed as follows, considering the definition (28).

$$\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} \begin{Bmatrix} d_{i-1} \\ d_i \end{Bmatrix} = \begin{bmatrix} -Q_{i-1} \\ Q_i \end{bmatrix} \tag{29}$$

d_{i-1}, d_i, Q_{i-1} and Q_i are defined below respectively.

$$d_{i-1} = \begin{Bmatrix} v(0) \\ v'(0) \end{Bmatrix} \tag{30}$$

$$d_1 = \begin{Bmatrix} v(1) \\ v'(1) \end{Bmatrix} \tag{31}$$

$$Q_{i-1} = \begin{Bmatrix} -F_1 \\ -F_2 \end{Bmatrix} \tag{32}$$

$$Q_i = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} \tag{33}$$

The equation (29) can be written as equations (34) and (35).

$$k_1 d_{i-1} + k_2 d_i = -Q_{i-1} \tag{34}$$
$$k_3 d_{i-1} + k_4 d_i = Q_i \tag{35}$$

The matrix equations (38) and (39) are written by applying the Ricatti transfer defined in equations (36) and (37) [33].

$$Q_{i-1} = T_{i-1} d_{i-1} \tag{36}$$
$$Q_i = T_i d_i \tag{37}$$

$$k_1 d_{i-1} + k_2 d_i = -T_{i-1} d_{i-1} \tag{38}$$
$$k_3 d_{i-1} + k_4 d_i = T_i d_i \tag{39}$$

d_{i-1} is calculated by using the matrix equation (38).

$$d_{i-1} = -[T_{i-1} + k_1]^{-1} k_2 d_i \tag{40}$$

By substituting equation (40) into equation (39), the matrix equation (41) is obtained.

$$-k_3 [T_{i-1} + k_1]^{-1} k_2 d_i + k_4 d_i = T_i d_i \tag{41}$$

From the matrix equation (41), T_i is obtained as follows,

$$T_i = -k_3 [T_{i-1} + k_1]^{-1} k_2 + k_4 \tag{42}$$

Table 1. T_1 and frequency equation for different boundary conditions

Support conditions	T_1	Stability equation
Cantilever rod 	k_4	$\lvert T_n \rvert = 0$
Pinned -Clamped rod 	$\begin{bmatrix} -\frac{K^1(3,2)*K^1(2,3)}{K^1(2,2)} + K^1(3,3) & -\frac{K^1(2,4)*K^1(3,2)}{K^1(2,2)} + K^1(3,4) \\ -\frac{K^1(4,2)*K^1(2,3)}{K^1(2,2)} + K^1(4,3) & -\frac{K^1(4,2)*K^1(2,4)}{K^1(2,2)} + K^1(4,4) \end{bmatrix}$	$\lvert T_n^{-1} \rvert = 0$
Pinned ended rod 	$\begin{bmatrix} -\frac{K^1(3,2)*K^1(2,3)}{K^1(2,2)} + K^1(3,3) & -\frac{K^1(2,4)*K^1(3,2)}{K^1(2,2)} + K^1(3,4) \\ -\frac{K^1(4,2)*K^1(2,3)}{K^1(2,2)} + K^1(4,3) & -\frac{K^1(4,2)*K^1(2,4)}{K^1(2,2)} + K^1(4,4) \end{bmatrix}$	$T_n(2,2) = 0$

Matrix equation (42) is 2×2 in size, and according to the nonlocal elasticity theory, the Euler beam's stability is the Ricatti transfer matrix.

For the n[th] element:

$$T_n d_n = Q_n \tag{43}$$

The buckling load is found by applying the boundary condition at the end of the beam using equation (43). Depending on the boundary conditions, T_1 and (43) stability equations are given in Table 1.

2. Results and Discussion

To investigate the suitability of the method, an example taken from the literature [37] was solved with the MFETM method. The results were compared with the literature for three different boundary conditions in Table 2, Table 3, and Table 4.

In the example, the modulus of elasticity was taken as 1 TPa, and the diameter value was taken as 1 nm.

Table 2. Comparison of critical buckling loads (nN) for cantilever rod

Cantilever rod						
(e_0a)	0		1.0		2.0	
L/d	This study	Wang et al. [37]	This study	Wang et al. [37]	This study	Wang et al. [37]
10	1.2112	1.2112	1.1818	1.18202	1.1022	1.1024
16	0.4731	0.4731	0.4685	0.4686	0.4555	0.4555
20	0.3028	0.3028	0.3009	0.3009	0.2955	0.2955

Table 3. Comparison of critical buckling loads (nN) for pinned ended rod

Pinned ended rod						
(e_0a)	0		1.0		2.0	
L/d	This study	Wang et al. [37]	This study	Wang et al. [37]	This study	Wang et al. [37]
10	4.8440	4.8447	4.4089	4.4095	3.4729	3.4735
16	1.8922	1.8925	1.8219	1.8222	1.6394	1.6396
20	1.2110	1.2112	1.1818	1.1820	1.1022	1.1024

Table 4. Comparison of critical buckling loads (nN) for clamped –pinned rod

Clamped -Pinned						
(e_0a)	0		1.0		2.0	
L/d	This study	Wang et al. [34]	This study	Wang et al. [34]	This study	Wang et al. [34]
10	9.9096	9.9155	8.2449	8.2461	5.4821	5.4830
16	3.8709	3.8715	3.5880	3.5885	2.9426	2.9431
20	2.4774	2.4789	2.3584	2.3957	2.0612	2.0615

The variation of the critical buckling loads obtained by the MFETM according to L/d and e_oa for three different boundary conditions is given in Figure 1, Figure 2, and Figure 3.

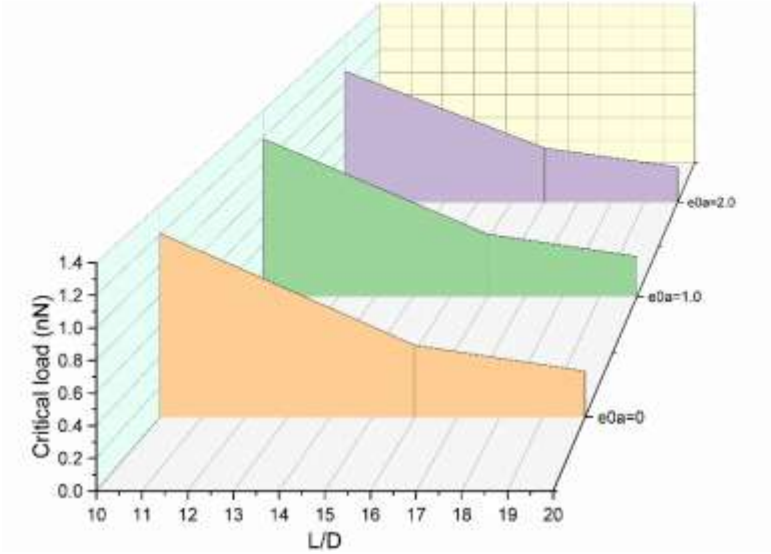

Figure 1. Critical buckling load vs. L/D for cantilever rod

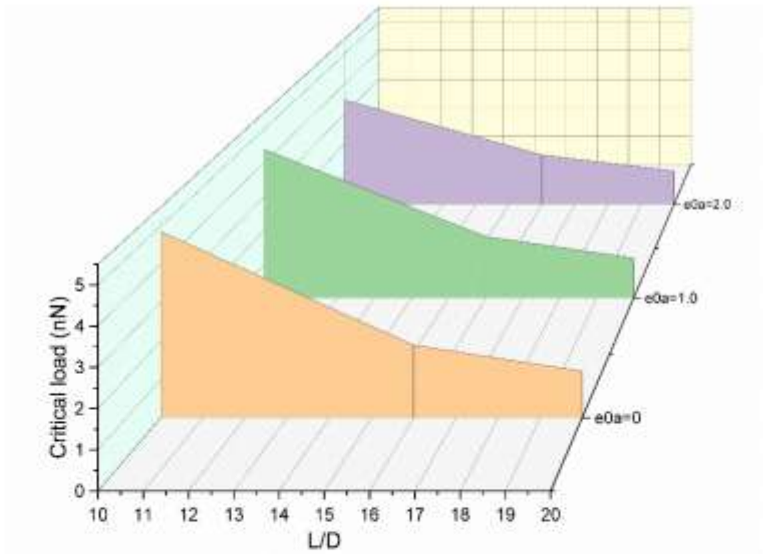

Figure 2. Critical buckling load vs. L/D for pinned ended rod.

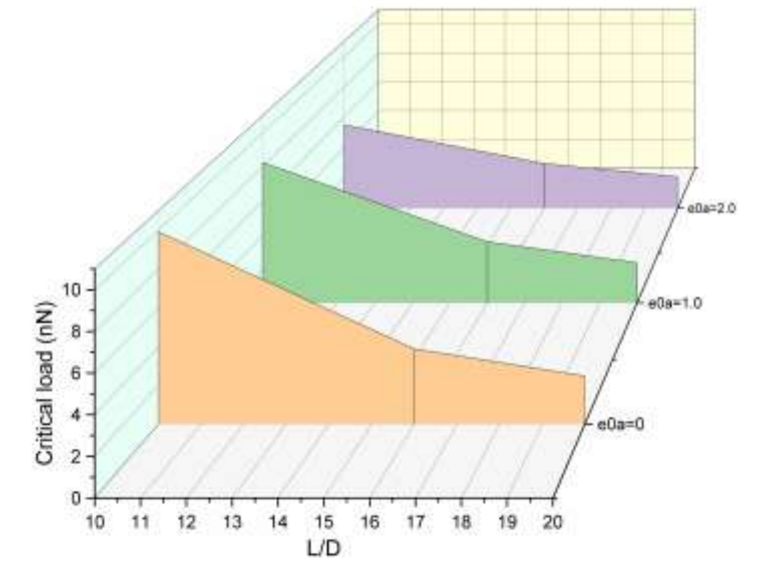

Figure 3. Critical buckling load vs. L/D for clamped-pinned rod.

As shown in Figure 1, Figure 2, and Figure 3, the critical buckling load decreases as the L/d ratio increases and the critical buckling load decreases as the $e_o a$ increases.

4. Conclusions

In this study, the MFETM method has been adapted for the stability analysis of nanobeams. In the study, the nanobeams' stability equation is written according to the nonlocal Euler beam theory. At the end of the study, the suitability of the presented method is shown from an example taken from the literature. In the finite element method, as the number of elements increases, the size of the system matrix increases, while in the MFETM method, the size of the system matrix remains constant regardless of the number of elements. The size of the system matrix to be solved in the MFETM method is 2×2 regardless of the element size.

In the presented method, the solution time is significantly reduced due to the small matrix dimensions. As a result, the MFETM method can be an excellent alternative to the transfer matrix method in beam’s and nanotube’s solutions.

Authors’ contributions

Each author's contribution to the study is 50%.

Competing interests

The authors declare that they have no competing interests.

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