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Nazim KEVEN

Dr. Öğr. Üyesi | Assist. Prof. Dr.
Bilkent Üniversitesi, İnsani Bilimler ve Edebiyat Fakültesi, Felsefe Bölümü, Ankara, TR
Bilkent University, Faculty of Humanities and Letters, Department of Philosophy, Ankara, TR
ORCID: 0000-0001-5368-8265
nazimkeven@bilkent.edu.tr

Boolean Algebra and Aristotelian Logic

Abstract

George Boole is one of the first logicians who offered a systematic formalization of language. He developed a notation to encode ordinary language sentences to algebraic symbols and developed algebraic methods to manipulate those symbols and to deduce results that are interpretable in the ordinary language. His methods formed the basis of modern logic. Boole applied his formal methods to many of the contemporary questions of his time, one of them being scholastic logic. In this paper, I explain how Boole deals with Aristotelian logic. I will start with his notation and algebraic methods, then apply them to Aristotelian conversions and syllogisms. It must be noted Boole has two versions of notation and methods, one is developed in *Mathematical Analysis of Logic* and the other is in his seminal book *The Laws of Thought*. I focus on the later version.

Keywords: George Boole, Laws of Thought, Boolean Logic, Aristotelian Syllogisms.

Boole Cebiri ve Aristoteles'in Mantığı

Öz

George Boole dili sistematik bir şekilde yapısallaştıran ilk mantıkçılardan biridir. Dildeki sıradan cümleleri cebirsel sembollerle ifade edebilmek için bir notasyon ve bu sembolleri sistematik bir şekilde değiştirerek gündelik dilde de yorumlananabilen sonuçlar bulmak için cebirsel yöntemler geliştirdi. Geliştiridiği bu yöntemleri modern mantığın temelini oluşturmuştur. Geliştirdiği metotları skolastik mantık da dahil zamanındaki bir çok probleme uygulamıştır. Bu makalede Boole'un metotlarının Aristoteles'in mantığına nasıl uygulandığını inceliyorum. Öncelikle notasyonuna ve cebirsel metotlarına değinip, daha sonra bunları Aristoteles'in evirmelerine ve kıyaslarına uygulanışını gösteriyorum. Boole'un notasyon ve metotlarının iki versiyonu bulunmaktadır, bir tanesi *Mantığın Matematiksel Analizi'*nde diğeri ise *Düşüncenin Yasaları'*nda geliştirilen. Bu makalede *Düşüncenin Yasaları*'nda geliştirdiği versiyon esas alınmıştır.

Anahtar Kelimeler: George Boole, Düşüncenin Yasaları, Boolean Mantık, Aristoteles'in Kıyasları.

1. Boolean Algebra

Let me start with Boole’s notation. Boole uses x, y, z, etc. to represent classes, where ‘class’ means a group of individuals defined by a particular description. “Nothing” and “universe” are also considered as a class. In a sense, a class is akin to a set in modern terms, where “nothing” can be considered as the empty set and “universe” can be considered as the universal set. xy denotes to the intersection of two classes. Idempotency and commutation are defined over classes. So,

$$xx = x \text{ or } x2 = x$$

and

$$xy = yx$$

Boole defines three operations on classes: summation, subtraction, and equality. Summation stands for the conjunctions “and”, “or” etc., and denoted by + sign. Subtraction stands for “except”, and denoted by – sign. Both commutation and distribution are defined for summation and subtraction. Equality is represented by = and stands for the copula “is” or “are”. Transposition, i.e. equal things that can be added or subtracted from both sides of an equation, is defined over equality. Multiplying both sides of an equation with the same class is also defined, however, the converse does not hold. It is not permissible to divide both sides of an equation with the same class.

In Boolean algebra, there are two numerical values that a class can take: 1 or 0. 1 is interpreted as “universe” and 0 is interpreted as “nothing”. So, for any class y, the following holds:

$$1 \times y = y, \text{ or } 1y = y$$

and

$$0 \times y = 0, \text{ or } 0y = 0$$

1-x represents the supplementary class of x in Boolean algebra. It does the job of negation. If x represents the class of man, then 1-x represents the class of not-man. Let me give a couple of translation examples. Let x = elastic, y= hard, z =metals. "Non-elastic metals" can be translated as z(1-x), "Elastic substances with non-elastic metals" can be translated as x +z(1-x), and "Metallic substances, except those which are neither hard nor elastic" can be translated as z – z(1 –y)(1-x) in Boolean Algebra.

Boole has two rules for the translation of universal and particular subjects and predicates. When both of them are universal, then they are simply written with an equality sign between them. For instance, consider "all fixed stars are suns". If x denotes to fixed stars and y denotes to suns, then the translation is:

$$x = y$$

When, on the other hand, either subject or predicate is particular, he equates them by attaching to the indefinite symbol v to the particular one. Note that the symbol v is identical to the symbols x, y, z, etc. kind and the general idempotency law applies to it. For instance, consider **"**All men are mortal", which is actually "All men are some of the mortal beings". If y represents men, x represents mortal beings, and v stands for an indefinite class that has some mortal members, then the translation is:

$$y = vx$$

Or, consider the statement "Some men are not wise". If men is represented by y, wise beings are represented by x, and an indefinite class v:

$$vy = v(1-x)$$

Boole deals with negative universals by translating them into positive ones. For instance, consider "no men are perfect beings", which can be converted to "All men are not perfect". If men is represented by y and perfect beings are represented by x, then the translation is:

$$y = v(1-x)$$

2. Boole's Methods

Boole develops three methods for the manipulation of symbols: The method of development, reduction, and of elimination. I will explain them in turns in what follows (cf. Ural 2011).

2.1 The Method of Development or Expansion

Boole's first method to manipulate interpretable symbols is called the method of development or expansion. Roughly, the method of development is partitioning given equality and then interpreting coefficients of the expanded equality according to interpretation rules so as to get an interpretable result. Partitioning works on the basic idea that, for any given class x, we can find two other classes u and v such that:

$$ux + v(1-x)$$

gives us the totality of class x, i.e., things that have the property x and that do not have the property x. According to Boole, f(x) represents an algebraic expression that involves a symbol x as a function of x. As the class symbols x, y, etc. can only take 0 and 1 values, f(x) can be f(0) or f(1) accordingly. Consequently, the development of a function is defined as "any function f(x), in which x is a logical symbol, is said to be developed when it is reduced to the form ax + b(1-x), a and b are so determined as to make the result equivalent to the function from which it was derived." (Boole 1951. 72) The a or b part of a partitioning of a function is called the coefficient and the class part is called the constituent.

Here the crucial thing is to find the coefficient values of partitions of the function. Boole calculates them by simply equating x to 1 and 0 accordingly. So, if we suppose:

$$f(x) = bx + a(1-x)$$

and make x equal to 1, we get

$$f(1) = b$$

If we make x equal to 0, this time we get

$$f(0) = a$$

Hence, we can determine the values of a and b. If the first equation is substituted with them:

$$f(x) = f(1)x + f(0)(1-x) \qquad (1)$$

gives us the development of f(x). The second part of the equation adequately represents the function f(x) as a proper partitioning of it, whatever the form of that function may be.

A function with two symbols can be similarly expanded. Suppose f(y, x) stand for that function. First, if we consider f(y, x) as only a function of y and expand it by general theorem (1), we get:

$$f(y, x) = f(1, x)\, y + f(0, x)(1-y) \qquad (2)$$

Now, if we take the coefficient f(1, x), consider it as a function of x and expand it accordingly we get

$$f(1, x) = f(1, 1)x + f(1, 0)(1-x) \qquad (3)$$

Similarly, expansion by the coefficient f(0, x) gives:

$$f(0, x) = f(0, 1)x + f(0,0)(1-x) \qquad (4)$$

If we substitute f(1,x) and f(0,x) in (2) with their values in (3) and (4), we get

$$f(y, x) = f(1, 1)yx + f(1,0)y(1-x) + f(0,1)(1-y)x + f(0,0)(1-y)(1-x) \quad (5)$$

In a similar manner, we can expand functions with three or more symbols.

Interpreting the expanded function depends on the value of the coefficients. As the symbols for classes can only take 0 and 1 values, the coefficients can give 1, 0, 0/0, or some other indefinite cases like 1/0 or 0/1. When a coefficient is 1, the constituent to which it is prefixed is taken in its entirety. When it is 0, the constituent to which it is prefixed is dropped. When a coefficient is 0/0, an indefinite amount of the constituent to which it is prefixed should be taken. Boole does not strictly define how to proceed in these cases. So, depending on the context, we are allowed to take some, none, or all of the members of the constituent to which the coefficient is prefixed. When any other symbol is used as a coefficient, this means that its prefixed constituent must be equal to 0.

In general, for a solution to a problem by development we have:

$$w = E + 0F + 0/0\ G + 1/0\ I$$

This solution can be separated into two equations as follows:

$$w = E + vG$$

$$I = 0$$

Let me give an example (Hailperin 2004. 354) to clarify the development of a function and then interpreting it. If class X consists of all Ys which are not-Zs and all Zs which are not-Ys, what is the class Z? Let us translate the definition of class x into Boole's symbolic notation:

$$x = y(1\text{-}z) + z(1\text{-}y)$$

As we want to get the definition of the class z, let us reorder them by common algebra to solve for z:

$$z = y – x / 2y – 1$$

Now, if we expand the right-hand side, we have:

$$z = f(1, 1)xy + f(1,0)x(1\text{-}y) + f(0,1)(1\text{-}x)y + f(0,0)(1\text{-}x)(1\text{-}y)$$

It is now possible to calculate the values of coefficients by putting x and y values into our function f(x,y):

$$= 0/1\ xy + \text{-}1/\text{-}1\ x(1\text{-}y) + 1/1(1\text{-}x)y + 0/0\ (1\text{-}x)(1\text{-}y)$$

When we drop the constituents with coefficient values of 0/1 and 0/0, we have

$$z = x(1\text{-}y) + (1\text{-}x)y$$

which is interpreted as the class Z consists of all Xs which are not a member of Ys and all Ys which are not a member of Xs.

2.2 The Method of Elimination

The limitations of the method of development are apparent. The development method can only rearrange what is already present in the premises. If you start with classes x,y, and z, for instance, you end up with x, y, and z in a different ordering. However, it does not let us conclude any relation between just two of them. In order to deal with it, Boole develops a way called the method of elimination. It gives a way to eliminate one, or more, of the classes in the original premises. The method hinges on the basic intuition that the class where x is true and false at the same time is empty. So, the basic intuition is:

$$x(1-x) = 0 \quad (6)$$

However, in the method of elimination, we consider any logical equation f(x)=0 and claim that:

$$f(1)f(0) = 0 \qquad (7)$$

will be true, independent from the interpretation of x or any other classes in f(x). Accordingly, to eliminate x from possible equations of the form f(x)=0, we consecutively change x to 1 and 0 and then the resulting equations are multiplied. But first, let us prove that (7) holds for any f(x)=0. In order to prove it, let us start with the development of (6), which is:

$$f(1)x + f(0)(1-x) = 0$$

as I showed it in (1). If algebraically rearrange it, we have:

$$\{f(1) – f(0)\}x + f(0) = 0$$

If we solve for x, we have:

$$x = f(0)/\ f(0) – f(1)$$

Moreover, if we solve for x-1, we have:

$$x - 1 = -f(1)/\ f(0) - f(1)$$

Putting x and x-1 values into our original equation (6), gives us:

$$- f(0)f(1)/\{f(0)-f(1)\}^2 = 0$$

When simplified, it is equal to:

$$f(1)f(0) = 0$$

which is the result we are trying to show. So, we have proved that for any given function $f(x) = 0$, (6) holds. Consequently, in order to eliminate a class from an equation, firstly, we take all the classes in the equation to the first side by transposition to equate them to 0. Secondly, we give 1 and 0 values to the class that we want to eliminate to get f(1) and f(0). Lastly, we multiply the resulting equations. As $f(1)f(0) = 0$, the resulting equation would again be equal to 0 without the class that we want to eliminate.

Let me clarify the method of elimination with a couple of examples. First, consider "All men are mortal", which is translated as:

$$x = vy$$

in which men are represented by x, mortals are represented by y and v being an indefinite class that we want to eliminate. First, let us bring the classes to the left side:

$$x - vy = 0$$

In case v = 1, we have:

$$x - y = 0 \qquad (8)$$

and in case v = 0, we get:

$$x = 0 \qquad (9)$$

Multiplying (8) and (9) gives us:

$$x^2 - xy = 0$$

By idempotency $x^2 = x$, and distributing out x we have:

$$x(1-y) = 0$$

which is interpreted as “Men who are not mortal do not exist”.

Now let us consider “No men are perfect”, which is translated as:

$$x = v(1-y)$$

where “men” is represented by x and “perfect beings” is represented by y. Again, the indefinite class v needs to be eliminated. First let us take all the classes to the left side:

$$x - v(1-y) = 0$$

By using the elimination rule , we get:

$$\{x - (1-y)\} \times x = 0$$

which is equal to:

$$x - x(1-y) = 0$$

or

$$xy = 0$$

This means “Perfect man does not exist” when it is translated back to English. We can use the elimination method in order to eliminate any other class, or classes, by applying the method in the same way.

2.3 The Method of Reduction

The method of development and elimination gives all the necessary tools to manipulate symbols, however, we need another method to be able to work on more than one premise and to deduce a conclusion from them. Boole’s method for dealing with more than one premise is called the method of reduction. Here I will present a brief version of the method of reduction and skip Boole’s proofs for his method, as they are lengthy and are not essential for our examination of Boole’s analysis of syllogisms.

The basic idea behind the method of reduction is working backward on a set of expanded equations to reduce the original premises into a single equation. When a set of premises are turned into equations and developed, or some classes are eliminated, we end up with various other equations, depending on the vanishing and not-vanishing constituents. So, if we want to combine those premises into a single premise without losing any information, then the combined equation should also give the same equations as before when developed. Therefore, an easier way to achieve this is to work backward instead and sum up the developed equations in a way as to get a single equation that has all the relevant information present in our initial premises. Boole proves that simple addition can be used to combine equations that have the form V = 0 that also satisfy the fundamental law of duality V(1-V) = 0. His proof hinges on the idea that as long the coefficients are positive, they follow the fundamental duality law. The remaining equations of the form V = 0 that do not satisfy the fundamental law of duality, can be turned into such a form by algebraically squaring them such that mere addition is applicable again. Here, the idea is equations do not satisfy the fundamental law of duality because they have negative coefficients. Given idempotency, we have that $x^2 = x$ for any class x. Therefore, squaring them makes sure that we have positive coefficients without changing classes.

Let me clarify the method of reduction with an example. Suppose we have the premises:

1st: All figures that have equal corresponding angles and proportional corresponding sides are similar.

2nd: Triangles that have equal corresponding angles also have proportional corresponding sides, and vice versa.

In order to translate these premises, let s denote “similar”, t denote “triangles”, q denote “having corresponding angles equal” and r denote “having corresponding sides proportional”. Now, we can translate the premises as:

$$s = qr \qquad (10)$$

$$tq = tr \qquad (11)$$

As neither of our equations satisfies the fundamental law of duality, we will proceed by squaring them. So, if we take all the terms to the left side, then square them, and then add them we get:

$$s + qr - 2qrs + tq + tr - 2tqr = 0 \qquad (12)$$

We managed to reduce our premises into a single premise. Now we can proceed by using the method of development and elimination to deduce whatever description we want. Suppose, we need to derive a definition of dissimilar figures that consists of the terms triangles, having corresponding angles equal and having corresponding sides proportional. From (12), we have:

$$s = (tq + qr + rt - 2tqr) / (2qr - 1)$$

from which we can get 1- s by multiplying both sides by minus -1 and adding 1 to both sides:

$$1\text{-}s = (qr - tq - rt + 2tqr - 1) / (2qr - 1) \qquad (13)$$

By developing (13) fully in a similar way to (5) but this time for three classes, we get:

$$1\text{-}s = 0tqr + 2tq(1\text{-}r) + 2tr(1\text{-}q) + t(1\text{-}q)(1\text{-}r) + 0(1\text{-}t)qr + (1\text{-}t)q(1\text{-}r) + (1\text{-}t)r(1\text{-}q)+(1\text{-}t)(1\text{-}q)(1\text{-}r) \quad (14)$$

According to our interpretation rules for coefficients, when we drop the terms with 0 coefficients and equate the ones with 2 coefficients to zero, we get:

$$1\text{-}s = t(1\text{-}q)(1\text{-}r) + (1\text{-}t)q(1\text{-}r) + (1\text{-}t)r(1\text{-}q)+(1\text{-}t)(1\text{-}q)(1\text{-}r) \qquad (15)$$

$$tq(1\text{-}r) = 0 \qquad (16)$$

$$tr(1\text{-}q) = 0 \qquad (17)$$

The equation (15) can be interpreted as "dissimilar figures consist of all triangles which have not their corresponding angles equal and sides proportional, and of all figures not being triangles which have either their angles equal, and sides not proportional, or their corresponding sides proportional, and angles not equal, or neither their corresponding angles equal nor corresponding sides proportional." This definition of dissimilar figures may seem overly complex at first sight; however, it gives a complete definition of dissimilar figures from the given premises and can be further simplified to get any specific part of it by using (16) and (17) if some shorter definition is required.

Boole sums up in a shorthand rule the expressions that are commonly found in scholastic logic and their transformed forms that obey the law of duality so as to make them addable. So, we change

$$Y = vX \text{ into } Y(1-X) = 0$$

$$Y = X \text{ into } Y(1-X) + X(1-Y) = 0$$

$$vY = vX \text{ into } vY(1-y) + vX(1-Y) = 0$$

in order to make the equations summable by reduction. $X = 0$ type forms do not need any further transformation, and $X = 1$ can be replaced by $X- 1 = 0$. Notice that not only these changed forms but also anything that follows from them by development or elimination can be added.

3. Aristotelian Logic

Armed with Boolean algebra and methods, we can proceed to scholastic logic. It is beyond the scope of this paper to explain and discuss Aristotelian logic in detail. There are many sources of this kind (Smith 2020, Ural 2017, Oralgul 2018). Here my aim is to only explain how Boolean algebra can be applied to Aristotelian logic, especially to his conversions and syllogisms.

Boole does not give a lengthy space to scholastic logic in *The Laws of Thought,* but only deals with them as a side issue. His reason for that is not only because he thinks scholastic logic is well studied by many other logicians, especially by his contemporaries De Morgan and Hamilton, but also because Boole thinks that scholastic logic "is not a science, but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest." (Boole 1951. 241).

3.1 Conversions

Boole analyses scholastic logic by looking at conversion and syllogisms individually and in a general manner. Conversion is reversing a single proposition to an equivalent form. A syllogism is deducing a conclusion from two premises that have a common term. Eight fundamental types of propositions in scholastic logic with their translations to Boolean algebra are as follows:

1. All X’s are Y’s $x = vy$

2. No X’s are Y’s $x = v\,(1-y)$

3. Some X’s are Y’s $vx = vy$

4. Some X’s are not-Y’s $vx = v(1-y)$

5. All not-X’s are Y’s $1 - x = vy$

6. No not-X’s are Y’s $1-x = v(1-y)$

7. Some not-X’s are Y’s $v(1-x) = vy$

8. Some not-X’s are not Y’s $v(1-x) = v(1-y)$

Conversion is simply done by eliminating the indefinite class v and then developing the resulting equation. Let me give a couple of examples to illustrate it. Let us look at a negative conversion like if All X’s are Y’s, then all not-Y’s are not-X’s. All X’s are Y’s can be written as $x = vy$, if we eliminate v by taking all the terms to the first side, giving 0 and 1 values to v, and multiplying the resulting equations:

$$x^2 - xy = 0$$

Given that $x^2 = x$ and distributing out y we have:

$$x(1-y) = 0$$

When this is solved with reference to 1-y,

$$1 - y = 0/1x + 0/0\,(1-x)$$

which can be simplified by dropping the constituent with 0/1 coefficient:

$$1 - y = 0/0\ (1-x)$$

the interpretation of which is All not-Y's are not-X's[1].

If we consider a simple conversion example like if No X's are Y's, then No Y's are X's. No X's are Y's can be written as $x = v(1-y)$, if we eliminate v:

$$yx = 0$$

if we solve it with reference to y:

$$y = 0/0\ (1-x)$$

the interpretation of which is No Y's are X's.

3.2 Syllogisms

Syllogisms, on the other hand, require the use of all three methods. An example of a syllogism is as follow:

All Y's are X's

All X's are Z's

Hence, All Y's are Z's

The terms Y and Z are referred to as the extremes here, and X is referred to as the middle term. If we translate the premises, we have:

$$y = vx$$

$$x = v'z$$

If we transform these into an addable form by the shorthand rule of reduction, we have

$$y(1-x) = 0 \text{ and } x(1-z) = 0$$

Now, we can simply add these forms. However, we also need to get rid of x, so after adding the equations, eliminating x by giving 0 and 1 values, and multiplying the resulting equations, we have:

$$y(1-z) = 0$$

By development on y, we have:

$$y = 0/0\ z$$

which is interpreted as All Y's are Z's.

Above is an example of how to deal with a particular example of a syllogism. First, reduce the premises into a single premise, then by applying the method of development or elimination get the conclusion in the terms you seek. Instead of considering all different examples of syllogisms, Boole devises a general shorthand method for syllogisms by categorizing them into two cases and applying three inference rules accordingly. The cases depend upon the middle term. If the middle terms are of like quality, i.e. they are both either positive or negative, then the syllogism belongs to Case 1. If they are of unlike quality, then the syllogism belongs to Case 2. Notice that Boole reserves the term 'quality' for being positive (affirmative) or negative and the term 'quantity' for being universal or particular.

Boole deduces the general inference rules for these cases by simply applying the methods of reduction, development, and elimination as in the above example, but this time in a more general form. So, for case 1 he considers the general forms:

$$vy = v'x$$

$$wz = w'x$$

Here x, y, z represents the extremes and the middle terms. The symbol y, for instance, may stand for either All not-Y's or All Y's, as it is purely conventional how to interpret the symbol. v, v', w, w' represents possible varieties of quantity, i.e., whether it is universal or particular. So, for instance, if we take v = 1 and denote an indefinite class by v', our first equation would represent a universal proposition. In his proof, Boole assumes that v and v', or w and w', cannot be both universal. He analyses these general forms, by reducing them into a single premise, and then determines the expressions of x, x-1, and vx by development. After a careful examination of these three descriptions in all possible values of v, v’, w, w’ under the conditions of syllogistic logic, he concludes with one simple inference rule for Case 1. We simply equate the extremes when the middle terms are similar quality.

For case 2, he considers the general forms:

$$vy = v'x$$

$$wz = w'(1-x)$$

He analyses these general forms, by reducing them into a single premise, and then determining the expressions of y, y-1, and vy by eliminating x and developing them. In a similar way, after a careful examination of these three descriptions in all possible values of v, v’, w, w’ under the conditions of syllogistic logic, he deduces two different inference rules depending on the quantity of the middle terms or the extremes for case 2. The first inference rule is if there is at least one universal extreme, then the quality and quantity of that extreme needs to be changed and equated to the other extreme. The second inference rule is if there are two universal middle terms, then the quality and quantity of either extreme needs to be changed and the result needs to be equated to the other extreme.

Let me clarify the rules of syllogisms with a couple of examples. First, let us consider:

All X’s are Y’s

All Z’s are X’s

As the middle terms are of like quality, i.e., both of them are X’s, this belongs to case 1. We simply equate the two extremes and conclude that:

All Z’s are Y’s

Notice that the second premise is translated into Boolean notation as $Z = vX$. Here Boole interprets the indefinite class v as referring to All. However, he does not give any reason why we did not interpret it as referring to None or Some. As I pointed above, Boole does not have any strict rule for the interpretation of indefinite class v and interprets it differently depending on the context.

Second, let us consider:

All Y's are X's

No Z's are X's

If we rewrite the second premise in the universal form:

All Y's are X's

All Z's are not-X's

As the middle terms X are of unlike quality, i.e. one is X and the other is not-X, this belongs to case 2. And as we have at least one universal extreme, we should use the first rule of inference. As we have two universal extremes here, we can pick either one of them. Let's take the extreme All Z's and change its quantity and quality, then we have Some not-Z. Equating it to the other extreme gives:

All Y's are not-Z's

which is equal to

No Y's are Z's

Notice that we would have the same conclusion if we proceed similarly with the other universal extreme.

Third, finally, let us consider

All X's are Y's

All not-X's are Z's

This one also belongs to case 2, as the middle term X's are of unlike quality. However, in this example, we do not have any universal extremes[2]. So, we apply the second rule of inference. We can pick either one of the extremes. Let's take the extreme (some) Y's and by changing its quantity and quality we have All not-Y's. If we equate it to the other extreme, we get:

All not-Y's are Z's

We would get an equivalent result if we process it in the reverse order:

All not-Z's are Y's[1*]

[*] 1) Recall that 0/0 indicates that an indefinite portion of the constituent to which it is prefixed should be taken. Boole does not strictly define how to proceed in these cases. So, depending on the context, we are allowed to take some, none, or all of the members of the constituent to which the coefficient is prefixed.
2) Notice that here we have another example of a deliberate interpretation of the indefinite class v. In this case Boole interprets both of them as referring to Some.

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