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Nonexistence of Solutions for a Logarithmic m-Laplacian Type Equation with Delay Term

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Abstract

In this work, we consider a logarithmic m-Laplacian type equation with delay term with initial and boundary conditions. Under suitable conditions on the initial data, we study the nonexistence of solutions in a finite time with negative initial energy $E(0) < 0$ in a bounded domain.

Keywords: *Delay term, logarithmic source term, m-Laplacian equation, nonexistence of solutions.*
2010 Mathematics Subject Classification: *35B05, 35B44, 35L05.*

1. Introduction

In this article, we consider the logarithmic m-Laplacian type equation with delay term and initial-boundary conditions as follows:

$$\begin{cases} u_{tt} - \operatorname{div}\left(|\nabla u|^{m-2} \nabla u\right) + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) & \\ = u|u|^{p-2} \ln |u|^k, & x \in \Omega,\ t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, \\ u_t(x,t-\tau) = f_0(x,t-\tau), & \text{in } (0,\tau), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^n$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$. $p > m \geq 2$, k, μ_1 are positive constants, μ_2 is a real number, $\tau > 0$ represents the time delay. The term $\Delta_m u = \operatorname{div}\left(|\nabla u|^{m-2} \nabla u\right)$ is called m-Laplacian. u_0, u_1, f_0 are the initial data functions to be specified later.

- **Logarithmic nonlinearity:**

The logarithmic nonlinearity generally seems in super symmetric field theories and in cosmological inflation. From Quantum Field Theory, that such kind of $(u|u|^{p-2} \ln |u|^k)$ logarithmic source term seems in nuclear physics, inflation cosmology, geophysics and optics (see [1, 6]). From the literature review, we begin with the study of Birula and Mycielski [2, 3]. The authors investigated the equation with logarithmic term as follows

$$u_{tt} - u_{xx} + u - \varepsilon u \ln |u|^2 = 0. \tag{1.2}$$

This type of logarithmic equation is a relativistic version of quantum mechanics. They are the pioneer of these kind of problems.
In 1980, Cazenave and Haraux [4] studied the logarithmic equation of type

$$u_{tt} - \Delta u = u \ln |u|^k, \tag{1.3}$$

and the authors proved existence and uniqueness of the equation (1.3).
In [11], Liu introduced the plate equation with logarithmic term as follows:

$$u_{tt} + \Delta^2 u + |u_t|^{m-2} u_t = |u|^{p-2} u \log |u|^k. \tag{1.4}$$

The author proved the local existence by the contraction mapping principle. Also, he studied the global existence and decay results. Moreover, under suitable conditions, the author proved the blow up results with $E(0) < 0$.

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Piskin and Irkıl [17], investigated the following equation

$$u_{tt}-\operatorname{div}\left(|\nabla u|^{m-2}\nabla u\right)-\Delta u+u_t=ku\ln|u|, \tag{1.5}$$

and they obtained the local existence result.

- **Time delay:**

Problems about the mathematical behavior of solutions for PDEs with time delay effects have become interesting for many authors mainly because time delays often appear in many practical problems such as thermal, economic phenomena, biological, chemical, physical, electrical engineering systems, mechanical applications and medicine. Moreover, it is well known that delay effects may destroy the stabilizing properties of a well-behaved system. In the literature, there are several examples that illustrate how time delays destabilize some internal or boundary control system [7].
In 1986, Datko et al. [5] indicated that a small delay is a source of instability in a boundary control. In [14], Nicaise and Pignotti investigated the following equation

$$u_{tt}-\Delta u+\mu_1u_t(x,t)+\mu_2u_t(x,t-\tau)=0. \tag{1.6}$$

Under the condition $0<\mu_1<\mu_2$, they proved the stability.
In [15], Nicaise et al. studied the wave equation in one space dimension in the presence of time-varying delay. In this article, the authors showed that the exponential stability results with the condition

$$a\le\sqrt{1-d}a_0,$$

here d is a constant and

$$\tau'(t)\le d<1,\ \forall t>0.$$

In [9], Kafini considered the wave equation with logarithmic nonlinearity with distributed delay as follows:

$$u_{tt}-\Delta u+\mu_1u_t(x,t)+\int_{\tau_1}^{\tau_2}\mu_2(s)u_t(x,t-s)=u|u|^{p-2}\ln|u|^k, \tag{1.7}$$

the author established the local and global existence. Moreover, he proved the exponential decay of solutions for the equation (1.7).
When $m=2$, then the problem (1.1) can be reduced the following equation

$$u_{tt}-\Delta u+\mu_1u_t(x,t)+\mu_2u_t(x,t-\tau)=u|u|^{p-2}\ln|u|^k. \tag{1.8}$$

Kafini and Messaoudi [8], studied the local existence result and they proved the blow-up result in a finite time for the equation (1.8). When $p=2$, Park [16] obtained local and global existence of solutions by using Faedo-Galerkin's method and the logarithmic Sobolev inequality. Then, the author investigated the decay rates and infinite time blow-up results by using the potential well and perturbed energy methods of the equation (1.8). In recent years, some other authors investigate hyperbolic type equations (see [7, 10, 12, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]).
Inspired by these results, we consider the nonexistence of solutions for the problem (1.1). The main goal of this paper is to establish the sufficient conditions for the nonexistence of solutions for the logarithmic ($u|u|^{p-2}\ln|u|^k$) m-Laplacian ($\operatorname{div}\left(|\nabla u|^{m-2}\nabla u\right)$) type equation (1.1) with delay term ($\mu_2u_t(x,t-\tau)$).
The outline of this paper is as follows: Firstly, in Sect. 2, we present some materials that shall be used in order to establish the main result. In Sect. 3, we state and prove the nonexistence results.

2. Preliminaries

In this part, we give some lemmas that we will use later. Firstly, as in [13], we introduce the new variable

$$z(x,\rho,t)=u_t(x,t-\tau\rho),\ x\in\Omega,\ \rho\in(0,1),\ t>0.$$

Therefore, we get

$$\tau z_t(x,\rho,t)+z_\rho(x,\rho,t)=0,\ x\in\Omega,\ \rho\in(0,1),\ t>0.$$

Hence, problem (1.1) can be transformed as follows

$$\begin{cases} u_{tt}-\operatorname{div}\left(|\nabla u|^{m-2}\nabla u\right)+\mu_1u_t(x,t)+\mu_2z(x,1,t) & \\ =u|u|^{p-2}\ln|u|^k, & \text{in } \Omega\times(0,\infty)\\ \tau z_t(x,\rho,t)+z_\rho(x,\rho,t)=0, & \text{in } \Omega\times(0,1)\times(0,\infty)\\ z(x,\rho,0)=f_0(x,-\rho\tau), & \text{in } \Omega\times(0,1)\\ u(x,t)=0, & \text{on } \partial\Omega\times[0,1)\\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), & \text{in } \Omega. \end{cases} \tag{2.1}$$

We define the energy functional of (2.1) by

$$\begin{aligned} E(t) &= \frac{1}{2}\|u_t\|^2+\frac{1}{m}\|\nabla u\|_m^m+\frac{k}{p^2}\|u\|_p^p-\frac{1}{p}\int_\Omega|u|^p\ln|u|^k dx\\ &\quad+\frac{\xi}{2}\int_\Omega\int_0^1|z(x,\rho,t)|^2d\rho dx, \end{aligned} \tag{2.2}$$

where

$$\tau|\mu_2|<\xi<\tau(2\mu_1-|\mu_2|)\text{ and }\mu_1>|\mu_2|. \tag{2.3}$$

Lemma 2.1. *Suppose that (2.3) holds and $\mu_1 > |\mu_2|$. Then, for $C_0 \geq 0$, we obtain*

$$E'(t) \leq -C_0 \int_\Omega \left(|u_t|^2 + |z(x,1,t)|^2\right) dx \leq 0. \tag{2.4}$$

Proof. We multiply the first equation in (2.1) by u_t and integrate over Ω, we have

$$\begin{aligned} & \frac{d}{dt}\left(\frac{1}{2}\|u_t\|^2 + \frac{1}{m}\|\nabla u\|_m^m + \frac{k}{p^2}\|u\|_p^p - \frac{1}{p}\int_\Omega |u|^p \ln|u|^k dx\right) \\ & +\mu_1\|u_t\|^2 + \mu_2\int_\Omega u_t z(x,1,t)\,dx \\ = & \ 0. \end{aligned} \tag{2.5}$$

Later, we multiply the second equation in (2.1) by $(\xi/\tau)z$ and integrate over $\Omega \times (0,1)$, $\xi > 0$, we obtain

$$\frac{\xi}{2}\frac{d}{dt}\int_\Omega\int_0^1 z^2(x,\rho,t)\,d\rho dx + \frac{\xi}{\tau}\int_\Omega\int_0^1 z(x,\rho,t)\,z_\rho(x,\rho,t)\,d\rho dx = 0. \tag{2.6}$$

We note that

$$\begin{aligned} & -\tfrac{\xi}{\tau}\int_\Omega\int_0^1 z(x,\rho,t)\,z_\rho(x,\rho,t)\,d\rho dx \\ & = -\tfrac{\xi}{2\tau}\int_\Omega\int_0^1 \tfrac{\partial}{\partial\rho}z^2(x,\rho,t)\,d\rho dx \\ & = \tfrac{\xi}{2\tau}\int_\Omega \left(z^2(x,0,t) - z^2(x,1,t)\right) dx \\ & = \tfrac{\xi}{2\tau}\left(\int_\Omega u_t^2 dx - \int_\Omega z^2(x,1,t)\,dx\right). \end{aligned} \tag{2.7}$$

By combining (2.5) and (2.6) and taking into consideration (2.7), we get

$$\begin{aligned} E'(t) = & -\left(\mu_1 - \frac{\xi}{2\tau}\right)\int_\Omega |u_t(x,t)|^2 dx - \frac{\xi}{2\tau}\int_\Omega |z(x,1,t)|^2 dx \\ & -\mu_2\int_\Omega z(x,1,t)\,u_t(x,t)\,dx, \end{aligned} \tag{2.8}$$

for $t \in (0,T)$.
Thanks to Young's inequality, we get the estimate as follows

$$-\mu_2\int_\Omega z(x,1,t)\,u_t(x,t)\,dx \leq \frac{|\mu_2|}{2}\int_\Omega \left(|u_t(x,t)|^2 + |z(x,1,t)|^2\right) dx.$$

Hence, by (2.8), we obtain

$$E'(t) \leq -\left(\mu_1 - \frac{\xi}{2\tau} - \frac{|\mu_2|}{2}\right)\int_\Omega |u_t(x,t)|^2 dx - \left(\frac{\xi}{2\tau} - \frac{|\mu_2|}{2}\right)\int_\Omega z^2(x,1,t)\,dx. \tag{2.9}$$

From (2.3), we get, for some $C_0 > 0$,

$$E'(t) \leq -C_0\int_\Omega \left(u_t^2 + z^2(x,1,t)\right) dx \leq 0.$$

□

Lemma 2.2. *Let $C > 0$, $u \in L^{p+1}(\Omega)$, $2 \leq s \leq p$, and $\int_\Omega |u|^p \ln|u|^k dx \geq 0$. Then,*

$$\left(\int_\Omega |u|^p \ln|u|^k dx\right)^{s/p} \leq C\left[\int_\Omega |u|^p \ln|u|^k dx + \|\nabla u\|_m^m\right].$$

Proof. In [8] from Lemma 3.2 we know that $\left(\int_\Omega |u|^p \ln|u|^k dx\right)^{s/p} \leq C\left[\int_\Omega |u|^p \ln|u|^k dx + \|\nabla u\|_2^2\right]$ is satisfied, by using Sobolev Embedding Theorem we get this result. □

Similar to the [8], we also get the following lemmas:

Lemma 2.3. *Let $C > 0$ and $\int_\Omega |u|^p \ln|u|^k dx \geq 0$. Then,*

$$\|u\|_2^2 \leq C\left[\left(\int_\Omega |u|^p \ln|u|^k dx\right)^{2/p} + \|\nabla u\|_m^{4/p}\right]. \tag{2.10}$$

Lemma 2.4. *Let $C > 0$, $u \in L^p(\Omega)$ and $2 \leq s \leq p$. Then,*

$$\|u\|_p^s \leq C\left[\|u\|_p^p + \|\nabla u\|_m^m\right]. \tag{2.11}$$

Firstly, to get the nonexistence result, we define

$$\begin{aligned} H(t) = & -E(t) = -\frac{1}{2}\|u_t\|^2 - \frac{1}{m}\|\nabla u\|_m^m - \frac{k}{p^2}\|u\|_p^p + \frac{1}{p}\int_\Omega |u|^p \ln|u|^k dx \\ & -\frac{\xi}{2}\int_\Omega\int_0^1 |z(x,\rho,t)|^2 d\rho dx. \end{aligned}$$

3. Nonexistence results

In this part, we establish the nonexistence results of (2.1).

Theorem 3.1. *Assume that (2.3) holds. Suppose further that*

$$\begin{cases} m < p \leq \frac{mn}{n-m}, & \text{if } n > m \\ p > m, & \text{if } n \leq m, \end{cases}$$

and

$$E(0) < 0. \tag{3.1}$$

Then, the solution of (2.1) blows up in finite time T^* and

$$T^* \leq \frac{1-\alpha}{\Lambda \alpha L^{\alpha/(1-\alpha)}(0)}.$$

Proof. From (2.4), we get

$$E(t) \leq E(0) < 0.$$

So,

$$H'(t) = -E'(t) = C_0 \int_0^1 \left(u_t^2 + z^2(x,1,t)\right) dx \geq C_0 \int_0^1 z^2(x,1,t)\, dx \geq 0 \tag{3.2}$$

and

$$0 < H(0) \leq H(t) \leq \frac{1}{p} \int_\Omega |u|^p \ln |u|^k \, dx. \tag{3.3}$$

We introduce

$$L(t) = H^{1-\alpha}(t) + \varepsilon \int_\Omega u u_t dx + \frac{\mu_1 \varepsilon}{2} \int_\Omega u^2 dx, \; t \geq 0,$$

where $\varepsilon > 0$ to be specified later and

$$0 < \alpha \leq \frac{mp-4}{mp}. \tag{3.4}$$

Utilizing the first equation in (2.1), we obtain

$$\begin{aligned} L'(t) &= (1-\alpha) H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^2 + \varepsilon \int_\Omega u u_{tt} dx + \varepsilon \mu_1 \int_\Omega u u_t dx \\ &= (1-\alpha) H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\nabla u\|_m^m - \varepsilon \mu_2 \int_\Omega u z(x,1,t)\, dx \\ &\quad + \varepsilon \int_\Omega |u|^p \ln |u|^k \, dx. \end{aligned} \tag{3.5}$$

By using

$$-\varepsilon \mu_2 \int_\Omega u z(x,1,t)\, dx \leq \varepsilon |\mu_2| \left(\delta \int_\Omega u^2 dx + \frac{1}{4\delta} \int_\Omega z^2(x,1,t)\, dx \right), \; \forall \delta > 0, \tag{3.6}$$

we obtain, by (3.5),

$$\begin{aligned} L'(t) &\geq \left[(1-\alpha) H^{-\alpha}(t) - \frac{\varepsilon |\mu_2|}{4\delta C_0} \right] H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\nabla u\|_m^m \\ &\quad + \varepsilon \int_\Omega |u|^p \ln |u|^k \, dx - \varepsilon \delta |\mu_2| \, \|u\|^2. \end{aligned} \tag{3.7}$$

By taking δ so that $|\mu_2| / 4\delta C_0 = \kappa H^{-\alpha}(t)$, for large κ to be specified later and substitute in (3.7), we obtain

$$\begin{aligned} L'(t) &\geq [(1-\alpha) - \varepsilon\kappa] H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\nabla u\|_m^m - \frac{\varepsilon |\mu_2|^2}{4\kappa C_0} H^\alpha(t) \|u\|^2 \\ &\quad + \varepsilon \int_\Omega |u|^p \ln |u|^k \, dx. \end{aligned}$$

We get, for $0 < a < 1$,

$$\begin{aligned} L'(t) &\geq [(1-\alpha) - \varepsilon\kappa] H^{-\alpha}(t) H'(t) + \varepsilon a \int_\Omega |u|^p \ln |u|^k \, dx + \varepsilon \frac{p(1-a)+2}{2} \|u_t\|^2 \\ &\quad + \varepsilon \left(\frac{p(1-a)}{m} - 1 \right) \|\nabla u\|_m^m + \frac{\varepsilon (1-a) k}{p} \|u\|_p^p - \frac{\varepsilon |\mu_2|^2}{4\kappa C_0} H^\alpha(t) \|u\|^2 \\ &\quad + \varepsilon p (1-a) H(t) + \frac{\varepsilon (1-a) p \xi}{2} \int_\Omega \int_0^1 z^2(x,\rho,t)\, d\rho dx. \end{aligned} \tag{3.8}$$

From (2.10) and (3.3), we have

$$\begin{aligned} H^{\alpha}(t)\|u\|_2^2 &\leq \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha}\|u\|_2^2 \\ &\leq \left[\left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha+2/p} + \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha}\|\nabla u\|_m^{4/p}\right]. \end{aligned}$$

By using Young inequality, we obtain

$$\begin{aligned} H^{\alpha}(t)\|u\|_2^2 &\leq \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha}\|u\|_2^2 \\ &\leq \left[\begin{array}{l} \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{(p\alpha+2)/p} + \frac{4}{mp}\|\nabla u\|_m^m \\ +\frac{mp-4}{mp}\left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha mp/(mp-4)} \end{array}\right]. \end{aligned}$$

Therefore, we have

$$\begin{aligned} H^{\alpha}(t)\|u\|_2^2 &\leq \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha}\|u\|_2^2 \\ &\leq C\left[\begin{array}{l} \left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{(p\alpha+2)/p} + \|\nabla u\|_m^m \\ +\left(\int_{\Omega}|u|^p\ln|u|^k dx\right)^{\alpha mp/(mp-4)} \end{array}\right], \end{aligned}$$

where $C=\max\left\{\frac{4}{mp},\frac{mp-4}{mp}\right\}$. From (3.4), we obtain

$2<\alpha p+2\leq p$ and $2<\alpha mp\leq mp-4$.

Therefore, lemma 2.2 provides

$$H^{\alpha}(t)\|u\|_2^2 \leq C\left(\int_{\Omega}|u|^p\ln|u|^k dx + \|\nabla u\|_m^m\right). \tag{3.9}$$

Combining (3.8) and (3.9), we get

$$\begin{aligned} L'(t) \geq\ & [(1-\alpha)-\varepsilon\kappa]H^{-\alpha}(t)H'(t)+\varepsilon\left(a-\frac{\varepsilon|\mu_2|^2}{4\kappa C_0}\right)\int_{\Omega}|u|^p\ln|u|^k dx \\ &+\varepsilon\left(\frac{p(1-a)-m}{m}-\frac{\varepsilon|\mu_2|^2}{4\kappa C_0}\right)\|\nabla u\|_m^m+\frac{\varepsilon(1-a)k}{p}\|u\|_p^p \\ &+\varepsilon\frac{p(1-a)+2}{2}\|u_t\|^2+\varepsilon p(1-a)H(t) \\ &+\frac{\varepsilon(1-a)p\xi}{2}\int_{\Omega}\int_0^1 z^2(x,\rho,t)\,d\rho dx. \end{aligned} \tag{3.10}$$

Since, choosing $a>0$ small enough, such that

$$\frac{p(1-a)+2}{2}>0,$$

and k large enough so that

$$\begin{cases} \frac{p(1-a)-m}{m}-\frac{\varepsilon|\mu_2|^2}{4\kappa C_0}, \\ a-\frac{\varepsilon|\mu_2|^2}{4\kappa C_0}>0. \end{cases}$$

Picking ε small enough, once κ and a are fixed, such that

$$(1-\alpha)-\varepsilon\kappa>0,$$

$$H(0)+\varepsilon\int_{\Omega}u_0u_1dx>0.$$

Therefore, for $\lambda>0$, from (3.10), we have

$$\begin{aligned} L'(t) \geq\ & \lambda\left[H(t)+\|u_t\|^2+\|\nabla u\|_m^m+\|u\|_p^p\right] \\ &+\lambda\left[\int_{\Omega}\int_0^1 z^2(x,\rho,t)\,d\rho dx+\int_{\Omega}|u|^p\ln|u|^k dx\right] \end{aligned} \tag{3.11}$$

and

$$L(t)\geq L(0)>0,\ t\geq 0, \tag{3.12}$$

then, from the embedding $\|u\|_2 \leq C\|u\|_p$ and Hölder's inequality, we get

$$\int_\Omega uu_t dx \leq \|u\|_2 \|u_t\|_2 \leq C\|u\|_p \|u_t\|_2,$$

and exploiting Young's inequality, we obtain

$$\left|\int_\Omega uu_t dx\right|^{1/(1-\alpha)} \leq C\left(\|u\|_p^{\mu/(1-\alpha)} + \|u_t\|_2^{\theta/(1-\alpha)}\right), \text{ for } 1/\mu + 1/\theta = 1. \tag{3.13}$$

From Lemma 2.4, we take $\theta = 2(1-\alpha)$ which gives $\mu/(1-\alpha) = 2/(1-2\alpha) \leq p$. So, for $s = 2/(1-2\alpha)$, estimate (3.13) yields

$$\left|\int_\Omega uu_t dx\right|^{1/(1-\alpha)} \leq C\left(\|u\|_p^s + \|u_t\|_2^2\right).$$

Hence, Lemma 2.4 gives

$$\left|\int_\Omega uu_t dx\right|^{1/(1-\alpha)} \leq C\left[\|\nabla u\|_m^m + \|u_t\|^2 + \|u\|_p^p\right]. \tag{3.14}$$

Therefore,

$$\begin{aligned}
L^{1/(1-\alpha)}(t) &= \left(H^{1-\alpha}(t) + \varepsilon\int_\Omega uu_t dx + \frac{\mu_1\varepsilon}{2}\int_\Omega u^2 dx\right)^{1/(1-\alpha)} \\
&\leq C\left[H(t) + \left|\int_\Omega uu_t dx\right|^{1/(1-\alpha)} + \|u\|_2^{2/(1-\alpha)}\right] \\
&\leq C\left[H(t) + \left|\int_\Omega uu_t dx\right|^{1/(1-\alpha)} + \|u\|_p^{2/(1-\alpha)}\right] \\
&\leq C\left[H(t) + \|\nabla u\|_m^m + \|u_t\|^2 + \|u\|_p^p\right], \ t \geq 0.
\end{aligned} \tag{3.15}$$

Combining (3.11) and (3.15), we obtain

$$L'(t) \geq \Lambda L^{1/(1-\alpha)}(t), \ t \geq 0, \tag{3.16}$$

where Λ is a positive constant. An integration of (3.16) over $(0,t)$ yields

$$L^{\alpha/(1-\alpha)}(t) \geq \frac{1}{L^{-\alpha/(1-\alpha)}(0) - \Lambda\alpha t/(1-\alpha)}.$$

Hence, $L(t)$ blows up in time

$$T \leq T^* = \frac{1-\alpha}{\Lambda\alpha L^{\alpha/(1-\alpha)}(0)}.$$

As a result, the solution of problem (1.1) blows up in finite time T^*. □

4. Conclusions

In recent years, there has been published much work concerning the wave equations (Kirchhoff, Petrovsky, Bessel,... etc.) with different state of delay time (constant delay, time-varying delay,... etc.). We have been obtained the nonexistence of solutions for the logarithmic m-Laplacian type equation with delay term in a finite time for negative initial energy.

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