A Study Of The Light Curves Of M Type Variables

by

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Özet : Bir çok uzun periyodlu değişen yıldızların ışık eğrilerindeki birinci üst tonların izafi şiddetlerinin A(1) değerleri tayin edilmiştir.

Bu A(1) ve log (P) arasında, peryod arttıkça A(1) değerleri artacak şekilde açık bir münasebet vardır.

Alt seviyelerin mevcudiyetine ait bir emare yoktur. Bu sebepten A(1) değerlerinin herhangi hususi tip populasyonla alâkalı olmadığı aşikârdır. Bu neticenin tetkik edilen yıldızların radyal hız ve öz hareketlerinden çıkarılan neticelerle teyit edildiği görülür.

⁂

Abstract : For a large number of long period variables the values A(1) of the relative intensity of the first overtone in the light curve were determined.

These values A(1) were plotted against the corresponding values log P.

There is a clear correlation between A(1) and log (P) in such a way that A(1) increases with increasing period.

There are no indications of sublevels. It therefore appears that the values A(1) are not related to any particular type of population. This conclusion seems to be confirmed by the results derived from the proper motions and radial velocities of stars under consideration.

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§ 1. Introduction.

In the present article we have analysed the light curves of long period variables by the autocorrelation method and subsequent power analysis. This method as developed by *Ashbrook, Duncombe* and *von Woerkom* [1] was extensively used in the various preceding communications of this institute [2]. By this method we obtain the relative intensities of the various harmonic terms by which the shape of a light curve can be described. In the present paper only the relative intensity A(1) of the first overtone has been considered, e. g. the ratio between the intensity of the first overtone and that of the fundamental.

When the values A(1) obtained in this way are plotted against the corresponding logarithms of the period, we find that there is a correlation between the numerical value of A(1) and period. In the mean A(1) increases with the period, but on the other hand the shape of the light curve does not seem to be uniquely determined by the period, the scatter of the individual points in the A(1) — log P correlation plane being large.

There are no indications in figure 1 of sublevels which might correspond to the different population types, the stars being evenly distributed within a fairly wide belt. This is rather puzzling as various authors agree that the M type variables do not constitute a homogeneous group. *Merrill* [3] pointed out that variables with periods around 200 days have higher space velocities than variables with longer periods. This relation between period and velocity was studied more in detail by *Oort* and *van Tulder* [4] who find a variation of the relative solar velocity between 129 Km/sec. maximum and 7 Km/sec. minimum.

Kukardin [5] thinks that within our galaxy there are at least 3 subsystems of M stars, one being almost spherical, the other highly elliptical while a third is intermediate. So it is astonishing to see that in the A(1) — P correlation plane the distribution of the individual points seems to indicate the existance of only one rather homogeneous group with steadily increasing value A(1). As will appear later on, the probable error in the determination of A(1) is fairly large. It is conceivable that a subdivision into separate levels in our diagram has been obliterated by these errors in A(1). But this explanation is improbable. In the second part of this paper also the proper motions and the radial velocities were taken into consideration, after having grouped the stars according both the values A(1) and log P. We obtain the mean relative velocities of the various groups. The numbers of stars within each group are small so that the numerical values obtained for the velocities are not very reliable.

However, the trend of all values is the same as that observed by *Merrill* [3], *Oort* and *van Tulder* [4], while no evidence is found of any relation between mean velocity and A(1). The conclusion seems to be that A(1) is independent of population type.

§ 2. Observational data.

The stars which were analysed are collected in table I.

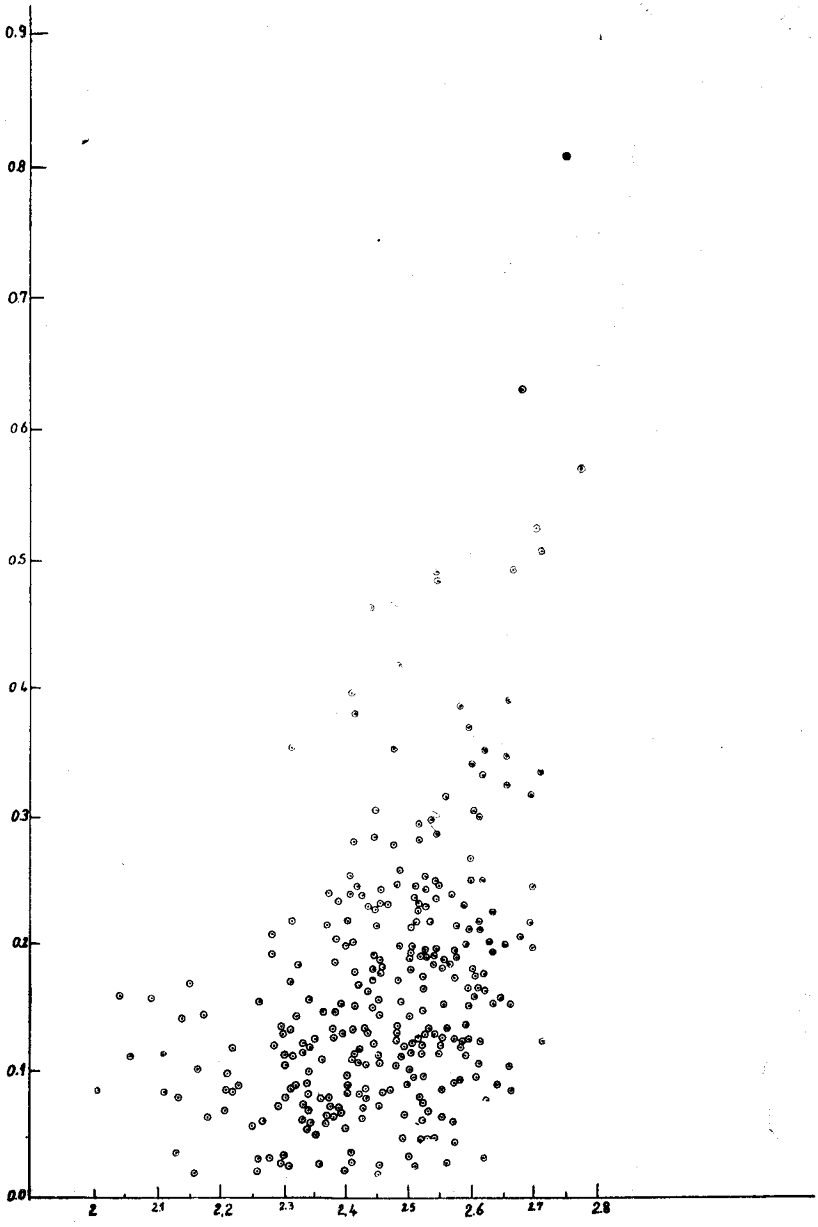
The light curves are from the work of *L. Campbell* as edited by *Miss Mayall* [6]. Not all stars collected by Campbell could be used. The autocorrelation method can only be applied when the whole range of the light curve is well observed. Of the long period variables many are faint and difficult to observe near minimum light. As a result it would seem that several curves are badly observed near minimum, several gaps occurring, so that here the mean light curve is difficult to draw with certainty. A certain number of the stars had to be entirely rejected while others, which are contained in table I, had to be indicated as uncertain. Even though the probable error in the determination of A(1) remains comparatively high. For all stars in table I a mean light curve was drawn by each of the two authors separately and from these light curves two sets of value A(1) are obtained. Table I only gives the mean values. It appeared that the probable error in one single determination is $\pm 0 \cdot 040$, so that the one in the tabulated values is $\pm 0 \cdot 040 : \sqrt{2}$ It is to be emphasized that the two mean curves were based on one and the same set of data. It is to be feared that if the values A(1) were derived from different light curves the the probable error might still be larger.

For future reference we have also given in this same table the apparent magnitude at maximum light as read from the variable star catalogue by *Kukarkin* and *Paranago*, the radial velocity as taken from *Wilson's* General Catalogue of radial velocities [7] and the two components of proper motions as read from the general *Albany* catalogue.

§ 3. The distribution of the values A(1).

In figure 1 the numerical values of A(1) in table I are plotted against the corresponding values of log P. There is a general tendency of the values A(1) to increase with period, but apparently A(1) is not uniquely determined by the period.

The scatter of the indivial values A(1) is rather large so that the individual points are scattered within a fairly wide belt. With each period only an upper limit of A(1) seems to be fixed and this upper limit clearly is determined by the pe-


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8

riod. On the other hand the lower limit is only vaguely determined and even with the longest periods small values A(1) occur. There is no indication that this is due to additional levels which start of farther towards the right hand side of the figure. This failure of a grouping to show up in figure 1, must either be ascribed to the fact that any tendency of grouping is masked by the scatter resulting from probable errors or to the fact that the relation between the shape of the light curve and period is the same with variables of all population types. As will appear from a study of the radial velocities and the proper motions of the stars under consideration, the first explanation is improbable. It would seem that the relation between A(1) and log P is pretty much the same for the different population groups. This can also mean that a possible evolution from one type into another is closely associated with a change in the shape of the light curve.

§ 4. The relative solar velocity as derived from the radial velocities.

If the arrangement of the individual points in figure [1] along different levels were only masked by the influence of probable error some grouping into different population types ought to show up if the variables are arranged in order of increasing values of A(1). This is to say, if between certain fixed limits of log P it were actually possible to split up the stars into different population types. As the space velocities of stars belonging to different population types are largely different, the numerical values of the relative solar velocity derived from the different groups of variables which are grouped both according to their values A(1) and their periods ought to be different. If for each group separately the relative solar velocity and the solar apex are derived, the results are as appears in table II. It is obvious that no high accuracy can be claimed for any of these values as the numbers of stars which are available within each group are small. Especially with the values derived for α and δ this lack of accuracy is very apparent. On the other hand the values in the table clearly indicate that the velocities do not systematically vary with A(1) while the general trend of the

the numbers is the same as that found by *Oort Tulders* [4]. When deriving the results of table II corrections were applied for the differential effect of galactic rotation but the influence of absorption was neglected.

Therefore a second solution was made, in which as coordinates of the apex the values given at the bottom of table II were adopted and in which the absorption is taken into account. The equations were used which *Kreiken* [8] has discussed in some previous articles.

$$\left.\begin{aligned}[v_R y^2 R]+v_{\odot}[y^2 R\lambda]_1-[y^3R^2]f-[y^4R^2]a'(y)f^2=0\\ [v_R\lambda_1]+v_{\odot}[\lambda_1^2]-[yR\lambda_1]f-[y^2R\lambda_1]a(y)f^2=0\end{aligned}\right\}\quad\cdots(1)$$

where λ_1 is the directional cosine of the solar velocity, $v_{\odot}$ the velocity of the sun, $a(y)$ the galactic absorption according to the relation $x = fy \exp\{y \cdot a(y)\}$ while f is given by $\Delta M = -5 \log f$ where ΔM is the possible zero point correction which must be applied to the period luminosity relation given by *Kukarkin* [9].

It would of course be possible for $v_{\odot}$ to adopt the values given by *Oort* and *Tulders* and to determine f and $a(y) f^2$. It appears however, that in all cases the determinant of the coefficients of f and $a(y)\ f^2$ is small, so that the resulting values of f would be very uncertain.

Therefore with the radial velocities it was supposed that $f = 3$ and now the equations (1) can be solved for $v_{\odot}$ and $a(y)$. The results appear in table III. The general trend of the numbers in table III is the same of that in table II. As far as evidence goes, there seems therefore to be no direct relation between the value of A(1) and population type.

§ 5. The intersteller absorption.

From the radial velocities it would appear that the coefficient of interstellar absorption per parsec is $a(y) = -0.00058 \pm 000014$(*pe*) where the probable error only indicates the internal consistency of the different values $(a)y$ which were obtained.

The true degree of uncertainty must be larger. Not only are the values $a(y)$ derived from the equations, influenced by the remaining uncertainties in the determination of the constant

Explanation of Table I

Column (1): Convential designation of variable.

Column (2): Logarithm of Period.

Column (3): Relative intensity of first overtone A(1). $\{A(o) = 1.00\}$.

Column (4): Apparant fotographic magnitudes at maximum light. These magnitudes have been taken from the Catalogue of variable stars by *Kukarkin* and *Paranago.*

Column (5): Radial velocity if available as obtained from the General Catalogue by *Wilson.*

Column (6): Values of $15\,\mu_\alpha \cos\delta$ (Unit $= 0''.001$) as obtained from General Catalogue and after correction for the effects of precession and differential galactic rotation.

Column (7): Values of μ_δ (Unit $= 0''.001$) obtained in the same way.

Column (8): The numbers in this column refer to the remarks at the bottom of the table.

Table I List of stars used in the present publications

Design.	log P	A (1)	m_f (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ ″.001	$\mu\delta$ ″.001	Remarks
SS Cas	2.15	0.170	8.8				
S Scl	2.56	0.029	7.0		+0.082	+0.013	
X And	2.54	0.248	8.1	−4			
T And	2.45	0.110	7.3	−90	+0.048	+0.047	
T Cas	2.65	0.326	6.7	−12	+0.054	−0.010	
R And	2.61	0.165	5.0	−11	−0.007	−0.026	
S Cet	2.51	0.025	7.0	+33	+0.016	+0.053	
T Scl	2.31	0.087	8.7				
Y Cep	2.52	0.197	8.3				
U Cas	2.44	0.151	7.6	−45			
V And	2.41	0.151	8.3	+16			
RR And	2.52	0.076	8.4	−71			
RV Cas	2.52	0.231	7.4				
W Cas	2.61	0.123	7.8	−39			
U Tuc	2.42	0.119	8.5				
U And	2.54	0.289	9.0	−4			(a)
S Cas	2.71	0.124	6.2	−32			
U Psc	2.24	0.107	9.3				
RZ Per	2.55	0.126	8.6	−10			
R Psc	2.54	0.197	7.0	−45	+0.012	−0.003	
RU And	2.38	0.146	9.5	−43			(a)
Y And	2.34	0.100	8.1	−7			
X Cas	2.63	0.153	8.4	−55			
U Per	2.50	0,194	7.5	+17			
S Ari	2.47	0.354	9.1	−27			
R Ari	2.27	0,061	7.2	+114	+0.031	−0.017	
Z Cet	2.26	0.014	8.4				
W And	2.60	0.160	6.5	−29			
Z Cep	2.45	0.186	9.3				(a)
o Cet	2.52	0.254	2.0		−0.007	−0.288	
R Cet	2.22	0.119	7.2	+42	+0,013	+0.007	
RR Per	2.59	0.251	7.9	+9			
R For	2.59	0.137	8.1	+30			
U Cet	2.37	0.066	6.7	−27			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design.	log P	A (1)	m_f (max)	V_R km/sec	$15\,\mu_\alpha \cos\delta$ ″.001	$\mu\,\delta$ ″.001	Remarks
RR Cep	2.59	0.166	9.2				
R Tri	2.42	0.085	5.4	+67			
T Ari	2.51	0.284	7.4	+7	−0.035	−0.007	
R Hor	2.61	0.176	6,8	+60			
T Hor	2.53	0.066	9.2				
U Ari	2,57	0.173	6.4	−37			
X Cet	2.25	0.192	7.9	+59			
Y Per	2.40	0.056	8.2	−9			
R Per	2.32	0.009	7.7	−79			
T Eri	2.40	0.096	7.1	+42			
R Tau	2,51	0.247	7.4	+82	+0.019	−0.003	
W Tau	2.42	0.240	8.5				
T Cam	2.57	0,215	6.4				
RX Tau	2.52	0.165	9.1				
R Ret	2.44	0.173	8.5	+26	−0.012	+0.030	
X Cam	2.16	0.101	7.3	0			
V Tau	2.23	0.089	8.5	+78	+0.008	+0.004	
R Ori	2.58	0.045	8.5	+36			
R Lep	2.64	0.091	5.5	+32			
V Ori	2.43	0.079	8.4	+22			
T Lep	2.57	0.061	7.5	−4	+0.001	−0.037	
R Aur	2.66	0.105	6.6	+8			
T Pic	2.30	0.029	9.2				
T Col	2.35	0.009	6.6				
W Aur	2.44	0.180	8.4	−132			
S Ori	2.62	0.077	7.5	+22	+0.049	+0,004	
S Cam	2.51	0.228	7.6	−13	−0.024	−0.040	
U Aur	2.61	0.253	7.4	+15			
Z Tau	2.69	0.525	9.2				
RU Tau	2.76	0.571	9.7				(a)
U Ori	2 57	0.194	5.2	−21	+0.004	−0.016	
Z Aur	2.06	0.113	8.9	−165			
R Oct	2.61	0.351	8.0				
X Aur	2.21	0.100	7.9	−18			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A (1)	m_f (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ $''.001$	μ_δ $''.001$	Remarks
V Aur	2.55	0.086	8.1	+6			
V Mon	2.52	0.129	6.0		+0.082	−0.067	
U Lyn	2.48	0.258	8.8	−16			
S Lyn	2.47	0.281	8.4				
X Gem	2.42	0.082	7.5	+75			
Y Mon	2.36	0.110	8.3	+71			
X Mon	2.09	0.159	6.8	+160			
R Lyn	2.57	0.127	6.5	+28	−0.004	−0.003	
R Gem	2.56	0.186	5.9	−41			
R CMi	2.52	0.147	7.0	+48			
R Vol	2.65	0.200	8.8				
V Gem	2.43	0.086	7.5	+22			
S CMi	2.52	0.063	6.9	+68	−0.017	−0.011	
T CMi	2.50	0.115	8.9	+35			
Z Pup	2.70	0.38	7.4	+26			
S Vol	2.59	0.269	8.5				
U CMi	2.61	0.335	8.0	+56			
T Gem	2.45	0.114	7.6	+22	−0.007	+0.004	
R Cnc	2.55	0.117	6.1	+32	+0.016	−0.017	
V Cnc	2.43	0.131	6.8	−1	+0.007	+0.001	
RT Hya	2.40	0.085	8.7	+40			
R Cha	2.52	0.200	8.5				
U Cnc	2.48	0.419	8.4	+72	+0.014	0.000	
S Hya	2.41	0.030	7.2	+74	−0.005	+0.007	
T Hya	2.46	0.089	6.8	−3	−0.014	−0.001	
S Pyx	2.31	0.111	8.3	+100			
W Cnc	2.59	0.214	7.4	+49			
Y Vel	2.65	0.417	9.7				(a)
R Cor	2.49	0.112	5.6	+28	−0.026	+0.005	
X Hya	2.48	0.137	7.2	+42	−0.087	−0.062	
Y Dra	2.51	0.291	7.8	+23			
R LMi	2.57	0.190	6.0	+10			
R Leo	2.50	0.102	4.4	+13			
S LMi	2.37	0.240	7.7	−2			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table 1 (Continued)

Design	log P	A (1)	m_f (max)	V_R km/sec.	$15\,\mu_\alpha \cos\delta$ ″.001	$\mu\,\delta$ ″.001	Remarks
U LMi	2.44	0.286	9.3				
V Leo	2.44	0.306	7.9	—23			
W Vel	2.59	0.114	9.5				
S Leo	2.28	0.037	9.1	+106			
W Leo	2.59	0.268	8.7	+54			
R UMa	2.48	0.185	7.6	−5			
S Sex	2.42	0,086	7.6	−5			
X Cen	2.50	0.181	7.0	+38			
W Cen	2.30	0.105	8.6				
SU Vir	2.32	0.186	8.4	+22			
T Vir	2.53	0.324	8.2	+22	+0.004	—0.008	
SS Vir	2.55	0.065	5.9	+2			
T Cun	2.46	0.233	8.6				
Y Vir	2.34	0.055	8.8	+9			
U Cen	2.34	0.089	9.3				
T UMa	2.41	0.109	6.4	—91			
R Vir	2.16	0.020	6.2	—25	—0.024	—0.016	
RS UMa	2.41	0.180	8.2	—26			
S UMa	2.35	0.126	7.1	+8			
U Vir	2.32	0.144	7.3	—46	+0.003	—0.001	
U Oct	2.36	0,078	8.9				
V CVn	2.29	0.077	7.0				
V Vir	2.40	0.219	8.2	+33			
S Vir	2.58	0.099	6.0	+10	+0.048	—0.018	
RV Cen	2.65	0.350	9.2				
T UMi	2.50	0.198	8.4	—3			
T Cen	1.96	0.294	5.2	+28			
RT Cen	2.40	0.086	8.1				
R CVn	2.51	0.000	6.1		+0.025	—0.018	
RR Vir	2.34	0.059	10.3				(a)
Z Boo	2.45	0.231	8.4	+40			
RU Hya	2.53	0.219	7.6	+2			
R Cen	2.73	0.810	7.7	−20	—0.014	—0.030	(b)
U UMi	2.52	0.046	7.6	—26			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A (1)	mf (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ ″.001	$\mu\delta$ ″.001	Remarks
S Boo	2.43	0.063	7.2	—17	+0.013	—0.036	
RS Vir	2.54	0.195	7.3	—26			
V Boo	2.41	0.112	6.4	—38			
R Cam	2.44	0.193	6.8	—33	+0.010	+0.014	
R Boo	2.35	0.050	5.9	—58	—0.006	+0.002	
U Boo	2.30	0.111	9.4	+19	+0.028	—0.040	(a)
Y Lib	2.44	0.229	7.5	—7			
S Lib	2.28	0.190	7.7	+294	—0.052	+0.003	
S Ser	2.56	0.185	7.6	+12	+0.002	—0.009	
S CrB	2.56	0.233	5.8	—1			
RS Lib	2.34	0.117	6.5	—5			
RU Lib	2.50	0.090	7.2	—47			
R Nor	2.70	0.504	8.5				
X Lib	2.22	0.072	9.4	—31			
W Lib	2.31	0.168	10.3				(a)
S UMi	2.52	0.075	7.5	—40			
T Nor	2.39	0.129	7.2	—31	—0.014	—0.030	
X CrB	2.38	0.202	8.0	—104			
R Ser	2.55	0.124	5,6	+24	—0.005	—0.052	
V CrB	2.55	0.180	6.8	—115			
R Lup	2.37	0.077	9.4				
Z CrB	2.40	0.200	8.9	—81			
RZ SCo	2,21	0.085	8.0	—174			
R Her	2.51	0.217	7.3	—30	+0.030	—0.019	
U Ser	2.38	0.065	7.6	—31			
X Sco	2.30	0.030	9.6				
RU Her	2.69	0.221	6.9	—25			
S Sco	2,25	0.054	9.3	+85			
W CrB	2.38	0.126	7.6	+20			
V Oph	2.47	0.082	6.9	—37	+0.007	—0.018	
U Her	2.61	0.161	6.2	—28	+0.022	—0.011	
SS Her	2.03	0.080	7.6	—46			
S Oph	2.37	0.214	8.2	—9	—0.011	—0 009	
W Her	2 45	0.073	7.3	—51	—0.026	—0.000	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A(1)	mf (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ ″.001	$\mu\delta$ ″.001	Remarks
R UMi	2.51	0.095	8.7	−22			
R Dra	2.39	0.066	6.3	−133			
S Her	2.49	0.060	5.9	−10	+0.005	−0.010	
RS Sco	2.50	0.215	6.0	+7	+0.043	−0.035	
RR Sco	2.45	0.014	5.0	−36			
SS Oph	2.26	0.154	8.0	−34			
RV Her	2.31	0.220	9.0	−40			
R Oph	2.48	0.126	6.2	−47	−0.021	−0.017	
RT Oph	2.47	0.214	9.0				
Z Oph	2.55	0.151	7.5	−78			
RS Her	2.39	0.072	7.2	−41			
S Oct	2.41	0.201	8.8				
RU Oph	2.31	0.026	8.6	−65			
RU Sco	2.43	0.133	7.8				
RT Oph	2.63	0.201	9.0	−40			
T Dra	2.63	0.212	7.2	−23			
RY Her	2.34	0.070	8.2	−39			
V Dra	2.43	0.107	8.8	+13			
R Pav	2.36	0.148	8.2		−0.010	+0.017	
T Her	2.22	0.068	6.8		+0.014	+0.006	
W Dra	2.41	0.133	8.7	−21			
X Dra	2.41	0.381	9.2				(a)
RY Oph	2.18	0.061	7.2	−65			
W Lyr	2.29	0.134	7.2	−174			
RV Sgr	2.51	0.120	7.2				
SV Her	2.38	0.186	8.9	−25			
T Ser	2.53	0.046	8.8	−122			
SV Dra	2.41	0.245	8.0	+22			
X Oph	2.52	0.190	5.9	−71			
RS Dra	2.45	0.246	8.3	−29			
RY Lyr	2.51	0.232	9.1	−19			
RW Lyr	2.70	0.336	9.2				
RX Lyr	2.40	0.395	10.8				(a)
Z Lyr	2.45	0.144	9.2	+5			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A(1)	mf (max)	V_R km/sec	$15\,\mu_\alpha \cos\delta$ ″.001	μ_δ ″.001	Remarks
RT Lyr	2.40	0.257	9.3	—94			
R Aql	2.49	0.115	5.1	+32	+0.003	—0.070	
RS Lyr	2.48	0.158	9.9	—18			(a)
RW Sgr	2.28	0.210	8.7				
RU Lyr	2.57	0.135	9.4				(a)
U Dra	2.50	0.140	9.0				
W Aql	2.69	0.223	7,2	—18	+0.029	+0.005	
T Sgr	2.59	0.178	7.0	+2			
R Sgr	2 43	0.068	6.6	—45	+0.006	+0.015	
S Sgr	2.36	0.027	9.1	+35			
U Lyr	2.66	0.152	7.9	—3			
TY Cyg	2.54	0.131	8.5				
RT Aql	2.51	0.237	6.7	—41			
R Cyg	2.63	0.226	5.9	—25	+0.001	—0.007	
RV Aql	2.34	0·158	8,1	—74			
T Pav	2.39	0.151	8.8	+68			
RT Cyg	2.28	0.122	6.2	—116			
TU Cyg	2 34	0.074	8.1	—80			
X Aql	2.54	0.195	8.2	+24			
Chi Cyg	2.61	0.106	2 3	—2			
S Pav	2.59	0.200	8.6	—22			
RR Sgr	2,52	0.113	5.5	+85			
RU Sgr	2.38	0.132	6.3	—68	—0.013	—0.127	
RR Aql	2.60	0.306	8,4	+11			
Z Cyg	2.42	0.110	7.0	—166			
SY Aql	2.55	0.315	8.5	—68			
R Cap	2.54	0.250	8.7		+0.005	0.007	
S Aql	2.17	0.142	8.4	—113	+0 017	—0.024	
R Tel	2.67	0.204	8.4				
RU Aql	2.43	0.163	7.9	+20			
Z Aql	2.11	0,081	8.5				
RS Cyg	2.66	0.494	6,7	—50	—0.009	—0.008	
R Del	2.45	0.175	7.1	—46	—0.014	—0.029	
SX Cyg	2.61	0.300	8.0	—8			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A(1)	mf (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ ″.001	$\mu\delta$ ″.001	Remarks
RT Sgr	2.48	0.105	7.2	+35			
WX Cyg	2.61	0.034	8.6	+32			
U Cyg	2.67	0.083	6.1	+10	−0.049	+0.028	
Z Del	2.48	0.048	8.2	+34			
ST Cyg	2.53	0.070	8.8	−14			
R Mic	2.14	0.140	9.8	+10			(a)
Y Del	2.67	0.633	8.7				
S Del	2.44	0.122	8.2	−13	+0.020	+0.029	
V Cyg	2.62	0.037	6.8	+3	−0.015	+0.024	
T Del	2.52	0.245	8.2	−10	−0.054	+0.018	
W Aqr	2.58	0.236	7.0	−15			
U Cap	2.31	0.357	9.9				(a)
T Aqr	2.30	0.076	6.7	−39	−0.016	−0.015	
RZ Cyg	2.74	1.180	9.0	−47			(b)
X Del	2.45	0.189	8.0	−57			
R Vul	2.13	0.037	7.0		+0.023	+0.005	
TW Cyg	2.53	0.131	8.8				
RS Aqr	2.34	0.088	9.1				
Z Cap	2.26	0.023	8.7	−64			
R Equ	2.42	0.168	8.5	−54			
T Cep	2.59	0.150	5.2	−12	−0.033	−0.064	
RR Aqr	2.26	0.035	8.6	−182			
X Peg	2.30	0.128	8.8	−56			
T Cap	2.43	0.229	10.2	+42			(a)
S Mic	2.33	0.114	9.9	+49			
S Cep	2.69	0.198	7.1	−34			
RU Cyg	2.67	6.000	6.9		−0.027	−0.005	(b)
RR Peg	2.42	0.283	8.5	−30			
R Gru	2.52	0.091	8.7				
V Peg	2.48	0.173	7.8	−25			
RZ Peg	2.64	0.161	8.0	−27			
T Peg	2.57	0.092	8.4	−10			
Y Peg	2.31	0.130	9.7	−85			
RS Peg	2.61	0.214	8.8	−28			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Table I (Continued)

Design	log P	A(1)	mf (max)	V_R km/sec	$15\mu_\alpha \cos\delta$ ″.001	$\mu\delta$ ″.001	Remarks
RS Lac	2.38	0.235	8.8				
X Aqr	2.49	0.189	9.7	+10			(a)
T Gru	2.13	0.078	9.3	+1			
RV Peg	2.59	0.366	9.0	−32			
S Lac	2.38	0.074	7.3	−60			
R Ind	2.33	0.120	8.0				
T Tuc	2.40	0.022	9.3				
R Lac	2.48	0.249	7.8	+18	−0.017	+0.007	
S Aqr	2.45	0.155	9.3	−58	+0.021	−0.047	
RW Peg	2.32	0.085	8.9	−76			
R Peg	2.58	0.118	6.9	+20	−0.004	−0.035	
V Cas	2.37	0.064	6.7	−31			
W Peg	2.54	0.048	7.3	−21			
S Peg	2.51	0.125	7.2	+5			
ST And	2.52	0.121	8.3	+32			
R Aqr	2.59	0.340	6.7	−22	+0.030	−0.024	
Z Cas	2.69	0.318	8.5	−32			
RR Cas	2.48	0.131	9.5	−46			(a)
R Phe	2.43	0.026	9.2	+23			
V Cet	2.41	0.244	8.6	+51			
R Cas	2.63	0.195	4.8	+21	+0.082	+0.007	
Z Peg	2.50	0.035	7.9	−31			
W Cet	2.54	0.234	9.2	+13			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(a) Around mininum the shape of the light curve is uncertain and consequently the value A(1) can only be accepted with reserve.

(b) The value of (1) is abnormal. The variable has two almost equal maxima and minima and in many respects resembles the light curve of a RV Tauri star.

Table II Values of the Solar velocity and of the coordinates of the Solar Apex as derived from the Radial velocities of different groups of M variables. The interstellar absorption is neglected in this solution

1=° $\log P < 2.176$ all values of A(1) combined				
	V=62 Km ;	$\alpha = 28°$;	$\delta = +55°$;	n=10
2=° $2.176 < \log P < 2.300$				
$A(1) < 0.100$	V=90 Km/sec	$\alpha = 258°$;	$\delta = +\ 3°$;	n=11
$0.100 < A(1) < 0.200$	V=98 Km/sec	$\alpha = 302°$;	$\delta = +47°$;	n=9
all values of A(1)	V=78 Km/sec	$\alpha = 287°$;	$\delta = +21°$;	n=20
3=° $2.300 < \log P < 2.476$				
$A(1) < 0.100$	V=50 Km/sec	$\alpha = 296°$;	$\delta = +35°$;	n=31
$0.100 < A(1) < 0.200$	V=39 Km/sec	$\alpha = 330°$;	$\delta = +52°$;	n=38
$A(1) > 0.200$	V=46 Km/sec	$\alpha = 236°$;	$\delta = +73°$;	n=18
all values A(1)	V=43 Km/sec	$\alpha = 300°$;	$\delta = +48°$;	n=87
4=° $2.476 < \log P < 2.543$				
$A(1) < 0.100$	V=31 Km/sec	$\alpha = 227°$;	$\delta = +65°$;	n=13
$0.100 < A(1) < 0.200$	V=16 Km/sec	$\alpha = 284°$;	$\delta = +48°$;	n=25
$A(1) > 0.200$	V=34 Km/sec	$\alpha = 229°$;	$\delta = +\ 8°$;	n=15
all values A(1)	V=23 Km/sec	$\alpha = 270°$;	$\delta = +41°$;	n=53
5=° $\log P < 2\ 543$				
$A(1) < 0.100$	V=17 Km/sec	$\alpha = 264°$;	$\delta = -50°$;	n=11
$0.100 < \log P < 0.200$	V=34 Km/sec	$\alpha = 254°$:	$\delta = +67°$;	n=29
$0.200 < \log P < 0.300$	V=36 Km/sec	$\alpha = 309°$;	$\delta = -\ 5°$;	n=13
$\log P > 0.300$	V=36 Km/sec	$\alpha = 318°$;	$\delta = +\ 3°$;	n=11
all values A(1)	V=26 Km/sec	$\alpha = 295°$;	$\delta = +38°$;	n=64

Values adopted in subsequent part of this paper.

$\log P < 2.176$	dropped as entirely unreliable	
$2.176 < \log P < 2.300$	$\alpha = 287°$	$\delta = +21°$
$\log P < 0.300$	$\alpha = 288°$	$\delta = +42°$

§ 6. The tangential velocities.

With the tangential velocities we use the equatiens :

$$\left.\begin{aligned} f[y^3\mu_\alpha{}^2] + [y^2\mu_\alpha\lambda_2]\, v_\odot + af^2[y^4\mu_\alpha{}^2] &= 0 \\ f[y\mu_\alpha\lambda_2] + [\lambda_2{}^2]\, v_\odot + af^2[y^2\mu_\alpha\lambda_2] &= 0 \end{aligned}\right\} \quad \cdots(2)$$

and a similar set of equations for $\mu\delta$ Before using the equations the equations the observed values $\mu\alpha$ and $\mu\delta$ were corrected for the influence of precession and of differential galactic rotation. Also before solving the equations for $\mu\alpha$ and $\mu\delta$ were combined into one set. When for the $v_\odot$ and λ_2 values given by *Oort* and *Tulders* are used, the unknown quantities are f and af^2 which can be solved. Next the values $a(y)$ and f are obtained. The results appear in table IV.

Table IV. Values for f and $a(y)$ as derived from the tangential velocities

log P	f	a	n
2.30 — 2.40	2.36	−0.00069	7
2.40 — 5.50	4.33	−0.00024	21
2.50 — 2.60	0.73	+0.00014	27
2.60	2 63	−0.00015	12
Mean	2.37	−0.00012	67

Just as with the radial velocities the values of f and $a(y)$ in table IV are bound to be affected by the probable error in the true distribution of the absolute magnitudes, the probable error in the true distribution of the absolute magnitudes, the probable error in the determination of the apparent magnitudes. If the numerical amount of these probable errors were known; their influence on the numerical amount of the coefficients $[y^3\mu_\alpha{}^2]$; $[y^2\mu_\alpha\lambda_2]$ etc. in the equations 2 could be evaluated.

For the present however these probeble errors are unknow and no corrections can be applied.

Therefore it is doubtful whether any significance at all can be attached to the values f appearing in the table. It is conceivable that the actual value of f is around unity and that the deviations from that value are to be ascribed to probable

A of differential galactic rotation ($\Delta V = A\cos^2 b \sin 2(l - l_0)$). but also bv the uncertainties in the determination of the zero point of the period luminosity relation. Even more serious may be the scatter of the individual values of the absolute magnitude around the mean absolute magnitude corresponding to their period. To this latter influence must be added the deviations due to probable error in the determination of the apparent magnitudes.

The two effects will cause error in the determination of y of the shape $y = y_0\,10^{-0.2\Delta M}$ and as is generally known in this case $\overline{\Delta y} \neq 0$ and $\bar{y} > y_0$. But the equations (1) contain terms of y up till the fourth power, so that the results may be strongly influenced by the errors in the determination of the distance parameter y.

Consequently although the value $a(y) = -0.00058$ seems to be quite reasonable, it can only be accepted with reserve.

Table III. Values of the solar velocity as derived from the radial velocities of M type variables. As coordinates of the solar Apex, the values adopted at the bottom of table II were used. The interstellar adsorption has been taken into account. The values of the coeffieients of absorption $a(y)$ as derived from the present material are given only for all stars together within a given period interval.

log P	A(1)	V_0 km/sec	$a(y)$ mag/psc	n
2.176—2.300	all values	71	—0.00116	20
2.300—2.476	<·100	60	—	31
	·100—·200	56	—	38
	>.200	48	—	18
	all values	57	– 0.00011	87
2.476—2.543	<.100	26	—	13
	·100 – ·200	22	—	25
	>.200	27	—	15
	all values	26	—0.00002	53
2.543	<.100	7	—	11
	.100—·200	30	—	29
	·200—·300	13	—	13
	>.300	32	—	11
	all values	25	—0.00151	64
Weighted mean value of $a(y) = 0{,}00058 \pm 0.00014$ (p.e)				

error. From the shape of the equations (2) it is evident that the numerical value of $a(y)$ is inversely proportional to that of f. So if it is assumed that the actual value of f is $f = 1$ in the mean for $a(y)$ the the value

$$a(y) \text{ is} = -0.00028 \text{ magn/parsec}$$

is obtained. The corresponding value as obtained from the radial velocities was $a(y) = -0.00058$ magn/parsec.

This is a clear indication that as yet no high accuracy has been obtained. Notwithstanding this lack of accuracy in the determination of f and $a(y)$ the results obtained from the proper motions have confirmed our conclusion as draw from the A(1) — log P correlation plane: viz the numerical value of A(1) and probably the shape of the light curve of a long period variable is not related to population type.

§ 7. Summary.

1. The numerical values **A(1)** of the relative intensities of the first overtone in the light curves of long period variables were determined. As far as possible all stars in the compilation by *Campbell* were used.

2. The numerical values A(1) are correlated with period. In the mean they increase with the longer periods.

3. The scatter of the individual points in the A(1) - period diagram is fairly large. It cannot be said that the individual points are arranged along a certain level. They seem to occupy a fairly large belt of which only the upper limit is sharply defined.

4. There is no indication of a subdivision in various levels. The relation between A(1) and period seems to be free from the influence of population type.

5. This is confirmed when the radial velocities and proper motions of the stars under consideration are studied. No subdivision into population levels other than based upon period seems possible.

6. The values f, $a(y)$ and $v_\odot$ which are obtained for the different groups are inaccurate. Even the final values $a(y) = -0.00028$ and $a(y) = -0.00058$ magn/parsec cannot be accepted without reserve.

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