A Discussion of the light curve of the RV Tauri variable AC Herculis

by

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Özet: *Payne - Gaposchkin, Brenton* ve *Gaposchkin* [1] RV Tauri dedeğişken yıldızların ışık eğrilerindeki karışık değişimlerin alt tonların pulzasyonundan ileri gelebileceği fikrini ileri sürdüler. Nazarî esas peryod «aslî» peryod ile kesişerek «klasik» peryodu verir. Esas peryodun diğer peryotlara oranı tam sayı değerlerine eşit değildir. R. Scuti'nin otokorelogramında bu esas peryod gözükmemiştir. 2 : 1 oranında iki peryod vardır. Birincisi klasik peryoda tekabül eder ve orijinal ışık eğrisinde otoregressiv tipin değişimlerinden meydana gelebilir. İkincisi «aslî» peryoda tekabül eder ve orijinal eğrideki harmonik terimlerden meydana gelir. Başka bir RV Tauri tipi değişken yıldızı için otokorelasyon metodunun tatbikinde lüzumlu olan tevsî edilmiş süreklilik ve homojenlik şartlarını karşılayan bir ışık eğrisi mevcut değildir.

Bir çok tevsî edilmiş ışık eğrileri mevcut olduğundan süreklilik şartının atılıp atılamıyacağı suali düşünülebilir. Bir misâl olarak AC Herculis'in ışık eğrisi tetkik edilmiştir. Korrelogramda ve mütekabil kuvvet spektrumunda iki peryod mevcuttur. Peryodlar 2 : 1 oranındadır. Birincisı klâsik peryoda tekabül eder ve otoregresiv değişmelerden meydana gelebilir. İkincisi «aslî» peryoda tekâbül eder ve orijinal ışık eğrisindeki harmonik değişimlerden ileri gelebilir. Esas peryod için hiç bir emare bulunmaz. Neticeler yalnız bazı ihtiyatlarla kabul edilebilir.

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Abstract: *Payne - Gaposchkin, Brenton* and *Gaposchkin* [1] have suggested that the complicated variations in the light curves of the RV Tauri variables may result from overtone pulsation. The hypothetical fundamental period interacts with the «essential» period thus causing the «classical» period. The ratios of the fundamental period to the other periods are equal to non integer values. In the autocorrelogram of R Scuti this fundamental period did not show up. There were two periods with a ratio 2 : 1. The first corresponded to the «classical» period and might be caused by variations of the autoregressive type in the original light curve. The second corresponded to the «essential» period and arose from harmonic terms in the original curve. For no other RV Tauri variables is a light curve available which meets the prerequisites of extended continuity and homogeneity which are essential for the application of the autocorrelation method.

The question is considered whether the condition of continuity can be dropped or not, many extended light curves being available. As an example the light curve of AC Herculis is considered. In the correlogram and the corresponding Power spectrum two periodicities are present. The periods are in the ratio 2 : 1. The first corresponds to the «classical» period and may be caused by autoregressive variations. The second corresponds to the «essential» period and may be due to harmonic variations in the original light curve. No indication is found for a fundamental period. The results can only be accepted with certain reserve.

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1. In their extensive study of the RV Tauri variables, *Payne - Gaposchkin, Brenton* and *Gaposchkin* [1] tentatively suggest that the observed irregularities in the light curves of the RV Tauri stars are the result of their undergoing an overtone pulsation. An hypothetical Cepheid like fundamental is introduced, while it is supposed that for most of the RV Tauri stars the amplitude of this fundamental is too small to effect appreciably the light curves and the velocity curves. From the relationship of the velocity curve and the light curve these same authors conclude that when the RV Tauri stars are to be considered as pulsating stars, the period of pulsating must be half the «classical» period. Here the classical period is the interval between successive similar minima. The period of pulsation which is the «essential» period represents the first overtone of the fundamental.

This suggestion is based on the results of the theoretical work of *Kluyver* [2], *Schwarzschild* [3] a. o., from which it has appeared that the ratio of the fundamental period and its first overtone depends on the ratio of the specific heats. If the ratio of the specific heats of the stellar material is between 1,43 and 1,67 the ratio of the periods is between 0,554 and 0,738. Generally this latter ratio is a non integer.

In a recent paper [4] the present autor has tried to ascertain whether the simultaneous existance of several periodicities in the atmosphere of RV Tauri stars could be confirmed by an analysis of the observed light curve.

In this analysis the method used was the one introduced by *Ashbrook Duncombe* and *van Woerkom* [5] in their study of the light curve of μ Cephei. To the light curve observed for this variable they first apply the autocorrelation method developed by *Kendall* [6] and thus obtain the correlogram. This correlogram is converted into the corresponding power spectrum. When the author applied this method to the light curve of the RV Tauri variable R Scuti an autocorrelation curve and a power spectrum were obtained in which two periods were clearly represented with a length of approximately 140 and 70 days. The 140 day period appeared to be strongly damped in the correlogram while as far as could be judged from the correlogram the 70 day period was undamped. The 70 day period, which obviously represents the «essential» period seemed to correspond to a harmonic term in the original light curve or a set of harmonic terms. The 140 day period represents the 146 day classical period and may not be a period in a strict sence. It may correspond to variations in the original light curve which are all of the same autoregressive type viz. $U_{t+2} = - a\ U_{t+1} - b\ U_t - \varepsilon_{t+2}$. In the power spectrum also only periods of 140 and 70 day respectively showed up and no definite evidence was obtained for the occurrence of an additional «fundamental» period, and so it was concluded that it either is non existent or has an amplitude which is too small to show up either in the correlogram or the power spectrum. The two periodicities which are present appeared to be exactly in the ratio 2 : 1. This conclusion based on the analysis of only one RV Tauri variable cannot as yet be considered as being definite. The question must be raised whether it can be extended to the other variables of this class as well. Obviously this question can only be settled by the analysis of additional light curves. But when we attempt to do this we are met by some difficulties.

2. The prerequisites of the autocorrelation method are extent continuity and reasonable homogeneity of the observed light curve. With the RV Tauri variables these specifications are completely met only by the light curve of R Scuti. In all other curves seasonal gaps occur. It is evident that especially the conditions of continuity drastically cuts down

the usefulness of the autocorrelation method in all astronomical work.

With the long period variables conditions of visibility and climatic conditions during part of the year will make observations impossibe or at least very difficult. As a result gaps will occur in the observed light curve while the spacing of these gasp is periodical. Similarly in the observed light curves of the short period variables is a series of interruption is bound to occur spaced at intervals of one day. Therefore in this case also there is a periodicity in the occurence of the interruptions. Consequently of the three afore mentioned prerequisites the one of continuity seems the most difficult to meet. The conditions of extent and homogeneity are satisfied in many instances. It therefore seems worth while to consider the question whether the condition of continuity is entirely indispensable. Normally when a continuous series of observations is avaiable we compute the numerical value of the autocorrelation coefficients r_k from the convenient approximation used by *Ashbrook, Duncombe* and, *van Woerkom* (1. c).

$$r_k = \left(\sum_i^N i\ U_i U_{i+K}\right)\left(\sum_i^N i\ U_i^2 - K U^{-2}\right) \qquad \cdots(1)$$

which is adequate for $N \ll k$. The correlogram is obtained by plotting the resulting values of r_k against the corresponding values k.

With the autocorrelation method aequidistant observations must be used which are obtained by dividing the observed light curve in a large number of aequidistant steps. When in the observed light curve a number of gaps occur it is still possible to divide the total time interval in aequidistant steps, but now a number of these steps coincide with the gaps and for those no observed values U_i are available. Now let N_1, N_2, . . . , N_{n+1} be the number of steps within the successive intervals covered by observations and G_1, G_2, . . . , G_n the number of steps within the successive gaps. Both with the seasonal interruptions and the diurnal ones the numbers $N_1 \ldots N_n$ and $G_1 - G_{n-1}$ will be more or less equal inter se. The question which must be considered is, whether we still obtain reliable results if when computing the correlation [1] we periodically drop a certain number of terms U_i. Provided that the intervals N are of sufficient extent and the intervals G are short, there do not in general seem to be many a priori objections. Obviously the periodicities to be expected in the observed light curve and in the correlogram should be of such a length, that the whole length of an oscillation is uniformly covered by the successive intervals N. The real danger is that we can never be entirely sure about this point. On the other hand, if relatively to the intervals N, the various periodicities are small and the ratio of the periodicities and the lengths of the intervals N and G can not be expressed as the ratios of small integer numbers we may be reasonably sure that by summation of the products derived from an intermittent curve we can obtain results which are not far inferior to those obtained from a continuous curve. The analytical expression for r_k derived from a light curve containing a number of gaps would be :

$$r_k = \left\{ \sum_{1}^{N} {}_i\, U_i U_{i+k} + \sum_{N_1+G_1+1}^{N_1+G_1+N_2-K} {}_i\, U_i U_{i+K} \cdots + \sum_{N_1+\ldots+N_n+G_1+\ldots+G_n+1}^{N_1+\ldots+N_{n+1}+G_1+\ldots+G_n-k} {}_i\, U_i U_{i+k} \right\} \times$$

$$\times \left\{ \sum_{1}^{N_1-k} U_i^2 \sum_{i}^{N_1-k} U_{i+k}^2 + \cdots \sum_{N_1+\ldots+N_n+G_1+\ldots+G_n+1}^{N_1+\ldots+N_{n+1}+G_1+\ldots+G_n-k} {}_i\, U_2^i \sum U_{i+k}^2 \right\}^{-1/2}$$

or written in a more compact form :

$$r_k = \left\{ \sum_{0}^{n} {}_j \sum_{L_j}^{A_j-k} {}_i\, U_i U_{i+k} \right\} \left\{ \sum_{0}^{n} {}_j \sum_{L_j}^{A_j-k} {}_i\, U_i^2 \times \sum_{0}^{n} {}_j \sum_{L_j}^{A_j-k} {}_i\, U_{i+k}^2 \right\}^{-1/2} \cdots (2)$$

where $L_j = 1 + \sum_{0}^{j} (N_j + G_j)$ and $A_j = \sum_{0}^{j} (N_{j+1} + G_j)$ with $N_0 = G_0 = 0$.

When the intervals N are of sufficient extent we could still use the approximation [1] so that instead of [2] we can write :

$$r_k = \left\{ \sum_{0}^{n} {}_j \sum_{L_j}^{A_{j-k}} U_i U_{i+k} \right\} \left\{ \sum_{0}^{n} {}_j \sum_{L_j}^{A_j} {}_i\, U_i^2 - k\, n\, U^{-2} \right\}^{-1} \cdots (3)$$

where L_j and A_j have the same meaning as before.

The relation (2) and its substitute (3) look much more formidable than the simple relation (1). In the actual computations the difference is less than would be expected. It simply means that the computations are carried out in the usual way, but that at given intervals a certain number of terms U_i and U_{i+k} has to be dropped.

We can also so say that the calculations are first carried out for the separate intervals while next the results for all intervals are summed. By a suitable arrangement of the observed values U_i these various operations can be carried out without any great difficulties. However, the values r_k derived from (3) will be less reliable than those derived from a continous curve.

For evaluating the mean error affecting the values r_k derived from a continuous set of observations *Ashbrook*, *Duncombe* and *van Woerkom* use a reletion derived by *Bartlett* [7], viz.

$$\sigma_k^2 = N^{-1}(1 + 2r_1{}^2 + 2r_2{}^2 + \cdots + 2r_k^2) \qquad \cdots(4)$$

Here N is the number of steps used in the computation of r_k. With a continuous series of observations N decreases by unity with each successive value of k. That is to say that there is a slow but steady increase in the mean error with increasing value of k.

If the relation (4) is assumed to remain valid for an intermittent curve (and this may be true provided that the conditions are met stated in the beginning of this section), it is at once evident that the increase of $|\delta|$ is considerably larger than in the previous case. To an increase of k by one there now corresponds a decrease of n in N and for r_k the decrease is $n \cdot k$.

3. When the duration of the gaps is short the intervals G_k are short and especially with the higher values of k it may occur that $k > G_j$. In these cases some additional terms must be included in the relations (2) and (3). No further explanation is needed to see that now instead of (2) we have :

$$r_k = \left\{ \sum_0^n j \left[\sum_{L_j}^{A_j - k} i \, U_i U_{i+k} + \sum_{A_j+G_j+1-k}^{A_j} i \, U_i U_{i+k} \right] \right\} \left\{ \left[\sum_0^n j \left(\sum_{L_j}^{A_j-k} i \, U_i^2 + \sum_{A_j+G_j+1-k}^{A_j} U_i^2 \right) \right] \left[\sum_0^n j \left(\sum_{L_j}^{A_j-k} i \, U_{i+k}^2 + \sum U_{i+2}^2 \right) \right] \right\}^{-1/2} \qquad \cdots(5)$$

while instead of (3) we have :

$$r_k = \left\{ \sum_0^n j \left[\sum_{L_j}^{A_j - k} i \; U_i U_{i+k} + \sum_{A_j+G_j+1-k}^{A_j} U_i U_{i+k} \right] \right\} \left\{ \left[\sum_0^n j \left(\sum_{L_j}^{A_j-k} i \; U_i^2 + \sum_{A_j+G_j+1-k}^{A_j} i \; U_i^2 \right) \right] - \left(\sum G_j + k \right) U^{-2} \right\}^{-1} \quad \cdots (6)$$

Here again the calculations are far less complicated than suggested by this analytical form. They merely state that the normal calculations are to be carried out for each interval but an additional number of terms has to be included which result from the overplapping of the values U_{i+k} with the values U_i in the next interval. As before the difficulty is purely formal and can be largely reduced by a suitable tabulation of the observed values U_i. The work which is involved in the calculations of the values r_k is considerable, but not much in excess of that which is needed to find the values r_k from a continuous series of observations However, just as in the case considered in section (2) there is a danger that the length of the periodicities in the original light curve and in the correlogram are of the same order of magnitude as the periodicities in the occurrence of the gaps or that at least the ratios of the various periodicities can be expressed by simple integer values. The result would be that the correlogram is seriously distorted. If we allow k to increase further at a given moment we will have have $k \geq A_j$ and as a consequence the values U_i are in the one observed interval and the values U_{i+k} in the next one. It would not be difficult to draw up an additional analytical expression covering this case. It woud be hardly justified to do so. Such an expression would only indicate that for all possible combinations of the observed values we must compute :

a) The sum of the products $U_i \cdot U_{i+k}$

b) The sum of the squares of all values U_i^2 while from this sum we must subst substract as many times the mean value U_i^{-2} as we had to drop terms U_i and finally :

c) The ratio of these two sums.

Within this additional range of k, the number of values which can be included in the calculations at first increases but later on sharply decreases again. Consequently over the whole range of values k the number N varies in a rather complicated way. But even with the higher values of k the objection remains that for the steps which coincide with the gaps no observed values U_i are available. So just as in the previous cases periodically a number of terms U_i have to be omitted, So the danger remains that this periodicity may distort the normal periodicities to be expected in the correlogram.

Summarising we find that is may be worth while to compute the correlogram even when only an intermittent light curve given, but that caution is needed when interpreting the periodicities in the correlogram.

4. From the RV Tauri variables discussed by *Payne-Gaposchkin*, *Brenton* and *Gaposchkin* [1] AC Herculis is the one which has been most extensively observed.

The aforementioned authors give the light curve between J. D. 24 23 000 and J. D. 24 30 580. The period as determined from two successive mimina is 75 d. 24 and this therefore represents the «classical» period.

From examination of this light curve it is evident that it is a typical example of a light curve affected by seasonal circumstances. Notwithstanding the fact that the variable has been extensively observed, the light curve contains a series of gaps. These gaps are fairly regularly spaced at one year intervals. For the application of the autocorrelation method aequidistant observations are required. With a view to the observed period the interval between J. D. 24 23 000 and J. D. 24 30 580 was divided in aequidistant steps of five days each. Obviously the gaps had to be included. Next from the observed curve given by *Payne - Gaposchkin at all.* the values U_i corresponding to each step were read. Here $U_i = m_i - \overline{m}$ and $\Sigma U_i = 0$. Both from an inspection of the light curve and from the values U_i it appears that no secular trends are present. I have refrained from tabulating the individual values U_i as these can directly been read from the observed curve.

Obviously values U_i are available only for the intervals covered by observations. These intervals have been indicated by N_j, within the intervals G_j we do not have any values U_i. As stated before when splitting up the total light curve in aequidistant steps, the intervals G_j were included. If the step coinciding with J. D. 24 23 000 is taken to be zero, the intervals N_j and G_j expressed in aequidistant steps are as indicated in Table I. Excluding the last value N_j it appears from the table that the mean lengths of the intervals N_j, G_j and $N_j + G_j$ are 53, 20 and 73 respectively. As was to be expected the latter number of steps correspondons to a time interval of one year. It is also evident that the scatter of the individual values N_j, G_j and $(N_j + G_j)$ round these means is fairly large. The sum of the numbers N_j indicate the maximum number of values U_i which can be used in the calculations. This sum is equal to 1046. This is a satisfactory number of values U_i but it can only be used when computing the numerical value of r_k for $k = 0$ (which is equal to one by definition). As explained in sections (2) and (3), with increasing values k, the number of values N which can be used at first decreases rapidly, next remain constant or almost constant until an increase sets in.

After a certain maximum is attained a new rapid decrease of the available number of values U_i sets in.

The total numbers of values U_i which can be used in the calculations has been indicated by $N(k)$. The numerical values of $N(k)$ have been tabulated in Table II They are important for evaluating the mean error affecting the autocorrelation coefficients r_k which are also given in Table II. These autocorrelation coefficients have been computed in the way indicated in the preceeding sections. From Table II we obtain the correlogram by plotting the values r_k against the corresponding values of k. The results appear in Figure I. Before discussing the implications of the correlogram a few words mnst be be said about the mean error of the values r_k on which the correlogram is based. According to *Bartlett* [7] the variance of r_k is given by the equation (4) provided that there are no sampling errors As sampling errors are present, the mean error should be larger than the values $|\sigma|$ computed from the equation (4). The values $|\sigma|$ as computad from (4) are also indicated in Figure I. For this it had to be assumed that the relation given by *Bartlett*

Table I. The intervals N_j and G_j expressed in steps. Numbers of steps within the intervals (sections 2 and 4).

j	N_j	Σi	G_j	Σi	j	N_j	Σi	G_j	Σi
1	22—80	59	81–93	13	11	750—798	49	799—829	31
2	94—170	77	171—185	15	12	830—882	53	883—899	17
3	186—225	40	226—243	18	13	900—948	49	949—975	27
4	244—300	57	301—323	23	14	976—1027	52	1028—1056	29
5	324—376	53	377—385	9	15	1057—1092	36	1093—1123	31
6	386—440	56	441—459	19	16	1124—1174	51	1175—1201	27
7	460—512	53	513—527	15	17	1202—1247	46	1248—1262	15
8	528—588	61	589–605	17	18	1263—1320	58	1321—1349	29
9	606—656	51	657–679	23	19	1350—1400	51	1401—1417	17
10	680–741	62	742—749	8	20	1418—1452	35		

Table II. Tabulation of the values r_K and N(K)

k	r_K	N_K	k	r_K	N_K	k	r_K	N_K	k	r_K	N_K
0	1.00	1046									
1	+.70	1026	21	—.12	701	41	—.31	645	61	+.43	774
2	+.16	1006	22	+.03	693	42	—.22	647	62	+.16	787
3	—.30	986	23	+.04	685	43	+.01	650	63	—.15	800
4	—.47	966	24	—.10	678	44	+.28	653	64	—.36	815
5	—.35	946	25	—.28	671	45	+.43	656	65	—.31	828
6	—.10	926	26	—.36	664	46	+.39	660	66	—.15	839
7	+.09	906	27	—.23	657	47	+.17	664	67	+.01	849
8	+.10	886	28	+.07	652	48	—.12	668	68	+.04	859
9	—.06	867	29	+.39	642	49	—.30	674	69	—.09	867
10	—.27	849	30	+.50	640	50	—.29	680	70	—.33	874
11	—.34	831	31	+.35	638	51	—.16	688	71	—.38	879
12	—.24	813	32	+.06	638	52	—.03	697	72	—.32	884
13	+.07	796	33	—.21	638	53	+.01	707	73	—.06	887
14	+.35	780	34	—.32	638	54	—.08	718	74	+.28	889
15	+.49	764	35	—.24	638	55	—.20	729	75	+.50	891
16	+.39	751	36	—.09	638	56	—.29	740	76	+.47	889
17	+.13	738	37	+.07	638	57	—.22	726	77	+.20	885
18	—.17	727	38	+.08	639	58	+.04	738	78	—.14	879
19	—.32	718	39	—.06	641	59	+.34	749	79	—.36	870
20	—.29	709	40	—.22	643	60	+.52	761	80	—.37	860

could also be applied in the present case. The observed numbers $N(k)$ of values U_i being large I have considered the possibility of evaluating the mean error affecting the values r_k in a purely empirical way.

By combining the values U_i of the successive intervals of Table I, we can compute values r_k for each interval separately.

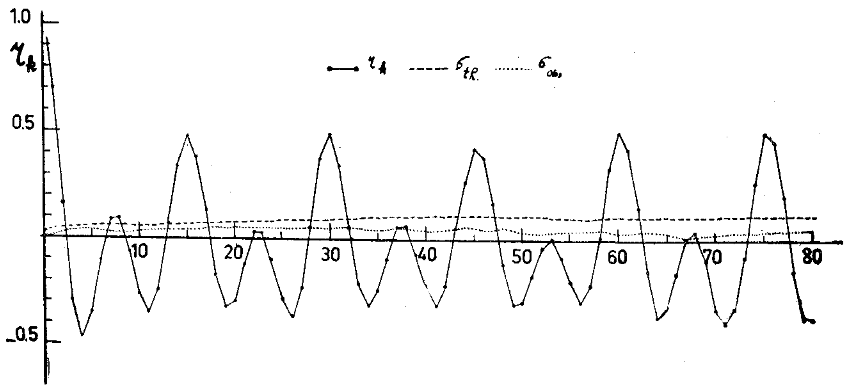

Fig. I. Autocorrelogram of the light curve of AC Herculis.

In that way 19 different sets of values r_k are obtained and by intracomparison we obtain a second set of values $|\sigma|$. In figure I this second set of values is indicated by the curve $|\sigma|$ obs. The results are not very encouraging For tho whole range of the values r_k we have $|\sigma|$ obs $<$ $|\sigma|$ th. With the smaller values of k this difference is small, but it becomes more and more pronounced as k increases. With the partiel curves r_k only short ranges of values U_i are used

Within these small ranges the different periodicities may not have sufficient opportunity to get out of tune. Consequently the small values $|\sigma|$ obs may be misleading. In the following only the values $|\sigma|$ as computed from theory are used and even these values indicate certain minimum values. The actual mean errors may bo even larger than these minima.

When considering the autocorrelation curve in Figure I, two periodicities are at once apparent. The lengths of the two periods are 15×5 and 7.5×5 days respectively. Apparently the 75 day period correspunds to the «classical» period and the 37,5 day period to the «essential» period of AC Herculis. Apart from the maximum at $k = 0$, which is equal to 1 by definition, the intensity of the successive maxima seems to remain constant for both periodicities. In this respect the correlogram of AC Herculis is different from that obtained for R Scuti (4). With R Scuti only the «essential» period seemed to be undamped, while the classical period was strongly damped and might correspond to variations of the autoregressive type in the original light curve. I do not think however that in the present case our conclusions can be very definite. With the higher values of k the correlogram in Figure I is affected by large uncertainties and here especially the periodic character of the successive gaps in the oberved light curve might affect its shape. The only conclusion which seems pretty certain is that just as with R Scuti, the two periodicities which occur in the light curve of AC Herculis are in the ratio 2 : 1.

5. By applying the relation :

$$\pi(f) = \frac{2}{N} \sum_{0}^{N} r_k \cos 2\,\pi\, f\, k \qquad \cdots(7)$$

the correlogram is converted into the corresponding power spectrum. Here f is the reciprocal of the trial period. In their study of the light curve of μ Cephei *Ashbrook, Dun ombe* and *van Woerkom* [5] sharply cut down the number of terms r_k which they use in the computation of the numerical values of $\pi(f)$. The reason is that they want to avoid the inclusion of terms which are statistically insignificant. The present author has shown [8] that there is some danger in using too small a number of values r_k. When the number of terms r_k is only slightly larger than the number of terms covering one complete period of the correlogram, some systematic effects may be intoduced which distort the power spectrum. The result may be that the maximum in the power spectrum corresponding to the principal period is broadened. At the same time some spurious maxima and minima may be thrown up. On the other hand when including the higher values r_k we do not only consider less reliable values of the autocorrelation coefficient, but there will also be an increasing danger that the periodical gaps in the observed light curve affect the shape of the power spectrum. It is therefore

rather difficult to fix the limit of the values r_k in a wholly unobjectionable way. Consequently I have preferred to compute the power spectrum using different limitting values of r_k. The result is that we obtain several power spectra and by comparing these different curves it will become apparent how the shape of the power spectrum is affected by the choice of the limit. In Table III I have tabulated the curves $\pi(f)$ which are obtained if for the limit of r_k we adopt the values $k = 20, 30, 40$ and 60 respectively. The first column of the table contains the reciprocal of the trial period A graphical representation of

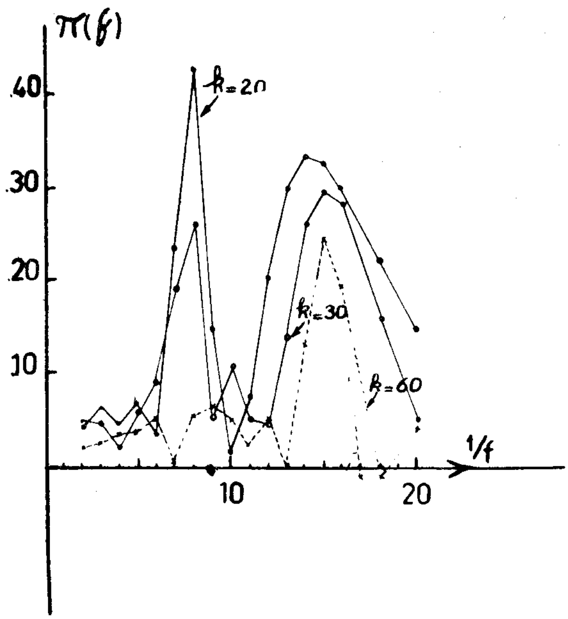

Fig. II. Power Spectrum of AC Herculis corresponding to the correlogram in Figure I.

some of the curves is given in Figure II From an inspection of the numbers in Table III and of the curves in Figure II it appears that in the power spectrum corresponding to $k = 20$ only two clear cut maxima occur. The approximate position of these maxima is at $1/f = 15$ and $1/f = 7.5$. They correspond to the periodicities which already occurred in the correlogram represented in Figure I. No further maxima are present. The maximum at $1/f = 7.5$ is sharp and narrow and may therefore correspond to a harmonic term in the original light curve. The maximum at $1/f = 15$ has a slightly greater intensity but appears to be somewhat broadened. This might be due to the fact that the 75 day period is not a period in a strict sense, but corresponds to a series of variations of the autoregressive type. In this case however, the evidence is hardly convincing, because the broadening may also be due to the fact that when computing this first power spectrum a limit of k was used, which is only slightly larger than the 75 day period.

When the power spectrum corresponding to $k = 30$ is considered it appears that the maximum around $1/f = 7.5$ starts to disintegrate. A faint secondary maximum appears at $1/f = 10$ exactly at the place where in the curve corresponding to $k = 20$ a minimum is found. With the curve $k = 30$, we are using for the first time values r_k which are derived from observed values U_i and U_{i+k} corresponding to two successive intervals N_j and N_{j+1} which are separated by a gap A_j. The mean length of G_j being 20 (see section 4), this secondary maximum may not be real. It is still apparent in the power spectrum corresponding to $k = 40$, but now the two principal maxima are of almost equal intensity. With $k = 60$ the disintegration of the power spectrum is almost complete. The secondary maximum at $k = 10$ has dissappeared, but at the same time the original maximum at $1/f = 7{,}5$ has split up into two parts. The maximum at $1/f = 15$ is flattened and the power spectrum contains an additional maximum (at $1/f = 24$) which is neither shown in Table III nor in Figure II. It was suppressed because it is evident that the power spectrum corresponding to $k = 60$ is valueless

The two only maxima thereforme which seem to be well established are those corresponding to the classical and essential period respectively. We do not obtain evidence for the presence of a third period, corresponding to the fundamental period.

6. In their original paper *Payne - Gaposchkin* etal. state that they expect the amplitude of the hypothetical fundamental period to be small Therefore the failure of such a fundamental period to show up in the correlogram and power spectrum of both R Scuti and AC Herculis does not prove that a funda-

mental period is not present. A more serious objection is that the periodicities which clearly show up in the correlograms and power spectra of these two RV Tauri variables are exactly in the ratio 2 : 1. With R Scuti we concluded that one of the periods (the longer one) is not a period in a strict sence. This may also be the case with AC Herculis, but with this latter

Table III. Values of $\pi(f)$ for different values of $1/f$.
The number of terms r_K included in the computations are 20, 30, 40 and 60 respectively.

$1/f$	$\pi(f)_{20}$	$\pi(f)_{30}$	$\pi(f)_{40}$	$\pi(f)_{60}$
20	+.153	+0.51	+.019	+.045
18	.221	+.158	.085	−.010
16	.302	.282	.215	+.188
15	.332	.296	.249	+.246
14	.332	.247	.215	.133
13	.300	.141	.108	.000
12	.205	.046	.005	.049
11	.077	.053	.026	.025
10	.011	.106	.113	.048
9	.149	.056	.027	.061
8	.432	.263	.203	.059
7	.239	.196	.117	.003
6	.038	.088	.090	.056
5	.075	.065	.010	.028
4	.052	.022	.016	.028
3	.064	.049	.025	.024
2	.042	.047	.015	.022

star the evidence is in no way conclusive For the present the question whether the longer period is a period in a strict sence or not remains debatable.

It seems dublious whether this question could be settled by an analysis of RV Tauri stars in the way described in the present paper. Not only does such an analysis require a considerable amount of calculations, but also the final results are not established with such a degree of reliability as must be desired.

On the other hand by now we can accept as a well established fact that only periodicities occur which are in the ratio 2 : 1. It may be difficult to reconcile this observational fact with a theory based on the simultaneous existence of a fundamental and its overtones in the atmosphere of a variable of which the ratios cannot as a matter of principle be represented by simple integer numbers.

For the present I would like to repeat the tentative conclusions derived as the result of an analysis of the light curve R Scuti. These conclusions were :

a) The atmospheres of the RV Tauri stars are affected by two periodicities which are in the ratio 2 : 1.

b) The shorter period, which corresponds to the essential period and to the period in the radial velocity curve, is the result of an harmonic term or the sum of harmonic terms in the original light curve

c) The longer period which is exactly twice as long as the previous one and which corresponds to the classical period may not be a period in a strict sence. It may result from a series of autoregressive variations in the original light curve. With AC Herculis this latter point is far less evident than with R Scuti.

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