THE DISTRIBUTION OF THE TANGENTIAL VELOCITIES WITH P II STARS

(With 3 tables)

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Abstract.

In this paper for P II stars the distribution of the velocities V_θ perpendicular to the radius vector from the galactic center to the sun is studied. The distribution curves V_θ were derived from the space velocity vectors of the stars, as far as these space velocity vectors are known. For stars in the vicinity of the sun, that is for all stars considered here, the space velocity component $V\eta$ can be taken to be identical with V_θ. The velocity criterium was applied in order to discriminate between P I and P II objects. The results seem to indicate that among the early spectral types only a few P II objects occur.

The frequencies of the P II objects increase both towards the later spectral types and towards lower luminosities. There is however little if any variation of the curve $V_\theta\eta$ among the different groups of P II stars. The final $V_{\theta_1}\eta$ curve is asymmetrical and has a poorly defined flat maximum around $V\eta$ = — 70 km/sec.

Özet

Bu kısımda güneş cıvarında bulunan Pop II yıldızları için yarıçap vektöre dik olan V_θ hızlarının dağılımı incelenecektir. V_θ hız eğrileri uzay hızı bilinen yıldızların uzaydaki vektörel bileşenlerinden elde edilmişlerdir. Güneş cıvarındaki yıldızlar için $V_\theta = V_r$ dır. Yıldızları P_I ve P_{II} gibi iki guruba ayırmak için total hız kullanılır. Neticeler genç yıldızlarda P_{II} tipine pek nadir rastlandığını göstermektedir.

$P_I{}^I$ tipine ait yıldızların frekansları spektral tipler ilerledikçe ve M mutlak parlaklık arttıkça büyür. Eğrinin şeklinin çeşitli spektral tip ve parlaklıklarla değişmediği görülmektedir Ortamala $V\theta, r$ eğrisi simetrik değildir ve tek yıldızların geniş dispersiyonunu gösterir ve bu son eğri $V r$ = — 70 km- sn cıvarında bir max'u haizdir.

§ 1. *Introduction.*

In stellar systems P II objects in the vicinity of the sun have velocities perpendicular to the radius vector equal to:

$$V\theta = \frac{2A}{S} = (1+e)^{1/2}\, k^{1/2} \quad Vs,c \quad \ldots\ldots(1.1)$$

where e = eccentricity of the instantaneous elliptical orbit; k is a constant defined in such a way that for the instantaneous orbit the distance of perigalacticum is equal to k S = (1–e) a; a is the semiaxis major of the instantaneous orbit; S is the radius vector from the centre of the galactic system to the sun (S=8.2 kpc) while finally Vs,c = the uniform velocity in a circular orbit of radius S. For a detailed discussion of the relation (1) the reader is referred to a previous article [1].

We adopt a system of rectangular coördinates ξ, η and ζ where the directions of ξ, η and ζ are defined in such a way that the positive η axis points towards the point on the sphere with galactic coördinates $l^{II} = 90°$; b = 0°, while the positive ζ axis is in the direction of the North galactic pole.

In the article mentioned before [1] for P II stars from a study of the radial velocities of stars situated on or at least near to the η axis, some partial information was obtained concerning the distribution of the velocities V_θ. Always taking the radial velocity to be positive in the direction of the positive η *axis* the radial velocity is almost identical to V_η the contribution of V_θ to the velocity η is:

$$V_{\theta,\eta} = (1+e)^{1/2}\, k^{1/2}\, V_{s,c} \cos i \quad \ldots\ldots\ldots \; (2.1)$$

where i is the the inclination between the instantaneous orbit and the galactic plane.

On the whole our results seemed to indicate that the orbital motion generally is direct so that $-90° > i > +90°$.

For stars situated exactly on the η axis the relation between the observed radial velocity $V\eta$ and $V\theta,\eta$ is given by:

$$V_R = V_{\theta,\eta} - V_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

where V_s is the component along the η axis of the velocity of the sun with respect to the galactic center.

As the velocity of the sun is almost exclusively directed along the η axis for stars at small angular distance D from the η axis the relation between V_R and $V_{\theta,\eta}$ is:

$$V_R = V_{\theta,\eta} - V_s \cos D \ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

Especially from this latter relation it is obvious that when using radial velocities, only stars with small values D can be considered. For this there are two reasons :

a) When D is large, the radial direction will not always coincide with tangential direction.

b) The numerical value of V_s is known with doubtful accuracy only.

Therefore the numerical values of the corrections $V_s \cos D$ also are uncertain. When only small values D are used $V_s \cos D \rightleftarrows V_s$ and all individual velocities are affected by approximately the same error. This may cause a shift in the zero point of the curve $V_{\theta,\eta}$ but the shape of the curve does not change in any way.

When only stars with small values D are considered, the unavoidable consequence is that only small numbers of stars are available.

As a consequence the actual shape of the curve $V\theta,\eta$ still remains uncertain.

For this reason in the present article it is attempted to obtain additional information by considering the velocity vectors of the stars.

§ 2. *The space velocity vectors.*

Recently *Eggen* [2] published a Catalogue which for 3483 stars give the components of the space velocity vector $V\xi$, $V\eta$ and $V\varsigma$. As all stars are in the vicinity of the sun, the component $V\eta$ can be taken to be identical to $V_{\theta,\eta} - V_s$ and so:

$$(1+e)^{1/2}\ k^{1/2}\ V_{s,c} \cos i = V\eta + V_s \quad \ldots . \quad (1.2)$$

where V_s is the component of the solar velocity relative to the galactic center, parallel to the η axis.

Therefore a study of the distribution of the values $V\eta$ will give information on the shape of the distribution curve $V_{\theta,\eta}$. Due to the remaining undertainties in the determination of the numerical value of V_s the zero point of the curve will remain somewhat indeterminate. It might seem that by using the total tangential velocities

$V_\theta = \sqrt{V_\xi^2 + (V_\eta + V_s)^2}$ it is possible to eliminate from (1.2) the influence of the term cos i. This certainly would be an advantage, but the result of this procedure would be that the uncertainties affecting the value V_s would now affect all values V_θ so that the gains would be entirely spurious.

When using the "observed" values $V\eta$ it is to be observed that apart from the radial velocities these depend on the proper motions. These proper motions were transformed into space velocities by the application of suitable luminosity criteria. In this respect Eggen (l.c.) states: "The assumed luminosities are much too heterogeneous to justify any detailed discussion of the distributions of all space motions in the table." So when in the subsequent sections of this paper from the data of Eggen the different curves $V_{\theta,\eta}$ are derived, it must be realised that these curves may strongly be affected by magnitude errors and have to be considered as representing first solutions only. In addition to this it must be remembered that the samples of stars contained in most catalogues are far from unbiassed. Actually the only justification for the present attempt to find the shape of the curve $V_{\theta,\eta}$ is that up till now our informations about this shape are next to nihil.

§ 3. *The star counts.*

The results of the star counts appear in table 1. For the different groups of stars the table gives the number of stars with $V\eta$ between definite limits. But for a constant the curves in the table correspond to the distribution curve $V_{\theta,\eta}$. With the B and A stars no attempt was made to subdivide these stars into different luminosity groups. So the numbers in the columns B and A indicate the distribution of all P I and P II stars respectively with spectral type B or A.

For discriminating between the P I and P II stars the velocity criterium was applied. All stars with a velocity relative to the sun

$V = \sqrt{V_\xi^2 + V_\eta^2 + V_\zeta^2} \geq |65|$ km/sec were taken to be P II stars.

The obvious advantage of using the space velocity vectors is that it is possible also to identify the possible P II objects in the velocity interval $V\eta = +65$ to -65 km/sec. When only using radial velocities in this interval it is not possible to discriminate between the P I and P II objects.

Table 1 :

Distribution of the velocities $V\eta$ for different groups of P_I and P_{II} stars. All stars with a velocity relative to the sun $|V| \geq 65$ Km/sec are considered to belong to P_{II}. The numbers in brackets are P_{II} stars. The roman numerals above columns indicate luminosity classes.

Type	B	A	F				G				
$V\eta$			I+II	III	IV	V+d	I+g	II	III	IV	V
Km-sec											
+50											
	0(0)	0(0)	0(0)	0(0)	0(0)	2(2)	0(0)	0(0)	0(0)	0(0)	0(1)
+40											
	0(0)	0(0)	0(0)	0(0)	0(0)	3(1)	1(0)	0(0)	1(0)	0(0)	1(0)
+30											
	0(0)	2(0)	0(0)	0(0)	2(0)	7(0)	4(0)	0(0)	3(0)	0(0)	6(0)
+20											
	2(0)	14(0)	0(0)	0(0)	3(0)	17(1)	20(0)	0(0)	6(0)	2(1)	5(1)
+10											
	5(0)	99(0)	6(0)	3(0)	14(0)	55(4)	49(0)	5(0)	34(1)	6(1)	24(2)
0											
	33(0)	163(0)	6(0)	4(0)	24(1)	99(10)	64(1)	3(0)	32(1)	10(1)	36(7)
—10											
	26(0)	178(1)	15(0)	3(0)	20(0)	165(4)	57(1)	2(0)	25(3)	13(2)	44(8)
—20											
	24(0)	85(0)	7(1)	2(0)	11(0)	82(5)	34(2)	1(0)	10(1)	8(2)	55(7)
—30											
	1(0)	13(1)	2(0)	0(0)	2(2)	24(6)	16(1)	0(0)	8(0)	4(2)	32(7)
—40											
	1(0)	1(1)	1(0)	0(0)	1(0)	15(8)	11(3)	0(0)	1(0)	9(2)	31(9)
—50											
	0(0)	0(0)	0(0)	0(0)	0(0)	3(2)	6(3)	0(0)	1(4)	0(2)	12(11)
—60											
	0(0)	0(0)	0(0)	0(0)	0(0)	0(1)	0(4)	0(0)	0(0)	0(6)	0(17)
—70											
	0(0)	0(0)	0(0)	0(0)	0(0)	0(1)	0(1)	0(0)	0(1)	0(2)	0(14)
—80											
	0(0)	0(0)	0(0)	0(0)	0(0)	0(1)	0(1)	0(0)	0(0)	0(0)	0(8)
—90											
	0(0)	0(1)	0(0)	0(0)	0(0)	0(3)	0(0)	0(0)	0(0)	0(1)	0(10)
<—100											
	0(0)	0(3)	0(1)	0(0)	0(1)	0(9)	0(1)	0(0)	0(6)	0(4)	0(13)

Not all P II objects singled out in this way will actually belong to P II. Remembering the remark by Eggen, quoted in the preceding section, it is apparent that some large velocities may be spurious and due to the application of erroneous luminosity criteria. For the present there is no possibility entirely to avoid this. However, from the following it would seem that the different distribution curves were not greatly affected by possible magnitude errors.

With the F, G, K and M stars a subdivision into the luminosity classes I., II – V was applied. Stars classified as gF, gK, gG, and gM have been included in the luminosity classes I. On the other hand the stars classified as dF, dG, dK and dM are included in the numbers FV, GV, KV, and MV. Obviously this procedure is not unobjectionable. Neither can all g stars be identified with luminosity class I nor all d stars with the luminosity class V. But in order to have numbers of stars sufficiently large to be statistically acceptable, for the present these rough identifications had to be accepted.

§ 4. *Relative frequencies of P II stars.*

The results given in table 2 are based on the data collected in table I. For each group of stars separately from the different columns in table 1, we derive:

a) The total number of P I stars within each group;

b) The total number of P II stars within the same group;

c) The number of P II stars expressed in percentages of the total numbers. From these percentages as given in the final column of table 2 some interesting conclusions may be drawn.

1) Among the stars of spectral type B no P II objects appear to be present. Among the stars of spectral type A also P II objects are rare. Concerning the few P II objects of type A which occur, it is difficult to state whether or not they can be considered as main sequence stars.

2) Fairly large numbers of P II objects show up for the first time among stars of spectral type F. However, there is a strong indication that this increase is more pronounced among the faint F stars than among the highly luminous ones.

3) These same trends show up even more strongly when proceeding to the G, F, K and M stars. The P II objects reach their maximum relative frequency among the M stars especially among the faint

ones. It is however, to be noticed that the total numbers of M stars are comparatively small so that the actual numerical values of the percentages remain open to considerable doubt. Also there may be effects due to the selection of the observational material.

4) The main trends however can hardly be doubled. The increase of the percentages of P II objects towards the later spectral types seems now to be well established, especially among the fainter stars, while the same effect exists, but to a lesser extent, among the giants.

5) In the present paper the percentages of P II stars among all groups of stars appear to be larger than the percentages which were derived from a consideration of the radial velocities only. This was to be expected. In the present counts also the P II objects are included with $V\eta$ in the interval $-65 \leqq V\eta \leq +65$ Km/sec while these latter escape detection when only radial velocities are taken into account.

6) The trend of the variations stated under 4) is identical to the trend obtained from a consideration of the radial velocities. As in the two cases different observational materials were used, the reality of these variations seems to be ensured.

7) It seems worth while to consider within each spectral type separately the surface distribution of the P I and the P II objects.

When the separation into P I and P II objects has correctly been made, different surface distributions should show up. Whether or not such is the case, can only appear from further investigations.

§ 5. *The frequency curves* $V\eta$ *for P II stars.*

With the P II stars the individual velocities $V\eta$ are distributed over a large range (from-320 to + 60 km/sec.).

So in order to have sufficient numbers in each separate interval of 20 Km width, for different groups of stars the data had to be collected into one set. Table 3 gives a survey of the results. The first part of the table gives the frequency curve V_{θ},η both in numbers and in percentages for the highly luminous stars (e.g. those classified as g I or II of all spectral types together). In the same way the second and third part of the table gives the distribution curves V_{θ},η for the stars of intermediate and weak luminosity respectively. Finally the last columns of the table give the distribution curves $V_{\theta,\eta}$ for all stars together, suspected to be P II objects. It is of course unfortunate that as yet it is not possible to consider the individual spectral types separately. However, the different sets of percentages given in table 3 suggest that the disadvantage is not too great. Although this result must be considered to be a provisional one, these percentages seem to indicate that the distribution curve $V_{\theta,\eta}$ does not vary much from one group of stars to the other. The impression is that the distribution curves $V_{\theta,\eta}$ are very similar for all different groups of P II stars.

This frequency curve $V_{\theta,\eta}$ seems to be asymmetrical. It extends far down to negative velocities around – 320 km/sec and up to positive velocities of + 60 km/sec. maximum.

At the same time the rather flat maximum in the frequency curve seems to be situated around – 70 km/sec., but the maximum is badly defined. Relative to the P I stars, the solar velocity being near 20 Km/sec., the distribution curves confirm that the instantaneous orbits described by the P I stars are such as could also be described by some P II objects. The distinguishing characteristic of the P I stars therefore seems to be that so many objects move along closely similar orbits. This also is a conclusion previously derived from the consideration of the radial velocities only.

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Literature:

[1] Kreiken, E.A.; Annales d'Astrophysique 1964, 27,6

[2] Eggen O; Royal Obser. Bulletins 51 1962.

Table 2:

Frequencies of PI and PII Stars among different types

Type	N(PI)	N(PII)	% PII
B	92	0	0.0
A	555	7	1.2
F+g	36	2	5.3
II	1	0	—
III	12	0	0.0
IV	77	4	4.9
V+d	490	54	9.9
G I+g	262	18	6.4
II	11	0	0.0
III	121	17	12.3
IV	52	26	33.3
V	246	115	31.9
K I+g	353	47	11.8
II	19	5	20.8
III	288	81	22.0
IV	31	20	29.2
V+d	96	51	34.7
M I+g	158	33	17.3
II	3	0	—
III	14	22	61.1
IV	0	0	—
V+d	10	8	44.4

Table 3:

Distribution of velocities $V\eta$ among stars of different brightness. Indifferent from spectral type all highly luminous stars classified as either g or I are compiled into one group. This same procedure was followed with the stars of luminosity classes II, III etc.

Typel / $V\eta$ Km/sec	g+I/II N($V\eta$)	%$V\eta$.	III+IV N($V\eta$)	%$V\eta$	V+d N($V\eta$)	%$V\eta$	all PII N($V\eta$)	%$V\eta$
+ 60								
	0	0.0	1	0.6	3	1.3	4	0.8
+ 40								
	1	1.0	0	0.0	1	0.4	2	0.4
+ 20								
	7	6.7	13	7.7	13	5.6	33	6.5
0								
	20	19.0	26	15.5	38	16.4	84	16.6
— 20								
	14	13.3	19	11.3	42	18.1	75	14.9
— 40								
	21	20.0	38	22.6	40	17.2	99	9.6
— 60								
	20	19.0	39	23.2	45	19.4	104	20.6
— 80								
	9	8.6	15	8.9	25	10.8	49	9.7
— 100								
≦— 100	13	12.4	17	10.2	25	10.8	55	10.9
Σ	105	100	168	100	232	100	505	100