

COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME 16 A

ANNÉE 1967

A characteristic property of a developable surface

by

GIACOMO SABAN

9

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

Comité de Rédaction de la Série A

S. Süray B. Tanyel A. N. Eskiöglü

Secrétaire de publication

B. C. Ünal

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifiques représentées à la Faculté: Mathématiques pures et appliquées, Astronomie, Physique et Chimie théoriques, expérimentales et techniques, Géologie, Botanique et Zoologie.

La Revue, à l'exception des tomes I, II, III, comprend trois séries

Série A: Mathématiques, Physique et Astronomie.

Série B: Chimie.

Série C: Sciences naturelles.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adres: Fen Fakültesi Tebliğler Dergisi, Fen Fakültesi, Ankara, Turquie.

A characteristic property of a developable surface*

GIACOMO SABAN

Department of Mathematics, Faculty of Science, University of Istanbul

(Received October 27, 1967)

The pitch of a closed developable surface is equal to the arclength of its edge of regression. The aim of this paper is to show that this is characteristic property of a closed developable, viz. that if the pitch of a closed ruled surface is equal to the arclength of its line of striction then the surface is developable.

1. The *Pitch* [1] of a *closed* [2] ruled surface R (*i. e.* a ruled surface whose directrix curve D and spherical indicatrix of the generators J are both simply connected closed curves) is defined as the line integral

$$(1.1) \quad P = \int_D \vec{g} \cdot d\vec{\delta}$$

where $\vec{\delta}$ is the vector describing D and \vec{g} is the unit vector on the generator of the surface. P is an integral invariant of R [3]

If \mathcal{L} denotes the line of striction of R , described by the vector function \vec{l} and if L is the length of \mathcal{L} (1.1) can be rewritten in the form

$$(1.2) \quad P = \int_{\mathcal{L}} \vec{g} \cdot d\vec{l}$$

and P satisfies the limitation (1)

$$(1.3) \quad P \leq L.$$

* This Note contains the answer to a question put to the Author during his stay at the Middle East Technical University in Ankara in June 1967.

If R is a closed developable, its line of striction reduces to the edge of regression and if λ is to indicate the arc-length of \mathcal{L}

$$\vec{g} = \frac{\vec{dl}}{d\lambda}$$

then

$$(1.4) \quad P = \int_{\mathcal{L}} \frac{\vec{dl}}{d\lambda} \cdot d\mathbf{i} = \int_{\mathcal{L}} \frac{(\vec{dl})^2}{d\lambda} = \int_{\mathcal{L}} d\lambda = L.$$

The object of this short note is to show that the converse of the statement implied by (1.4), viz. *the pitch of a closed developable equals the length of its edge of regression*, is equally true and that therefore the property mentioned above characterizes developables.

2. Suppose that the pitch of a closed ruled surface R is equal to the length of its line of striction.

$\vec{l} = \vec{l}(\lambda)$ being the vector function which describes the generic point Λ of \mathcal{L}

$$(2.1) \quad \int_{\mathcal{L}} \vec{g} \cdot \frac{\vec{dl}}{d\lambda} d\lambda = L$$

or

$$(2.2) \quad \int_{\mathcal{L}} \vec{g} \cdot \frac{\vec{dl}}{d\lambda} d\lambda = \int_{\mathcal{L}} \frac{\vec{dl}}{d\lambda} \cdot \frac{\vec{dl}}{d\lambda} d\lambda$$

or again

$$(2.3) \quad \int_{\mathcal{L}} \left(\vec{g} - \frac{\vec{dl}}{d\lambda} \right) \cdot \frac{\vec{dl}}{d\lambda} d\lambda = 0.$$

But $\frac{\vec{dl}}{d\lambda}$ is a unit vector in the tangent plane of R at the point Λ : if \vec{t} denotes the unit vector of the tangent to R

orthogonal to the generator

$$(2.4) \quad \frac{\vec{dl}}{d\lambda} = \vec{g} \cos \varphi + \vec{t} \sin \varphi$$

where $\varphi = \varphi(\lambda)$ is a periodic function of λ of period L .

Then

$$(2.5) \quad \left(\vec{g} - \frac{\vec{dl}}{d\lambda}\right) \cdot \frac{\vec{dl}}{d\lambda} d\lambda = (\cos \varphi - 1) d\lambda$$

so that the condition (2.3) reduces to

$$(2.6) \quad \int_{\mathcal{L}} (\cos \varphi - 1) d\lambda = 0.$$

3. Whatever the nature of the function $\varphi = \varphi(\lambda)$, $|\cos \varphi| \leq 1$ hence

$$(3.1) \quad -2 \leq \cos \varphi - 1 \leq 0$$

whereas λ is a monotonic increasing point function of the point Λ on \mathcal{L} . Therefore the integrand in the condition (2.6) does not change sign along \mathcal{L} : for the condition (2.6) to hold the integrand must therefore be identically nil along \mathcal{L} . This implies

$$(3.2) \quad \cos \varphi = 1$$

along \mathcal{L} or else

$$(3.3) \quad d\lambda = 0.$$

The first case obviously leads to R being developable whereas the second implies that \mathcal{L} reduces to a point and therefore that the surface R reduces to a conc, as a particular case of a developable.

Thus

$$(3.4) \quad \int_{\mathcal{L}} \left(\vec{g} \cdot \frac{\vec{dl}}{d\lambda} - 1\right) d\lambda = 0$$

characterizes closed developables.

REFERENCES

- [1] G. Saban, *Su alcune famiglie di geodetiche*, "Rend. di Mat. e delle sue Appl.", Serie 5a, Vol. XIII, pp. 115 - 124, (1954). Roma.
- [2] G. Saban, *Alcune limitazioni integrali nella teoria metrica delle rigate*, "Rend. Accad. Naz. Lincei", Cl. Sc. Fis. Mat. Nat., Serie 8a, Vol. V. pp. 24 - 30, (1948). Roma.
- [3] W. Blaschke, *Vorlesungen über Differentialgeometrie*, Bd. 1. p. 242. Julius Springer, (1930) Berlin.

ÖZET

Kapalı ve açılabilir bir regle yüzeyin adımı (the pitch) onun dayanak eğrisinin uzunluğuna eşittir. Bu yazının amacı, açılabilir yüzeyler için bu özelliğin karakteristik olduğunu yani kapalı bir regle yüzeyin adımı onun boğaz çizgisinin uzunluğuna eşit ise yüzeyin açılabilir olduğunu göstermekten ibarettir.

Prix de l'abonnement annuel 1967:

Turquie : 15 TL ; Etranger : 30 TL.

Prix de ce numéro : 5 TL (pour la vente en Turquie).

Prière de s'adresser pour l'abonnement à : Fen Fakültesi Dekanlığı

Ankara, Turquie.