COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME 15 A ANNEE 1966

# Invariant Amplitudes of SU(3)

by

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Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sicences
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# Invariant Amplitudes of SU(3)*†

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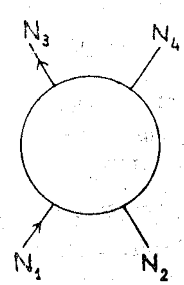
(Received 20 January 1966)

All the scattering amplitudes of SU (3) can be built up in terms of the infinitesimal operators of the representations of the group. By the recursion formula for three-particle reactions, C - G. coefficients belonging to higher dimensional representations are obtained from simple ones.

## I. INTRODUCTION

The aim of this work is to construct the amplitudes of SU(3) which are used in S-Matrix Theory. The construction is done by a similar way to that of isotopic spin amplitudes [1]. This systematic group theoretical method gives all the amplitudes for arbitrary isotopic spin and hypercharge values. The important point in this work is that the amplitudes of any group can be constructed in terms of the generators of the representations of the group. Indeed in the reference [1] the SU(2) amplitudes are in terms of the generators of the corresponding representations although this was not specified.

## II. SU(3) AMPLITUDES




* Supported in part by The Scientific and Technical Research Council of Turkey.
† A more detailed discussion is submitted as an M. S. thesis to The Faculty of Sciences of Ankara.

Let us consider, for simplicity, a four-particle reaction: $N_1 + N_2 \rightarrow N_3 + N_4$. Let $(I_1, I_{1z}, Y_1)$ and $(I_2, I_{2z}, Y_2)$ be the quantum numbers of two incoming particles (or outgoing anti-particles) which belong to the representations $N_1$ and $N_2$, and $(I_3, I_{3z}, Y_3)$ and $(I_4, I_{4z}, Y_4)$ be those of two outgoing particles (or incoming anti-particles) which belong to the representations $N_3$ and $N_4$, respectively. A scattering amplitude of SU(3) will be denoted by

$$R_{\nu_3\nu_4}{}^{\nu_1\nu_2} \tag{1}$$

where the indices are

$$\nu_i = (I_i, \; I_{iz}, \; Y_i)$$

According to the SU (3) invariance (the conservation of U-spin) and unitarity condition, the amplitudes (1) transform invariantly under the product of four unitarity representations D(p,q) = N of the group SU(3):

$$R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = N_{\nu_3}{}^{\alpha_3} N_{\nu_4}{}^{\alpha_4} N_1^{*\nu_1}{}_{\alpha_1} \; N_2^{*\nu_2}{}_{\alpha_2} \; R_{\alpha_3\alpha_4}{}^{\alpha_1\alpha_2} \tag{2}$$

Since we always consider the incoming and outgoing particles individually, we must distinguish them by some convention. The convention was given in (2). It says that the incoming particles or outgoing anti-particles transform according to D(p,q) = N* and the outgoing particles or incoming anti-particles according to $D(q,p) = N$.

The number of the linearly independent solutions of the transformation eq.(2) is equal to the number of the unit representations $D$ (0,0) = 1 included in the C–G. series of the product

$$N_3 \times N_4 \times N_1^\dagger \times N_2^\dagger$$

That is also the number of the possible U-spin channels. The coefficients in the most general solution of (2) are also called "scalar amplitudes" under the group SU(3).

There are two ways of finding the solutions. As it is known, one of these is the recursion formula (6) given in the reference [1]. This method was applied to SU(3) in the sec. V. The second is as follows: For the three-particle reactions $N^* + N \rightarrow 8$ and $N'^* + N' \rightarrow 8^*$ the corresponding transformation equations can be written respectively as :

$$N \times N^{*}\, G_{\alpha} = 8^{*\beta}{}_{\alpha}\, G_{\beta} \tag{3.a}$$

$$N' \times N'^{*}\, G'^{\alpha} = 8_{\beta}{}^{\alpha}\, G'^{\beta} \tag{3.b}$$

From these equations we obtain

$$N \times N^{*} \times N' \times N'^{*}\; G_{\alpha} \times G'^{\alpha} = G_{\beta} \times G'^{\beta} \tag{4}$$

Here, $G_{\alpha}$ and $G'_{\alpha}$ are generators of the $N$ and $N'$ dimensional representations of SU(3). We took by convention the $G\alpha$'s in the same order as the corresponding N-particle basis and $G'^{\alpha}$'s as the corresponding N′anti-particle basis. This is in agreement with de Swart's convention for the order of particles such as

$\Phi^{\alpha} = (p, n, \Sigma^{+}, \Sigma^{\circ}, \Sigma^{-}, \Lambda, \Xi^{\circ}, \Xi^{-})$ for the baryon octet and

$\Phi_{\alpha} = (\bar{p}, \bar{n}, \Sigma^{-}, \bar{\Sigma}^{\circ}, \Sigma^{+}, \bar{\Lambda}, \bar{\Xi}^{\circ}, \bar{\Xi}^{\circ})$ for the anti-baryon octet [2].

## III. THREE–PARTICLE REACTIONS

For three-particle reactions the following amplitudes are obtained by using the Clebsch-Gordan coefficients:

1) $3 + 3 \to 3^{*}$ : $$R^{\nu_3\nu_1\nu_2} = \varepsilon^{\nu_3\nu_1\nu_2} \tag{5}$$

2) $3^{*} + 3^{*} \to 3$ : $$R_{\nu_3\nu_1\nu_2} = \varepsilon_{\nu_3\nu_1\nu_2} \tag{6}$$

3) $3^{*} + 3 \to 8$ : $$R_{\alpha\nu_1}{}^{\nu_2} = H_{\alpha\nu_1}{}^{\nu_2} \tag{7.a}$$

Where $\varepsilon$'s are the totally anti-symmetrical tensors and $H_{\alpha}$'s are the infinitesimal operators of the fundamental representation D(1,0) = 3 in the following order:

$$H_{\alpha} = (-K_{-}, L_{-}, -I_{-}, \sqrt{2}\, I_{z}, I_{+}, \sqrt{2}\, M, L_{+}, K_{+}) \tag{7.b}$$

4) $3^{*} + 3 \to 8^{*}$ : $$R^{\alpha}{}_{\nu_1}{}^{\nu_2} = H^{\alpha}{}_{\nu_1}{}^{\nu_2} \tag{8.a}$$

Here, $H\alpha$ 's are $H\alpha = H^T_\alpha$, *i. e.*

$$H\alpha = (-K_+, L_+, -I_+, \sqrt{2}\, I_z, I_-, \sqrt{2}\, M, L_-, K_-) \qquad (8.b)$$

$$5)\ 6^* + 6 \rightarrow 8^* : \quad R^{\alpha\ \nu_2}_{\ \nu_1} = \mathcal{U}^{\alpha\ \nu_2}_{\ \nu_1} \qquad (9.a)$$

$\mathcal{U}^\alpha$'s are the generators of the representation D (2,0) = 6 in the same order as (7. b) :

$$\mathcal{U}^\alpha = (-\mathcal{K}_-, \mathcal{L}_-, -\mathcal{J}_-, \sqrt{2}\, \mathcal{J}_z, \mathcal{J}_+, \sqrt{2}\, \mathcal{M}, \mathcal{L}_+, \mathcal{K}_+) \qquad (9.b)$$

$$6)\ 6^* + 6 \rightarrow 8 : \quad R_{\alpha\nu_1}^{\ \ \nu_2} = \mathcal{U}_{\alpha\nu_1}^{\ \ \nu_2} \qquad (10)$$

Here, the order of $\mathcal{U}_\alpha$ 's is like (8.b).

$$7)\ 8^* + 8 \rightarrow 8^* : \quad R^{\alpha\ \nu_2}_{\ \nu_1} = A\, \mathcal{H}^{\alpha\ \nu_2}_{\ \nu_1} + B\, \Omega^{\alpha\ \nu_2}_{\ \nu_1} \qquad (11)$$

$\mathcal{H}^\alpha$'s are the generators of the representation $D(1,1) = 8$ in the same order as (7. b) and (9. b). Those are constructed by the C-G. table $(8^*\, 8\, 8^*_a)^{\nu}{}_{\nu_1}{}^{\nu_2}$. $\Omega^\alpha$'s are constructed by the table $(8^* 8\ 8^*_s)^{\nu}{}_{\nu_1}{}^{\nu_2}$.

$$8)\ 8^* + 8 \rightarrow 8 : \quad R_{\alpha\nu_1}^{\ \ \nu_2} = A\, \mathcal{H}_{\alpha\nu_1}^{\ \ \nu_2} + B\ \Omega_{\alpha\nu_1}^{\ \ \nu_2} \qquad (12)$$

Here, as one expects, we have $\mathcal{H}_\alpha = \mathcal{H}^{\alpha\, T}$ and $\Omega_\alpha = \Omega^{\alpha\, T}$.

$$9)\ 10^* + 10 \rightarrow 10^* : \quad R^{\alpha\ \nu_2}_{\ \nu_1} = G^{\alpha\ \nu_2}_{\ \nu_1} \qquad (13)$$

$G^\alpha$'s are the generators of the representation D (3,0) = 10 in the order (7.b) and (9. b).

$$10)\ 10^* + 10 \rightarrow 10 : \quad R_{\alpha\nu_1}^{\ \ \nu_2} = G_{\alpha\nu_1}^{\ \ \nu_2} \qquad (14)$$

## IV. FOUR – PARTICLE REACTIONS

The amplitudes of four-particle reactions are obtained as follows:

1) $3 + 3 \to 3 + 3$: There must be two linearly independent amplitudes. According to Schur's Lemma and eq. (4) we find four amplitudes:

$$\delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} \ , \ \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} \ , \ H_{\alpha\nu_3}^{\ \ \nu_1} H^{\alpha}_{\ \nu_4}{}^{\nu_2} \ , \ H_{\alpha\nu_3}^{\ \ \nu_2} H^{\alpha}_{\ \nu_4}{}^{\nu_1}$$

But two of them are linearly dependent :

$$H_{\alpha\nu_3}^{\ \ \nu_1} H^{\alpha}_{\ \nu_4}{}^{\nu_2} = \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} - \frac{1}{3} \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} \tag{15.a}$$

$$H_{\alpha\nu_3}^{\ \ \nu_2} H^{\alpha}_{\ \nu_4}{}^{\nu_1} = \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} - \frac{1}{3} \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} \tag{15.b}$$

So we obtain the exact amplitude in the following form :

$$R_{\nu_3\nu_4}^{\ \ \nu_1\nu_2} = A\, \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} + B\, \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} \tag{16}$$

2) $3^* + 3^* \to 3^* + 3^*$ : Similarly we get the following amplitude:

$$R^{\nu_3\nu_4}_{\ \ \nu_1\nu_2} = A\, \delta^{\nu_3}_{\ \nu_1} \delta^{\nu_4}_{\ \nu_2} + B\, \delta^{\nu_3}_{\ \nu_2} \delta^{\nu_4}_{\ \nu_1} \tag{17}$$

3) $3 + 6 \to 3 + 6$ :

$$R_{\nu_3\nu_4}^{\ \ \nu_1\nu_2} = A\, \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} + B\, H_{\alpha\nu_3}^{\ \ \nu_1} \mathcal{U}^{\alpha}_{\ \nu_4}{}^{\nu_2} \tag{18}$$

4) $6 + 6 \to 6 + 6$ :

$$R_{\nu_3\nu_4}^{\ \ \nu_1\nu_2} = A\, \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} + B\, \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} + C\, \mathcal{U}_{\alpha\nu_3}^{\ \ \nu_1} \mathcal{U}^{\alpha}_{\ \nu_4}{}^{\nu_2} \tag{19.a}$$

The possible fourth amplitude for the last reaction is linearly dependent:

$$\mathcal{U}_{\alpha\nu_3}^{\ \ \nu_2} \mathcal{U}^{\alpha}_{\ \nu_4}{}^{\nu_1} = \mathcal{U}_{\alpha\nu_3}^{\ \ \nu_1} \mathcal{U}^{\alpha}_{\ \nu_4}{}^{\nu_2} + \frac{4}{3} \delta_{\nu_3}^{\ \nu_2} \delta_{\nu_4}^{\ \nu_1} \tag{19.b}$$

5) $3 + 8 \to 3 + 8$ :

$$R_{\nu_3\nu_4}^{\ \ \nu_1\nu_2} = A\, \delta_{\nu_3}^{\ \nu_1} \delta_{\nu_4}^{\ \nu_2} + B\, H_{\alpha\nu_3}^{\ \ \nu_1} \mathcal{H}^{\alpha}_{\ \nu_4}{}^{\nu_2} + C H_{\alpha\nu_3}^{\ \ \nu_1} \Omega^{\alpha}_{\ \nu_4}{}^{\nu_2} \tag{20}$$

6) $8 + 8 \to 8 + 8$ : We can find ten amplitudes according to Schur's Lemma and eq. (4); but two of these must be linearly depen-

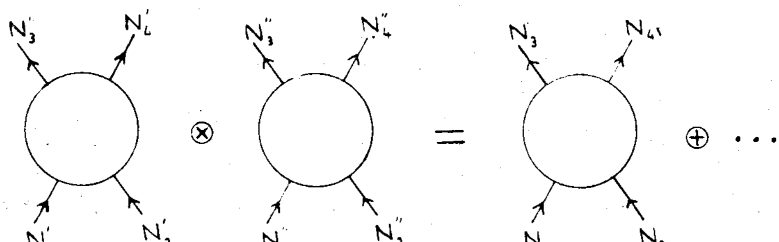
dent. Here, we shall write the resulting amplitude with ten parts:

$$
\begin{aligned}
R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = {} & A\,\delta_{\nu_3}{}^{\nu_1}\,\delta_{\nu_4}{}^{\nu_2} + B\,\delta_{\nu_3}{}^{\nu_2}\,\delta_{\nu_4}{}^{\nu_1} + C\,\mathcal{H}_{\alpha\nu_3}{}^{\nu_1}\mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_2} \\
& + D\,\mathcal{H}_{\alpha\nu_3}{}^{\nu_2}\,\mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_1} + \mathrm{E}\,\Omega_{\alpha\nu_3}{}^{\nu_1}\,\Omega^{\alpha}{}_{\nu_4}{}^{\nu_2} + F\,\Omega_{\alpha\nu_3}{}^{\nu_2}\,\Omega^{\alpha}{}_{\nu_4}{}^{\nu_1} \\
& + G\,\mathcal{H}_{\alpha\nu_3}{}^{\nu_1}\,\Omega^{\alpha}{}_{\nu_4}{}^{\nu_2} + I\,\mathcal{H}_{\alpha\nu_3}{}^{\nu_2}\,\Omega^{\alpha}{}_{\nu_4}{}^{\nu_1} + J\,\Omega_{\alpha\nu_3}{}^{\nu_1}\mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_2} \\
& + \mathrm{K}\;\Omega_{\alpha\nu_3}{}^{\nu_2}\,\mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_1}
\end{aligned}
\tag{21}
$$

7) $10 + 10 \rightarrow 10 + 10$ :

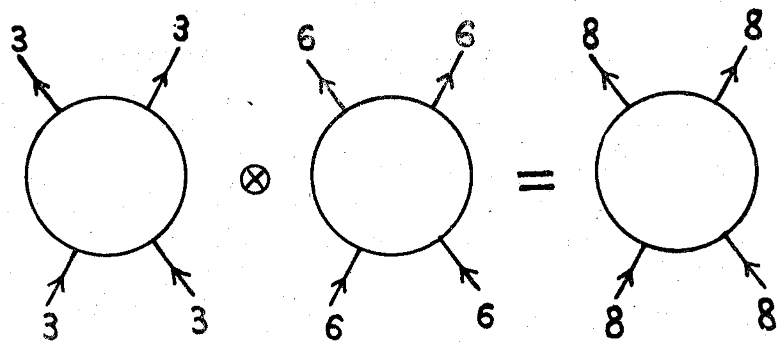
$$
\begin{aligned}
R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = {} & \mathrm{A}\,\delta_{\nu_3}{}^{\nu_1}\,\delta_{\nu_4}{}^{\nu_2} + B\,\delta_{\nu_3}{}^{\nu_2}\,\delta_{\nu_4}{}^{\nu_1} + C\;\mathcal{G}_{\alpha\nu_3}{}^{\nu_1}\;\mathcal{G}^{\alpha}{}_{\nu_4}{}^{\nu_2} \\
& + D\;\mathcal{G}_{\alpha\nu_3}{}^{\nu_2}\;\mathcal{G}^{\alpha}{}_{\nu_4}{}^{\nu_1}
\end{aligned}
\tag{22}
$$

## V. THE CONSTRUCTION OF AMPLITUDES BY RECURSION FORMULA




$$
\begin{aligned}
R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = {} & (N'_3 N''_3 N_3)_{\nu_3}{}^{\alpha_3\beta_3}\,(N'_4 N''_4 N_4)_{\nu_4}{}^{\alpha_4\beta_4}\,\{N'_1\,N''_1 N_1\}^{\nu_1}{}_{\alpha_1\beta_1} \\
& \{N'_2\,\mathrm{N}''_2\,N_2\}^{\nu_2}{}_{\alpha_2\beta_2}\;\;R'_{\alpha_3\alpha_4}{}^{\alpha_1\alpha_2}\,R''_{\beta_3\beta_4}{}^{\beta_1\beta_2}
\end{aligned}
\tag{23}
$$

It can be shown that the eq. (23) satisfies the fundamental eq.(2). Here, $(\ \ )_{\nu}{}^{\alpha\beta}$ and $\{\ \ \}^{\nu}{}_{\alpha\beta}$ are the C–G. coefficients of SU(3). By that recursion formula all the amplitudes for arbitrary isotopic spin and hypercharge values can be built too. We have done it for the reactions (III.7), (IV. 3), (IV. 4), (IV.5) and (IV. 6), and we found the same results. That is also a check up. As an example, we shall make it explicitly for the reaction (IV. 6).




$$R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = (368)_{\nu_3}{}^{\alpha_3\beta_3}\,(368)_{\nu_4}{}^{\alpha_4\beta_4}\,\{368\}^{\nu_1}{}_{\alpha_1\beta_1}\,\{368\}^{\nu_2}{}_{\alpha_2\beta_2}\,M_{\alpha_3\alpha_4}{}^{\alpha_1\alpha_2}\,N_{\beta_3\beta_4}{}^{\beta_1\beta_2} \qquad (24)$$

In the eq. (24) we know the amplitudes $M$, $N$ and the C-G. coefficients. Using them we obtain the amplitude in this form:

$$\begin{aligned} R_{\nu_3\nu_4}{}^{\nu_1\nu_2} = {} & A\,\delta_{\nu_3}{}^{\nu_1}\,\delta_{\nu_4}{}^{\nu_2} + B\,\delta_{\nu_3}{}^{\nu_2}\,\delta_{\nu_4}{}^{\nu_1} \\ & + C\,(368)_{\nu_3}{}^{\nu_2\beta_1}\,\{368\}^{\nu_1}{}_{\alpha_1\beta_1}\,(368)_{\nu_4}{}^{\alpha_1\beta_2}\,\{368\}^{\nu_2}{}_{\alpha_2\beta_2} \\ & + D\,(368)_{\nu_3}{}^{\alpha_1\beta_2}\,\{368\}^{\nu_2}{}_{\alpha_2\beta_2}\,(368)_{\nu_4}{}^{\alpha_2\beta_1}\,\{368\}^{\nu_1}{}_{\alpha_1\beta_1} \\ & + E\,(368)_{\nu_3}{}^{\alpha_1\beta_3}\,\{368\}^{\nu_1}{}_{\alpha_1\beta_1}\,(368)_{\nu_4}{}^{\alpha_2\beta_4}\,\{368\}^{\nu_2}{}_{\alpha_2\beta_2} \\ & \qquad \mathcal{U}_{\alpha\beta_3}{}^{\beta_1}\,\mathcal{U}^{\alpha}{}_{\beta_4}{}^{\beta_2} \\ & + F\,(368)_{\nu_3}{}^{\alpha_2\beta_3}\,\{368\}^{\nu_2}{}_{\alpha_2\beta_2}\,(368)_{\nu_4}{}^{\alpha_1\beta_4}\,\{368\}^{\nu_1}{}_{\alpha_1\beta_1} \\ & \qquad \mathcal{U}_{\alpha\beta_3}{}^{\beta_1}\,\mathcal{U}^{\alpha}{}_{\beta_4}{}^{\beta_2} \qquad (25) \end{aligned}$$

Some properties of the C-G. coefficients were given in the Appendix. Using them and the relations

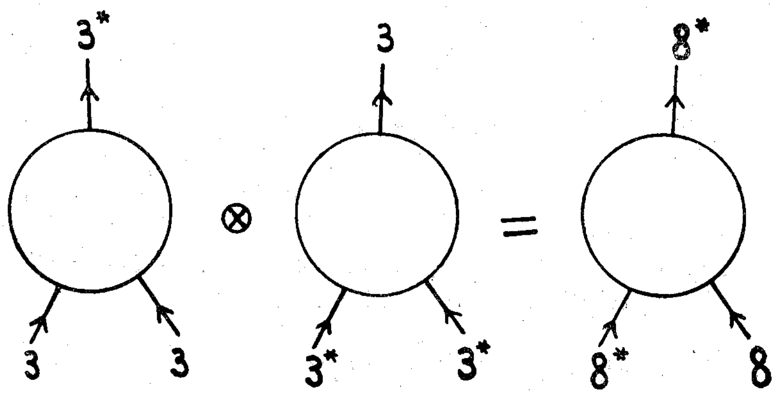
$$Tr\ H_{\alpha} \times H^{\beta} = \delta_{\alpha}^{\ \beta} \tag{26.a}$$

$$Tr\ \mathcal{U}_{\alpha} \times \mathcal{U}^{\beta} = 5\ \delta_{\alpha}^{\ \beta} \tag{26.b}$$

we get back the eq. (20). In addition one can get the same result starting by the following product :

$$(3 + 3 \rightarrow 3 + 3) \times (3^* + 3^* \rightarrow 3^* + 3^*) = (8 + 8 \rightarrow 8 + 8)$$

By the recursion formula for three-particle reactions, C-G. coefficients belonging to higher representations are also obtained from simple ones. Consider the following product as an example:




Its recursion formula is

$$R^{\alpha_3}{}_{\alpha_1}{}^{\alpha_2} = \{33^*\ 8^*\}^{\alpha_3}{}_{a}{}^{i}\ (33^*\ 8^*)_{\alpha_1}{}^{j}{}_{b}\ \{33^*8\}^{\alpha_2\ k}{}_{c}\ \varepsilon^{abc}\ \varepsilon_{ijk} \tag{27}$$

Using the relations

$$\varepsilon^{abc}\ \varepsilon_{ijk} = \delta^{abc}_{ijk} = \begin{vmatrix} \delta^a_i & \delta^b_i & \delta^c_i \\ \delta^a_j & \delta^b_j & \delta^c_j \\ \delta^a_k & \delta^b_k & \delta^c_k \end{vmatrix} \tag{28}$$

and

$$\sum_{i=1}^{3} (33^* \, 8)_{\nu}{}^{i}{}_{i} = 0, \quad \sum_{i=1}^{3} (33^* \, 8^*)_{\nu}{}^{i}{}_{i} = 0 \text{ (for all } \nu) \tag{29}$$

one gets $R^{\alpha}$ ($\alpha = 1,2,\ldots, 8$) matrices. The decomposition of $R^{\alpha}$'s into two parts is done by the complex conjugate operation which is not involved by SU (3):

$$R^{\alpha} = a \, \mathcal{H}^{\alpha} + b \, \Omega^{\alpha} \tag{30}$$

Thus we find $1/\sqrt{6} \; \mathcal{H}^{\alpha}$ and $\sqrt{30}/10 \; \Omega^{\alpha}$ matrices as the tables ($8^* \, 88^*_a$) and ($8^* \, 88^*_s$) respectively. This way may be useful for SU (4) and SU (6).

## Acknowledgment

I am indepted to A.O. Barut for suggesting this problem and would like to thank to B. C. Ünal for his continued interest while supervising this work.

### APPENDIX

We give some C-G. coefficients that are used in the paper.

1) The C-G. table $(33^* \, 8)_{\nu}{}^{\nu_1}{}_{\nu_2}$ :

$$\begin{aligned}
\Phi_1 &= -\psi_1 \psi^3 & \Phi_5 &= \psi_2 \psi^1 \\
\Phi_2 &= -\psi_2 \psi^3 & \Phi_6 &= \frac{1}{\sqrt{6}} (\psi_1 \psi^1 + \psi_2 \psi^2 - 2\psi_3 \psi^3) \\
\Phi_3 &= -\psi_1 \psi^2 & \Phi_7 &= -\psi_3 \psi^2 \\
\Phi_4 &= \frac{1}{\sqrt{2}} (\psi_1 \psi^1 - \psi_2 \psi^2) & \Phi_8 &= \psi_3 \psi^1
\end{aligned} \tag{A.1}$$

2) The C-G. table $(33^* \, 8^*)^{\nu\nu_1}{}_{\nu_2}$ :

$$\begin{aligned}
\Phi^1 &= -\psi_3 \psi^1 & \Phi^5 &= \psi_1 \psi^2 \\
\Phi^2 &= -\psi_3 \psi^2 & \Phi^6 &= \frac{1}{\sqrt{6}} (\psi_1 \psi^1 + \psi_2 \psi^2 - 2\psi_3 \psi^3)
\end{aligned} \tag{A.2}$$

$$\Phi^3 = -\psi_2 \psi^1 \qquad \Phi^7 = -\psi_2\psi^3$$

$$\Phi^4 = \frac{1}{\sqrt{2}}(\psi_1\psi^1 - \psi_2\psi^2) \qquad \Phi^8 = \psi_1\psi^3$$

3) The C-G. table $(333^*)^{\nu\nu_1\nu_2}$ :

$$\psi^1 = \frac{1}{\sqrt{2}}(\psi_2\psi_3 - \psi_3\psi_2)$$

$$\psi^2 = \frac{1}{\sqrt{2}}(\psi_1\psi_3 - \psi_3\psi_1) \tag{A.3}$$

$$\psi^3 = \frac{1}{\sqrt{2}}(\psi_1\psi_2 - \psi_2\psi_1)$$

4) The C–G. table $(336)_\nu^{\nu_1\nu_2}$ :

$$\varphi_1 = \psi_1\psi_1 \qquad \varphi_4 = \frac{1}{\sqrt{2}}(\psi_1\psi_3 + \psi_3\psi_1)$$

$$\varphi_2 = \frac{1}{\sqrt{2}}(\psi_1\psi_2 + \psi_2\psi_1) \qquad \varphi_5 = \frac{1}{\sqrt{2}}(\psi_2\psi_3 + \psi_3\psi_2) \tag{A.4}$$

$$\varphi_3 = \psi_2\psi_2 \qquad \varphi_6 = \psi_3\psi_3$$

5) The C–G. table $(368)_\nu^{\nu_1\nu_2}$ :

$$\Phi_1 = -\frac{1}{\sqrt{3}}\psi_1\varphi_2 + \frac{2}{\sqrt{6}}\psi_2\varphi_1 \qquad \Phi_5 = \frac{1}{\sqrt{3}}\psi_2\varphi_5 - \frac{2}{\sqrt{6}}\psi_3\varphi_3$$

$$\Phi_2 = -\frac{2}{\sqrt{6}}\psi_1\varphi_3 + \frac{1}{\sqrt{3}}\psi_2\varphi_2 \qquad \Phi_6 = -\frac{1}{\sqrt{2}}\psi_1\varphi_5 + \frac{1}{\sqrt{2}}\psi_2\varphi_4 \tag{A.5}$$

$$\Phi_3 = \frac{1}{\sqrt{3}}\psi_1\varphi_4 - \frac{2}{\sqrt{6}}\psi_3\varphi_1 \qquad \Phi_7 = \frac{2}{\sqrt{6}}\psi_1\varphi_6 - \frac{1}{\sqrt{3}}\psi_3\varphi_4$$

$$\Phi_4 = \frac{1}{\sqrt{6}}\psi_1\varphi_5 + \frac{1}{\sqrt{6}}\psi_2\varphi_4 - \frac{2}{\sqrt{6}}\psi_3\varphi_2 \qquad \Phi_8 = \frac{2}{\sqrt{6}}\psi_2\varphi_2 - \frac{1}{\sqrt{3}}\psi_3\varphi_6$$

6) The C-G. table $(6^*6\ 8^*)_\nu^{\nu_1\nu_2}$ :

$$\Phi^1 = \frac{1}{\sqrt{5}} \left( -\sqrt{2}\,\varphi^1 \varphi_4 - \varphi^2 \varphi_5 - \sqrt{2}\,\varphi^4 \varphi_6 \right)$$

$$\Phi^2 = \frac{1}{\sqrt{5}} \left( -\varphi^2 \varphi_4 - \sqrt{2}\,\varphi^3 \varphi_5 - \sqrt{2}\,\varphi^5 \varphi_6 \right)$$

$$\Phi^3 = \frac{1}{\sqrt{5}} \left( -\sqrt{2}\,\varphi^1 \varphi_2 - \sqrt{2}\,\varphi^2 \varphi_3 - \varphi^4 \varphi_5 \right)$$

$$\Phi^4 = \frac{1}{\sqrt{5}} \left( \sqrt{2}\,\varphi^1 \varphi_1 - \sqrt{2}\,\varphi^3 \varphi_3 + \frac{1}{\sqrt{2}} \varphi^4 \varphi_4 - \frac{1}{\sqrt{2}} \varphi^5 \varphi_5 \right) \qquad \text{(A.6)}$$

$$\Phi^5 = \frac{1}{\sqrt{5}} \left( \sqrt{2}\,\varphi^2 \varphi_1 + \sqrt{2}\,\varphi^3 \varphi_2 + \varphi^5 \varphi_4 \right)$$

$$\Phi^6 = \frac{1}{\sqrt{5}} \left( \frac{2}{\sqrt{6}} \varphi^1 \varphi_1 + \frac{2}{\sqrt{6}} \varphi^2 \varphi_2 + \frac{2}{\sqrt{6}} \varphi^3 \varphi_3 - \frac{1}{\sqrt{6}} \varphi^5 \varphi_5 - \frac{4}{\sqrt{6}} \varphi^6 \varphi_6 \right)$$

$$\Phi^7 = \frac{1}{\sqrt{5}} \left( -\varphi^4 \varphi_2 - \sqrt{2}\,\varphi^5 \varphi_3 - \sqrt{2}\,\varphi^6 \varphi_5 \right)$$

$$\Phi^8 = \frac{1}{\sqrt{5}} \left( \sqrt{2}\,\varphi^4 \varphi_1 + \varphi^5 \varphi_2 + \sqrt{2}\,\varphi^6 \varphi_4 \right)$$

7) The C-G. table $(8^* \ 88^*_a)^{\nu}{}_{\nu_1}{}^{\nu_2}$ :

$$\Phi^1 = \frac{1}{\sqrt{2}} \left( -\frac{1}{\sqrt{2}} \Phi^1 \Phi_4 - \frac{3}{\sqrt{6}} \Phi^1 \Phi_6 - \Phi^2 \Phi_5 - \Phi^3 \Phi_7 - \frac{1}{\sqrt{2}} \Phi^4 \Phi_8 - \frac{3}{\sqrt{6}} \Phi^6 \Phi_8 \right)$$

$$\Phi^2 = \frac{1}{\sqrt{6}} \left( \Phi^1 \Phi_3 + \frac{1}{\sqrt{2}} \Phi^2 \Phi_4 - \frac{3}{\sqrt{6}} \Phi^2 \Phi_6 - \frac{1}{\sqrt{2}} \Phi^4 \Phi_7 - \Phi^5 \Phi_8 + \frac{3}{\sqrt{6}} \Phi^6 \Phi_7 \right)$$

$$\Phi^3 = \frac{1}{\sqrt{6}} \left( -\Phi^1 \Phi_2 - \sqrt{2}\,\Phi^3 \Phi_4 - \sqrt{2}\,\Phi^4 \Phi_5 - \Phi^7 \Phi_8 \right)$$

$$\Phi^4 = \frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{2}} \Phi^1\Phi_1 - \frac{1}{\sqrt{2}} \Phi^2\Phi_2 + \sqrt{2}\ \Phi^3\Phi_3 - \sqrt{2}\ \Phi^5\Phi_5 + \frac{1}{\sqrt{2}} \Phi^7\Phi_7 - \frac{1}{\sqrt{2}} \Phi^8\Phi_8 \right) \qquad (A.7)$$

$$\Phi^5 = \frac{1}{\sqrt{6}} (\Phi^2\Phi_1 + \sqrt{2}\ \Phi^4\Phi_3 + \sqrt{2}\ \Phi^5\Phi_4 + \Phi^8\Phi_7)$$

$$\Phi^6 = \frac{1}{\sqrt{6}} \left( \frac{3}{\sqrt{6}} \Phi^1\Phi_1 + \frac{3}{\sqrt{6}} \Phi^2\Phi_2 - \frac{3}{\sqrt{6}} \Phi^7\Phi_7 - \frac{3}{\sqrt{6}} \Phi^8\Phi_8 \right)$$

$$\Phi^7 = \frac{1}{\sqrt{6}} \left( \Phi^3\Phi_1 + \frac{1}{\sqrt{2}} \Phi^4\Phi_2 - \frac{3}{\sqrt{6}} \Phi^6\Phi_2 - \frac{1}{\sqrt{2}} \Phi^7\Phi_4 + \frac{3}{\sqrt{6}} \cdot \Phi^7\Phi_6 - \Phi^8\Phi_5 \right)$$

$$\Phi^8 = \frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{2}} \Phi^4\Phi_1 + \Phi^5\Phi_2 + \frac{3}{\sqrt{6}} \Phi^6\Phi_1 + \Phi^7\Phi_3 + \frac{1}{\sqrt{2}} \Phi^8\Phi_4 + \frac{3}{\sqrt{6}} \Phi^8\Phi_6 \right)$$

8) The C-G. table $(8{*}88^*_s)^{\nu\ \nu_2}_{\nu_1}$ :

$$\Phi^1 = \frac{\sqrt{30}}{10} \left( \frac{1}{\sqrt{2}} \Phi^1\Phi_4 - \frac{1}{\sqrt{6}} \Phi^1\Phi_6 + \Phi^2\Phi_5 - \Phi^3\Phi_7 - \frac{1}{\sqrt{2}} \Phi^4\Phi_8 + \frac{1}{\sqrt{6}} \Phi^6\Phi_8 \right)$$

$$\Phi^2 = \frac{\sqrt{30}}{10} \left( -\Phi^1\Phi_3 - \frac{1}{\sqrt{2}} \Phi^2\Phi_4 - \frac{1}{\sqrt{6}} \Phi^2\Phi_1 - \frac{1}{\sqrt{2}} \Phi^4\Phi_7 - \Phi^5\Phi_8 - \frac{1}{\sqrt{6}} \Phi^6\Phi_7 \right)$$

$$\Phi^3 = \frac{\sqrt{30}}{10} \left( -\Phi^1\Phi_2 + \frac{2}{\sqrt{6}} \Phi^3\Phi_6 - \frac{2}{\sqrt{6}} \Phi^6\Phi_5 + \Phi^7\Phi_8 \right)$$

$$\Phi^4 = \frac{\sqrt{30}}{10} \left( \frac{1}{\sqrt{2}} \Phi^1\Phi_1 - \frac{1}{\sqrt{2}} \Phi^2\Phi_2 + \frac{2}{\sqrt{6}} \Phi^4\Phi_6 + \frac{2}{\sqrt{6}} \Phi^6\Phi_4 - \frac{1}{\sqrt{2}} \Phi^7\Phi_7 + \frac{1}{\sqrt{2}} \Phi^8\Phi_8 \right) \qquad (A.8)$$

$$\Phi^5 = \frac{\sqrt{30}}{10} \left( \Phi^2 \Phi_1 + \frac{2}{\sqrt{6}} \Phi^5 \Phi_6 - \frac{2}{\sqrt{6}} \Phi^6 \Phi_3 - \Phi^8 \Phi_7 \right)$$

$$\Phi^6 = \frac{\sqrt{30}}{10} \left( -\frac{1}{\sqrt{6}} \Phi^1 \Phi_1 - \frac{1}{\sqrt{6}} \Phi^2 \Phi_2 + \frac{2}{\sqrt{6}} \Phi^3 \Phi_3 + \frac{2}{\sqrt{6}} \Phi^4 \Phi_4 + \frac{2}{\sqrt{6}} \Phi^5 \Phi_5 - \frac{2}{\sqrt{6}} \Phi^6 \Phi_6 - \frac{1}{\sqrt{6}} \Phi^7 \Phi_7 - \frac{1}{\sqrt{6}} \Phi^8 \Phi_8 \right)$$

$$\Phi^7 = \frac{\sqrt{30}}{10} \left( -\Phi^3 \Phi_1 - \frac{1}{\sqrt{2}} \Phi^4 \Phi_2 - \frac{1}{\sqrt{6}} \Phi^6 \Phi_2 - \frac{1}{\sqrt{2}} \Phi^7 \Phi_4 - \frac{1}{\sqrt{6}} \Phi^7 \Phi_6 - \Phi^8 \Phi_5 \right)$$

$$\Phi^8 = \frac{\sqrt{30}}{10} \left( -\frac{1}{\sqrt{2}} \Phi^4 \Phi_1 - \Phi^5 \Phi_2 + \frac{1}{\sqrt{6}} \Phi^6 \Phi_1 + \Phi^7 \Phi_3 + \frac{1}{\sqrt{2}} \Phi^8 \Phi_4 - \frac{1}{\sqrt{6}} \Phi^8 \Phi_6 \right)$$

Some properties of the C-G. coefficients of SU (3) (say, semi-ortogonality relations) are given as follows :

$$(33^*8)_{\nu_4}{}^{\nu_1}{}_{\beta} \{33^*8\}^{\nu_2}{}_{\nu_3}{}^{\beta} = \frac{1}{3} \delta_{\nu_3}{}^{\nu_1} \delta_{\nu_4}{}^{\nu_2} + \frac{1}{2} H_{\alpha\nu_3}{}^{\nu_1} \mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_2} + \frac{1}{2} H_{\alpha\nu_3}{}^{\nu_1} \Omega^{\alpha}{}_{\nu_4}{}^{\nu_2} \qquad \text{(A.9)}$$

$$(336)_{\nu_4}{}^{\nu_1\beta} \{336\}^{\nu_2}{}_{\nu_3\beta} = \frac{1}{3} \delta_{\nu_3}{}^{\nu_1} \delta_{\nu_4}{}^{\nu_2} + \frac{1}{2} H_{\alpha\nu_3}{}^{\nu_1} \mathcal{U}^{\alpha}{}_{\nu_4}{}^{\nu_2} \qquad \text{(A.10)}$$

$$(368)_{\nu_4}{}^{\nu_1\beta} \{368\}^{\nu_2}{}_{\nu_3\beta} = \frac{1}{3} \delta_{\nu_3}{}^{\nu_1} \delta_{\nu_4}{}^{\nu_2} + \frac{1}{6} H_{\alpha\nu_3}{}^{\nu_1} \mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_2} - \frac{1}{2} H_{\alpha\nu_3}{}^{\nu_1} \Omega^{\alpha}{}_{\nu_4}{}^{\nu_2} \qquad \text{(A.11)}$$

Similar other relations can be built up easily. For example, we guess

$$( 368 )_{\nu_4}{}^{\beta\nu_1} \{368\}^{\nu_2}{}_{\beta\nu_3} = \frac{1}{6} \delta_{\nu_3}{}^{\nu_1} \delta_{\nu_4}{}^{\nu_2} + a\, \mathcal{U}_{\alpha\nu_3}{}^{\nu_1} \mathcal{H}^{\alpha}{}_{\nu_4}{}^{\nu_2} + b\, \mathcal{U}_{\alpha\nu_3}{}^{\nu_1} \Omega^{\alpha}{}_{\nu_4}{}^{\nu_2} \qquad (A.12)$$

and when verified one finds $a = \frac{1}{6}$ , $b = -\frac{1}{2}$ .

## REFERENCES

[1] A. O. Barut and B. C. Ünal, Nuovo Cimento **28**, 112 (1963)

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## Özet

Bu çalışmada, SU(3) grubunun S-Matrisi Teorisinde kullanılan saçılma genlikleri, grubun temsillerinin infinitesimal operatörleri cinsinden kuruldu. Bu teorik grup metodu, keyfi isopotik spin ve hiperyük değerleri için bütün genlikleri vermektedir. Çalışmanın önemli sonucu şudur: Elemanter parçacıkların kuvvetli interaksiyonlarının SU(4) ve SU(6) gibi herhangi bir üniter simetri grubuna göre invaryan saçılma genlikleri, o grubun generatör temsilleri cinsinden yazılabilir. Gerçekten belirtilmemiş olmasına rağmen, referans [1] deki SU(2) genlikleri uygun generatör temsilleri cinsindendir. Diğer bir sonuç da, üç - parçacık reaksiyonlarına ait rekürsiyon farmülünü kullanarak, basit temsillerden hareketle yüksek boyutlu temsillerin Clebsch-Gordan katsayılarını hesaplama yoludur.

## AVIS IMPORTANT

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" sera publiée dorénavant sous forme de fascicules à l'arrivée de chaque article. Le tome 15 de la série A, commençant par ce fascicule, sera complété par le dernier fascicule à paraître avant le 31 décembre 1966.

**Prix de l'abonnement annuel pour 1966 :**

Turquie : 15 TL ; Etranger : 30 TL.

Prix de ce numéro : 5 TL (pour la vente en Turquie).

Prière de s'adresser pour l'abonnement à : Fen Fakültesi Dekanlığı, Ankara, Turquie.

ANKARA ÜNİVERSİTESİ BASIMEVİ. 1966