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# Pion-Gauge Conditions for $N\rho \rightarrow N^*\pi$ Reaction

by

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# Pion-Gauge Conditions for $N\rho \rightarrow N^*\pi$ Reaction

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General self-consistency conditions are derived for $N^*$ $j = 1/2$ production amplitudes and coupling constants by the methods of pure S-matrix theory. The similarity to electroproduction is discussed.

We consider

$$N(k_1) + \varrho(k_2) \rightarrow N^*(k_3) + \pi(k_4) \qquad (1)$$

reaction (s-channel) with masses $k_1^{\,2} = m_1^{\,2}$, $k_2^{\,2} = \lambda^2$, $k_3^{\,2} = m_3^{\,2}$, and $k_4^{\,2} = \mu^2$. $N$ and $N^*$ are both spin $1/2$, isospin $1/2$ nucleon resonances. The spin basis for the reaction (1) is well known from electroproduction, only a slight modification is needed to take care of the unequal masses of the nucleons:

$$V_i = u(k_3)\, \gamma_5\, v_i^{\mu}\; u(k_1)\, \varepsilon_\mu (k_2)$$

where

$$\begin{aligned} v_{1\mu} &= \gamma_\mu\, k_2 & \qquad v_{5\mu} &= -\; \gamma_\mu \\ v_{2\mu} &= 2\, P_\mu & v_{6\mu} &= P_\mu\; \not{k}_2 \\ v_{3\mu} &= 2\, k_{4\mu} & v_{7\mu} &= k_{2\mu}\; \not{k}_2 \\ v_{4\mu} &= 2\, k_{2\mu} & v_{8\mu} &= k_{4\mu}\; \not{k}_2 \end{aligned}$$

with $P = (k_3 + k_1)/2$. The isospin basis is same as with isovector electroproduction basis. The number of independent spin amplitudes is reduced to six by the condition $k_2 \,.\, \varepsilon(k_2) = 0$.

In the limit $k_4 \rightarrow 0$, the undetermined part of the scattering amplitude will come from the contribution of the $N$ and $N^*$ in-

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termediate states to Born terms. We give in Table I these terms, where we introduced the following form factors.

$$g\ (\mu^2) \equiv g_{NN\pi}\ (m_1^2, m_1^2, \mu^2); \quad f_{1,2}(\lambda^2) \equiv f^{v}{}_{1,2NN\rho}\ (m_1^2, m_1^2, \lambda^2)$$
$$g'\ (\mu^2) \equiv g'_{NN^*\pi}\ (m_1^2, m_3^2, \mu^2)\ ;\ f'_{1,2}\ (\lambda^2) \equiv f'^{v}{}_{1,2NN^*\rho}\ (m_1^2, m_3^2, \lambda^2) \quad (2)$$
$$g''(\mu^2) \equiv g''_{N^*N^*\pi}\ (m_3^2, m_3^2, \mu^2)\ ;\ f''_{1,2}\ (\lambda^2) \equiv f''^{v}{}_{1,2N^*N^*\rho}\ (m_3^2, m_3^2, \lambda^2)$$

All form factors have been assumed to be real and symmetric in the first two variables, $m_1^2$ and $m_3^2$. The $N^*N\varrho$ vertex is written as

$$\bar{u}(k)\ [\gamma_\mu f_1'(\lambda^2) + i\,\sigma_{\mu\nu}\,k_2^\nu\, f_2'\ (\lambda^2)]\ u(k_1)\ \varepsilon^\mu\ (k_2)$$

TABLE I

$$\overset{\pm B}{B_1} = g'\ \left(\frac{f_1 + 2m_1 f_2}{s - m_1^2} \pm \frac{f_1'' + 2m_3 f_2''}{u - m_3^2}\right) + [f_1' + (m_3 + m_1) f_2'] + \left(\frac{g''}{s - m_3^2} \pm \frac{g}{u - m_1^2}\right)$$

$$\overset{\pm B}{B_2} = -\,g'\ \left(\frac{f_1}{s - m_1^2} \pm \frac{f_1''}{u - m_3^2}\right) - f_1'\ \left(\frac{g''}{s - m_3^2} \pm \frac{g}{u - m_1^2}\right)$$

$$\overset{\pm B}{B_3} = -\,\frac{1}{2}\,g'\left(\frac{f_1}{s - m_1^2} \mp \frac{f_1''}{u - m_3^2}\right) - \frac{1}{2}\,f_1'\ \left(\frac{g''}{s - m_3^2} \mp \frac{g}{u - m_1^2}\right)$$

$$\overset{\pm B}{B_5} = g'\ (f_2 \mp f_2'') + f_2'\ \left(\frac{s - m_1^2}{s - m_3^2}\,g'' \mp \frac{u - m_3^2}{u - m_1^2}\,g\right) + (m_3 - m_1)\, f_1'\ \left(\frac{g''}{s - m_3^2} \pm \frac{g}{u - m_1^2}\right)$$

$$\overset{\pm B}{B_6} = 2g'\ \left(\frac{f_2}{s - m_1^2} \pm \frac{f_2''}{u - m_1^2}\right) + 2\,f_2'\ \left(\frac{g''}{s - m_3^2} \pm \frac{g}{u - m_1^2}\right)$$

$$\overset{\pm B}{B_8} = g'\ \left(\frac{f_2}{s - m_1^2} \mp \frac{f_2''}{u - m_3^2}\right) + f'_2\ \left(\frac{g''}{s - m_3^2} \mp \frac{g}{u - m_1^2}\right).$$

The normalization is same as in the references.[1–2]

We shall continue the total amplitude $M = B_i V_i$ to the unphysical point:

$$s = m_3^2, \quad u = m_1^2, \quad t = \lambda^2 \qquad (k_4 \to 0) \tag{3}$$

For this purpose we separate the scalar $B_i$ amplitudes into Born terms coming from $N$ and $N^*$ intermediate states and the remainder part which is regular at the point (3).

$$M = (B_i^R + B_i^B)\, V_i \tag{4}$$

We also separate the basis functions $V_i$ into their finite and vanishing parts when $k_4 \to 0$, by the following identities:

$$\begin{aligned}
V_1 &= y_2 + \tilde{u}(k_3)\,\gamma_5(\gamma_\mu \not{k}_4 - k_{4\mu})\, u(k_1)\, \varepsilon^\mu(k_2) \\
V_2 &= y_2 - (m_3 - m_1)\, y_1 \\
V_3 &= 2\, \tilde{u}(k_3)\,\gamma_5\, k_{4\mu}\, u(k_1)\, \varepsilon^\mu(k_2) \\
V_5 &= -y_1 \\
V_6 &= -\frac{m_3 + m_1}{2}[y_2 - (m_3 - m_1) y_1] + \tilde{u}(k_3)\,\gamma_5 P_\mu \not{k}_4 u(k_1) \varepsilon^\mu(k_2) \\
V_1 &= \tilde{u}(k_3)\,\gamma_5\, k_\mu \not{k}_2\, u(k_1)\, \varepsilon_\mu(k_2)
\end{aligned} \tag{5}$$

Where

$$\begin{aligned}
y_1 &= \tilde{u}(k_3)\,\gamma_5\,\gamma_\mu\, u(k_1)\, \varepsilon^\mu(k_2) \\
y_2 &= -i\, \tilde{u}(k_3)\,\gamma_5\,\sigma_{\mu\nu}\, u(k_1)\, \varepsilon^\mu(k_2)\, (k_3 - k_1)^\nu
\end{aligned} \tag{6}$$

are the two spin amplitudes of the $NN^*\varrho$ vertex with a pion spurion.

We evaluate the limit of Eq. (4) as $k_4 \to 0$, using (5). From the regular parts, only $B_1^R$, $B_2^R$, $B_5^R$ and $B_6^R$ survive. In the Born-terms we use the method developped in references [1–2]: For the terms of the type $1/s - m_3^2$ we put first $k_4 = \alpha k_3$ and then $\alpha \to 0$; for the terms of the type $1/u - m_1^2$, we put $k_4 = -\beta k_1$ with $\beta \to 0$. We obtain by this method

$$M^{\pm}_{k4=0} = \Big\{ \overset{\pm R}{B_1} + \overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6} - \frac{g'(o)}{m_3+m_1} (f_2 \pm f_2'') - \frac{1}{2} f_2' \Big( \frac{g''(o)}{m_3} \pm \frac{g(o)}{m_1} \Big) \Big\} y_2$$

$$- \Big\{ \overset{\pm R}{B_5} + (m_3 - m_1) \Big( \overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6} \Big) - \frac{g'(o)}{m_3+m_1} \times (f_1 \mp f_2'') - \frac{1}{2} f_1' \Big( \frac{g''(o)}{m_3} \mp \frac{g(o)}{m_1} \Big) - f_2 (g''(o) \mp g(o)) \Big\} y_1 \quad (7)$$

The pion gauge condition[3] lim $M = 0$ gives the following consistency conditions at the point (3):

$$\overset{\pm R}{B_1} + \overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6} = \frac{g'(o)}{m_3+m_1} (f_1 \pm f_2'') + \frac{1}{2} f_2' \Big( \frac{g''(o)}{m_3} \pm \frac{g(o)}{m_1} \Big)$$

$$\overset{\pm R}{B_5} + (m_3 - m_1) \Big( \overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6} \Big) = \frac{g'(o)}{m_3+m_1} (f_1 \mp f_1'') + \frac{1}{2} f_1' \Big( \frac{g''(o)}{m_3} \mp \frac{g(o)}{m_1} \Big) + f_2' (g''(o) \mp g(o)). \quad (8)$$

As $B_1^-$, $B_2^-$, $B_5^+$ and $B_6^-$ amplitudes are antisymmetric under the interchange

$$s \leftrightarrow u, \qquad m_3 \leftrightarrow m_1 \quad (9)$$

they must be odd functions of the equally antisymmetric variable

$$\nu = \frac{2}{m_3+m_1} k_4 \,.\, P = \frac{1}{2(m_3+m_1)} (s - u - m_3^2 + m_1^2) \quad (10)$$

Hence in the limit $k_4 \rightarrow 0$, the constant terms in (8) must vanish. This implies the conditions:

$$\frac{g'(o)}{m_3+m_1}(f_2-f_2'') + \frac{1}{2} f_2' \left(\frac{g''(o)}{m_3} - \frac{g(o)}{m_1}\right) = o \qquad (11)$$

$$\frac{g'(o)}{m_3+m_1}(f_1-f_1'') + \frac{1}{2} f_1' \left(\frac{g''(o)}{m_3} - \frac{g(o)}{m_1}\right) + f_2'(g''(o) - g(o)) = o \qquad (12)$$

From Eq (11) we obtain

$$g'(m_3^2, m_1^2, o) = o; \quad \frac{g''(m_3^2, m_3^2, o)}{m_3} = \frac{g(m_1^2, m_1^2, o)}{m_1} \qquad (13)$$

and with these conditions the Eq. (12) gives:

$$f_2'^{v}(m_3^2, m_1^2, \lambda^2) = o \qquad (14)$$

The conditions (13) are common to all reactions of the type $NX \to N^*\pi$ with $X = \pi, \gamma, \varrho$ . [1-2]. But the last condition on the magnetic isovector form factor of the vertex $NN^*\varrho$ is new.

The self consistency conditions (8) become, using (13) and (14)

$$\left. \overset{\pm R}{B_1} + \overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6} \right|_{k_4 = o} = 0$$

$$\left. \overset{\pm R}{B_5} + (m_3-m_1)\left(\overset{\pm R}{B_2} - \frac{m_3+m_1}{2} \overset{\pm R}{B_6}\right) \right|_{k_4 = o} \qquad (15)$$

$$= \frac{1}{2} f_1' \left(\frac{g''(o)}{m_3} - \frac{g(o)}{m_1}\right)$$

In the equal mass case, we cannot directly take $m_3 \to m_1$ limit of Eq. (15) because $f_2'^{v}(m_3^2, m_1^2, \lambda^2)$ is discontinuous. Since, $f_2^{v}(m_1^2, m^2, \lambda^2) \neq 0$ in opposition to the condition (14) obtained as a consequence of the pion gauge condition and of the symmetry properties of the scattering amplitude under (9). We have to go back to the expressions of Born terms before $k_4 \to o$ limiting process. We have to take first $m_3 = m_1$ and then to go to $k_4 \to o$ limit.

The procedure is same, we give in Table II the Born terms due to $N$ and $N^*$ intermediate states.

TABLE II

$$\overset{\pm B}{B_1} = g\,(f_1 + 2m\,f_2)\,\left(\frac{1}{s-m^2} \pm \frac{1}{u-m^2}\right)$$

$$\overset{\pm B}{B_2} = -\,g\,f_1\,\left(\frac{1}{s-m^2} \pm \frac{1}{u-m^2}\right)$$

$$\overset{\pm B}{B_3} = -\,\frac{1}{2}\,g\,f_1\,\left(\frac{1}{s-m^2} \mp \frac{1}{u-m^2}\right)$$

$$\overset{\pm B}{B_5} = g\,f_2\,(1 \mp 1)$$

$$\overset{\pm B}{B_6} = 2g\,f_2\,\left(\frac{1}{s-m^2} \pm \frac{1}{u-m^2}\right)$$

$$\overset{\pm B}{B_1} = g\,f_2\,\left(\frac{1}{s-m^2} \mp \frac{1}{u-m^2}\right)$$

The pion gauge condition gives in this equal mass case the following consistency conditions:

$$\left.\overset{\pm R}{B_1} + \overset{\pm R}{B_2} - m\,\overset{\pm R}{B_6}\right|_{k_4=o} = \frac{1}{2\,m}\,g\,f_2\,(1 \pm 1)$$

(16)

$$\left.\overset{\pm R}{B_5}\right|_{k_4=o} = \frac{1}{2\,m}\,g\,f_1\,(1 \mp 1)$$

We observe that when $m_3 \neq m_1$, $\overset{\pm R}{B_1} + \overset{\pm R}{B_2} - \frac{m_3+m_1}{2}\,\overset{+R}{B_6} = o$ at the limit $k_4 = o$. But when $m_3 = m_1$, the same amplitude is not zero: $B_1^{+R} + B_2^{+R} = \frac{1}{m} g f_2$. This is similar to $A_6^{-}$ amplitude in electroproduction[2] where the discontinuity of $f_1'(m_3^2, m_1^2, \lambda^2)$ plays a similar role.

## CONCUSION

We obtained, starting from the physical amplitude two sets of Adler type consistency conditions (15) and (16). We notice once more that, $k_4 \to o$ and $m_3 \to m_1$ limits do not commute, due to the discontinuities of form factors.

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[1] A. O. Barut and B. C. Ünal, Lettere al Nuovo Cimento, **1**, 145 (1969).

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[3] For a critical discussion of this condition see G. Kramer and W. F. Palmer, Argonne preprint (Dec. 1968), and for a general group theoretical description we refer to A. O. Barut, Lettere al Nuovo Cimento, **2**, 94 (1969).

## ÖZET

Yalnız S-Matrisi teorisinin metotları kullanılarak $N^{*}j = 1/2$ prodüksiyon genliklerinin ve kuplaj sabitlerinin genel uygunluk (self-consistency) şartları elde edilmiştir. Elektroprodüksiyon ile olan benzerlik tartışılmıştır.

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