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by

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Application of Hansen's method to the Earth-Neptun system

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SUMMARY

"Hansen's method of perturbation has been applied to the motion of Mars by Clemence, to the first order in 1949 and to the second order in 1961. It has never been applied to the motion of the Earth. The current ephemeris of the Earth is based on the Newcomb theory given in the Astronomical Papers Vol. VI. In the present work we will apply Hansen's hethod to find the motion of the Earth perturbed by Neptun. A comparison of secular perturbations of the motion of the Earth given by Newcomb and Hansen methods shows a good agrreement."

The famous two body problem in celestial mechanics was solved by Kepler and Newton three centuries ago; such that, if there were only two bodies in space, for example: the Earth and the Sun, the former would decribe an ellipse around the latter and the latter would occupy one of the foci of this ellipse.

Now suppose that there is a third body disturbing the elliptic motion of the Earth. This problem known as the three body problem has no general solution. Therefore, approximate solutions are sought. In the following we will apply Hansen's method to find the motion of the Earth as disturbed by Neptun.

In Hansen's method we begin by taking an auxiliary ellipse in the plane of the instantaneous orbit of the Earth. The elements of this ellipse at an epoch are absolutely constant throughout in the theory. let $n\,\delta z$ show the perturbations in the mean anomaly, and r_0 the radius vector of the Earth in the auxiliary orbit at any time. Now if r shows the radius vector in the instantaneous orbit, i. e. perturbed radius vector, we have $r = r_0\,(1 + \nu)$.

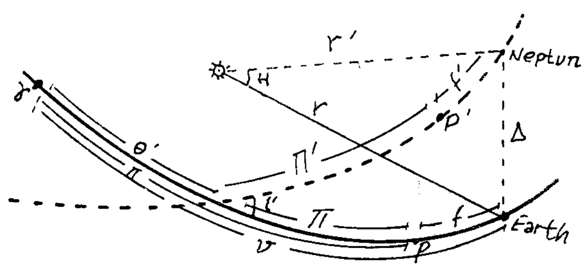
If we know the perturbation series for $n\delta z$ and ν in terms of the time and the constants (elements at epoch), the perturbed radius vector, r, and the perturbed longitude v, of the Earth can be found. These two coordinates give the motion of the Earth in the instantaneous plane of orbit, and it remains to find the motion of the plane itself. This is obtained by finding perturbation series, $u/\cos i$, giving the latitude, B, of the Earth above a fixed plane, i. e. ecliptic plane at epoch.

We will now give the steps for the calculations of the perturbation series for $n\delta z$, ν, and $u/\cos i$:

1. Elements of the Earth's and Neptun's orbit at the epoch 1850 January O, G.M.T.

Elements	Earth	Neptun
Semi-major axis	$a = 1.000000038$	$a' = 30.057342$
Eccentricity	$e = 0.01677128$	$e' = 0.0085082$
Inclination	$i = 0$	$i' = 1°47'01''.81$
Longitude of perihelion	$\pi = 100°21'38''.92$	$\pi' = 43°19'43''.7$
Longitude of node	$\theta = 0$	$\theta' = 130°08'00''.2$
Mean motion	$n = 1295977''.4496$ / Jul. year	$n' = 7864''.698$ / Jul. year
Mass ($m_\odot = 1$)	$m = 0.000003035915$	$m' = 0.000051775914$
Mean longitude at epoch	$\varepsilon_0 = 99°48'18''.56$	$\varepsilon'_0 = 335°05'38''.91$

2. $\Pi = \pi - \theta'$
$\Pi' = \pi' - \theta'$
gives the argument of the perihelion of the Earth, Π, and that of Neptun's, Π'.



3. Take 12 special values between 0° and 360° with the interval of 30°, of the mean anomaly of the Earth, l, and Neptun, l'

4. Solve Kepler's equatin $l = u - e \sin u$ for each special values of l to get $u_1, u_2, \ldots, u_{12}$ and $l' = u' - e' \sin u'$ for each special values of l' to get $u'_1, u'_2, \ldots, u'_{12}$

5. Solve $\cos f = \dfrac{\cos u - e}{1 - e \cos u}$ and $\sin f = \dfrac{\sqrt{1-e^2}\,\sin u}{1 - e \cos u}$

$$\cos f' = \frac{\cos u' - e'}{1 - e' \cos u'} \quad \text{and} \quad \sin f' = \frac{\sqrt{1-e'^2}\,\sin u'}{1 - e' \cos u'}$$

$$r = a \quad (1 - e\cos u)$$
$$r' = a' \quad (1 - e'\cos u')$$

to get $f_1, f_2, \ldots, f_{12}$; $f'_1, f'_2, \ldots, f'_{12}$

$r_1, r_2, \ldots, r_{12}$; $r'_1, r'_2, \ldots, f'_{12}$

6. For each pair of f and f' get cosH from

$\cos H = \cos(f + \Pi)\cos(f' + \Pi') + \cos J \sin(f + \Pi)\sin(f' + \Pi')$
and $\triangle$ from $\triangle^2 = r^2 + r'^2 - 2rr' \cos H$ where $J = i'$

Calculate $\dfrac{a'}{\triangle}$ and $\left(\dfrac{a'}{\triangle}\right)^3$, these will be in a matrix form having 144 values each.

7. Make double hormonic analysis of $\dfrac{a'}{\triangle}$ and $\left(\dfrac{a'}{\triangle}\right)^3$

using the following Fourier series:

$$F(1,1') = \sum_{k=0}^{6} \sum_{j=0}^{6} A_{k,j} \cos jl \cos kl' + \sum_{k=1}^{5} \sum_{j=1}^{5} B_{k,j} \sin jl \sin kl'$$

$$+ \sum_{k=0}^{6} \sum_{j=1}^{5} C_{k,j} \sin jl \cos kl' + \sum_{k=1}^{5} \sum_{j=0}^{6} D_{k,j} \cos jl \sin kl'$$

Make the trigonometric transformations to have the above series in terms of addition and subtraction of sine and cosine.

8. Calculate the following series

$$\left(\frac{r}{a}\right)^2 = 1 + \frac{3}{2} e^2 - \sum_{i=1}^{6} \frac{4}{i^2} J_i(ie) \cos il$$

$$\left(\frac{r'}{a'}\right)^2 = 1 + \frac{3}{2} e'^2 - \sum_{i=1}^{6} \frac{4}{i^2} J_i(ie') \cos il'$$

where

$$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304}$$

$$J_1(x) = \frac{x}{2}\left(1 - \frac{x^2}{8} + \frac{x^4}{192} - \frac{x^6}{9216}\right)$$

$$J_2(x) = \frac{x^2}{8}\left(1 - \frac{x^2}{12} + \frac{x^4}{384}\right)$$

$$J_3(x) = \frac{x^3}{48}\left(1 - \frac{x^2}{16} + \frac{x^4}{640}\right)$$

$$J_4(x) = \frac{x^4}{384}\left(1 - \frac{x^2}{20}\right)$$

$$J_5(x) = \frac{x^5}{3840}\left(1 - \frac{x^2}{24}\right)$$

$$J_6(x) = \frac{x^6}{46080}$$

$$J_7(x) = \frac{x^7}{645120}$$

are the Bessel functions.

9. For the Earth:

$$P_0 = -3e$$

$$Q_0 = 0$$

$$P_i = \frac{1}{i}\left[J_{i-1}(ie) - J_{i+1}(ie)\right]$$

$$Q_i = \frac{1}{i}\left[J_{i-1}(ie) + J_{i+1}(ie)\right]$$

for Neptun:

$$P'_0 = -3e'$$

$$Q'_0 = 0$$

$$P'_i = \frac{1}{i}\,[J_{i-1}\,(ie') - J_{i+1}\,(ie')]$$

$$Q'_i = \frac{1}{i}\,[J_{i-1}\,(ie') + J_{i+1}\,(ie')]$$

where $i = 1, 2, ..., 6$

$$\frac{r'}{a'}\sin(f'+\Pi') = \tfrac{1}{2}P_0'\sin\Pi' + \sum_{i=1}^{6}(P'_i\sin\Pi'\cos il' + Q_i'\sqrt{1-e'^2}\cos\Pi'\sin il')$$

$$\left(\frac{a'}{r'}\right)^2\sin(f'+\Pi') = \sum_{i=1}^{6}(i^2\,P'_i\sin\Pi'\cos il' + i^2Q'_i\sqrt{1-e'^2}\cos\Pi'\sin il')$$

10.Get k, k_1, K, K_1 from $k\cos(\Pi'-K) = \cos\Pi$, $k_1\cos(\Pi'-K_1) = \cos J\cos\Pi$ and $k\sin(\Pi'-K) = \cos J\sin\Pi$, $k_1\sin(\Pi'-K_1) = \sin\Pi$

Get p, P, v, V from

$$p\sin P = 2\alpha\left(\alpha\frac{e}{e'} - k\cos K\right)$$

$$p\cos P = 2\alpha k_1\sqrt{1-e^2}\,\sin K_1$$

$$v\sin V = 2\alpha k\sqrt{1-e'^2}\,\sin K$$

$$v\cos V = 2\alpha k_1\sqrt{1-e^2}\,\sqrt{1-e'^2}\,\cos K_1$$

Where $\alpha = \dfrac{a}{a'}$

Get $h, h_1, \bar{I}, \bar{I}_1$ from

$$h = \alpha\,k\cos K$$

$$h_1 = \frac{1}{2}\,v\cos V$$

$$\bar{I} = \tfrac{1}{2}\,p\cos P$$

$$\bar{I}_1 = \tfrac{1}{2}\,v\sin V$$

Using these, get the indirect part from

$$a' \frac{r}{r'^2} \cos H = \sum_{i=o}^{6} \sum_{k=o}^{6} \{ \tfrac{1}{2} i^2 [hP_kP'_i \pm h_1Q_kQ'_i] \cos (\pm il'-kl)$$
$$- \tfrac{1}{2} i^2 [\bar{I}Q_kP'_i \pm \bar{I}_1P_kQ'_i] \sin (\pm il'-kl)\}$$

11. Now we are ready to obtain the derivatives of the disturbing function in order to calculate the perturbations of the first order. It is desired to conduct the work with the second of arc as unit instead of the radian by multiplying $\frac{m'}{1+m}$ by 206264″.8

$$a\Omega = \frac{m'}{1+m} \alpha \left[\frac{a'}{\triangle} - \alpha \left(\frac{a'}{r'}\right)^2 \left(\frac{r}{a}\right) \cos H\right]$$

$$ar \frac{\delta\Omega}{\delta r} = \frac{m'}{1+m} \alpha\left[-\tfrac{1}{2}\left(\frac{a'}{\triangle}\right)^3\left(\alpha^2 \frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right) - \tfrac{1}{2} \frac{a'}{\triangle} - \alpha\left(\frac{a'}{r'}\right)^2 \left(\frac{r}{a}\right) \cos H\right]$$

$$a^2 \frac{\delta\Omega}{\delta z} = \frac{m'}{1+m} \alpha^2 \sin J \left[\left(\frac{a'}{\triangle}\right)^3 \left(\frac{r'}{a'}\right) \sin (f' + \Pi') - \left(\frac{a'}{r'}\right)^2 \sin(f'+\Pi')\right]$$

We put the restriction k, j ≤ 6, then the results will be in the folwing form: $\sum_{k=0}^{6} \sum_{j=0}^{6}$ coefficients $\begin{matrix}\sin \\ \cos\end{matrix}$ $(kl \pm jl')$

12. Derivation of $a\Omega$ with respect to l gives $a \frac{\delta\Omega}{\delta l}$

13. Get A, B, C from the relations

$$A = -3 + \frac{2}{1-e^2} \left[-\frac{5}{2} eP_1\cos\lambda + \sum_{j=1}^{6} \frac{1}{j} (P_1Q_j \mp Q_1P_j)\cos (jl \pm \lambda)\right]$$

$$B = \frac{1}{1-e^2} \left[-eQ_1\sin\lambda + \sum_{j=1}^{6} (P_1Q_j \mp Q_1P_j) \sin (jl \pm \lambda)\right]$$

$$C = -\tfrac{1}{2} \sum_{j=1}^{6} (P_1Q_j \mp Q_1P_j) \sin (jl \pm \lambda)$$

A,B,C will be in the following form $A = \sum_{k=0}^{6}$ coefficient $\cos(\lambda \pm kl)$

$B = \sum_{k=0}^{6}$ coefficient $\sin(\lambda \pm kl)$ and $C = \sum_{k=0}^{6}$ coefficient $\sin(\lambda \pm kl)$

14. $$T = \frac{1}{n}\frac{dW}{dt} = Aa\frac{\delta\Omega}{\delta l} + Bar\frac{\delta\Omega}{\delta r} \text{ and } R = \frac{1}{n}\frac{dR}{dt} = Ca^2\frac{\delta\Omega}{\delta z}$$

The arguments of the expressions for T and R are generally in the form of $(k\lambda + jl + il')$. $k = 0, \pm 1$ in T and $k = \pm 1$ in R, the other terms are added after integration

15. $W = \int T\, ndt$ and $R = \int R\, ndt$

The terms to be added after integration are:

for W: W is in the following form $\sum \alpha^{(k)} \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} (k\lambda + jl + il')$ after the integration we have $\alpha(\pm 1)$, then $\alpha(\pm k) = \eta^{(k)}\alpha(\pm 1) + \theta^{(k)}\alpha(\mp 1)$

where $\eta^{(k)} = \frac{1}{2}\left(\frac{P_k}{P_1} + \frac{Q_k}{Q_1}\right)$ and $\theta^{(k)} = \frac{1}{2}\left(\frac{P_k}{P_1} - \frac{Q_k}{Q_1}\right)$

for R : the same terms + the term for k = O, that is,

$$\alpha^{(0)} = \eta^{(0)}\left[\alpha^{(1)} + \alpha^{(-1)}\right]$$

where $\eta^{(0)} = \frac{1}{2}\frac{P_0}{P_1}$

16. Constants of integration (for mean elements):

We add to W the following series:

$$k_0 + k_1\left(\cos\lambda + \frac{P_2}{P_1}\cos 2\lambda + \ldots + \frac{P_6}{P_1}\cos 6\lambda\right) + k_2\left(\sin\lambda + \frac{Q_2}{Q_1}\sin 2\lambda + \ldots + \frac{Q_6}{Q_1}\sin 6\lambda\right)$$

where $k_0 = -$ { the sum of the coeff. of the terms cos (iλ-il) in W}
$k_1 = -$ { the sum of the coeff. of the terms cos [(i+1)λ-il] in W + the coeff. of sin λ in T}
$k_2 = -$ { the sum of the coeff. of the terms sin [(i+1)λ-il)] in W — the coeff. of cosλ in T}

We add to R the following series:

$$k_3\left(\frac{1}{2}\frac{P_0}{P_1} + \cos\lambda + \frac{P_2}{P_1}\cos 2\lambda + \ldots + \frac{P_6}{P_1}\cos 6\lambda\right) + k_4\left(\sin\lambda + \frac{Q_2}{Q_1}\sin 2\lambda + \ldots + \frac{Q_6}{Q_1}\sin 6\lambda\right)$$

where $k_3 = -$ {the sum of the coeff. of the terms cos [(i+1)λ-il)] in R}
$k_4 = -$ {the sum of the coeff. of the terms sin[(i+1)λ-il)] in R}

17. $$\frac{d}{dt}\delta z = \overline{W}, \quad \frac{1}{n}\frac{d\nu}{dt} = -\frac{1}{2}\left(\frac{d\overline{W}}{d\lambda}\right), \quad \frac{u}{\cos i} = \overline{R}$$

The sign — shows that λ has been changed to 1

18. $$n\,\delta z = \int \overline{W}\, ndt + C'$$

$$\nu = \int \left(\frac{1}{n}\frac{d\nu}{dt}\right) ndt + C''$$

where $C' = 0$ and $C'' = -\frac{1}{6}\left(k_0 + \frac{3}{2}\frac{e}{P_1}k_1\right)$

In this way we obtain the perturbation series for n δ z, ν and u / cosi. The results will be given in the following table:

secular terms for n δ z:

$(10^{-8})''$ (–18 nt sin 1 –984 nt cos 1 - 4 nt cos 21)

secular terms for ν:

$(10^{-8})''$ (9 nt cos 1 – 492 nt sin 1 – 4 nt sin 21)

secular terms for u / cosi:

$(10^{-8})''$ (–455 nt sin 1 – 798 nt cos 1 – 4 nt sin 21 – 7 ntcos 21)

Periodic terms for $n\delta z$, ν and $u/\cos i$

		$n\delta z$ $(10^{-8})''$		ν $(10^{-8})''$		$u/\cos i$ $(10^{-8})''$	
l	l'	sin	cos	sin	cos	sin	cos
0	0				6547—		8+
	1	158522—	9557—	1483+	1318+	66+	98—
	2	2884+	1401—	1565—	577+	1567+	1142+
	3	55+				28+	23+
1	-4		129+	64+			
	-3	3001—	7666+	3724+	1488+	1109+	943—
	-2	172650—	375302+	186739+	85893+	61508+	44929—
	-1	218109—	345149—	171740—	108526+	827+	496—
	0			6+	492+		
	1	2122—	652—	327—	1068+	1643—	821+
2	-4	7—	30+	22+	6+	1—	
	-3	557—	1528+	1118+	417+	16—	17—
	-2	23248—	52012+	38158+	17061+	502+	366—
	-1	822—	1661—	1597—	844+	16—	15—
	0	5+	22—	16—		16—	9+
	1	9—	3—	3—	9+	14—	7+
3	-4			4+	21+		
	-3	620—	110+	96+	518+		
	-2	205—	471+	554+	243+	6+	4—
	-1	7—	13—	20—	10+		
	0	1—			1+		
4	-5	1—	1—	1—	1+		
	-4	7—	8—	7—	7+		
	-3	13—	2+	2+	11+		
	-2	2—	5+	8+	3+		
5	-5	1—			1+		

The calculation of the perturbation series $n\,\delta\,z$, ν, $u\,/\,\cos i$ completes the first part of this work. As a check, we will now find the secular perturbations in the elements of Earth orbit due toNeptun's influence, and compare the results with those given by Newcomb's method.

It is known that, to the first order of disturbing forces, the secular variations of the eccentricity and perihelion are rigorously equivalent to the terms of $n\,\delta\,z$ factored by nt, and the variations of the inclination and node are rigorously equivalent to the similar terms of $u\,/\,\cos i$. Denoting the centennial variation of any element by prefixing D to it, we have

$$De = 166.1193 \times \text{coeff . of } nt\sin l \text{ in } n\,\delta\,z$$
$$eD\pi = -\,166.4835 \times \text{coeff . of } nt\cos l \text{ in } n\,\delta\,z$$
$$Di = (86.3506 \times \text{coeff . of } nt\sin l \text{ in } u\,/\,\cos i$$
$$-\,323.8895 \times \text{coeff . of } nt\cos l \text{ in } u\,/\,\cos i)$$

$\sin i D\theta = (-324.5996 \times$ coeff . of ntsinl in u / cosi
$- 86.1617 \times$ coeff . of nt cosl in u / cosi)

We take the coeffients from the secular terms and we get

$De = 0''.000$

$eD\pi = 0''.002$

$Di = 0''.002$

$\sin i D\theta = 0''.002$

corresponding values from Newcomb theory are:

$De = 0''.000$

$eD\pi = 0''.003$

$Di = 0''.004$

$\sin i D\theta = 0''.004$

This, indeed, is a very good agreement.

We will now give the steps to find the coordinates of the Earth as perturbed by Neptun:

1. Mean anomalies of the Earth and Neptun:

$l = nt + (\varepsilon_0 - \pi) = 1295977''.4496t + 1293999''.64$

$l' = n' t + (\varepsilon'_0 - \pi') = 7864''.698t + 1050355''.21$

where $t = t_{date} - 2396758.0$

Substituting these values in the table we get $n\delta z$, ν, u

2. $l + n\delta z = E - e\sin E$ gives E

3. $\bar{r} = a(1 - e\cos E)$ gives $\bar{r}$

4. $r = \bar{r}(1 + \nu)$ gives r

5. $\left.\begin{array}{l} r\cos f = a\cos E - ae \\ r\sin f = a\sqrt{1-e^2}\sin E \end{array}\right\}$ gives f

6. $v = f + \pi$ gives v

7. $\sin B = \dfrac{a}{r} u$ gives B

r, v and B are the radius vector, longitude and latitude of the Earth as disturbed by Neptun. It is obvious that the undisturbed coordinates can easily be found by taking $n\delta z$, ν and u equal to zero in the above formulae. The differences between the disturbed and undisturbed coordinates will be the perturbations in the coordinates. We call them $\triangle r$, $\triangle v$ and $\triangle B$. To see the variations of the perturbations we found them for the years 1850, 1932 and 2015, which means that during this interval Neptun makes one revolution around the Sun:

In the following table, the dates are given in julian days, $\triangle r$ in 9- decimals of astronomical unit and, $\triangle v$ and $\triangle B$ in 4– decimals of second of arc.

An inspection of the table shows that:

1. The perturbations are yearly periodic,
2. The zero points of the perturbations change from year to year due to the motion of Neptun,
3. Due to secular perturbation, after Neptun's one complete recolution about the sun, the amplitudes of the perturbations increase,
4. After 165 years from the epoch the maximum values of the perturbations are:

$$\triangle r \max \simeq 7 \text{ km.}$$
$$\triangle v \max \simeq 13 \text{ km.}$$
$$\triangle B \max \simeq 7 \text{ km.}$$

5. These first order perturbations are so small that it is not necessary to take into account the second order terms in the disturbing function.

1850				1932				2015			
date	Δr	Δv	ΔB	date	Δr	Δv	ΔB	date	Δr	Δv	ΔB
239				242				245			
6758.5	3—	66—	8+	6706.5	11+	78—	34—	7022.5	1+	72—	74—
68.5	6—	63—	7+	16.5	7+	85—	38—	32.5	7—	170—	82—
78.5	10—	58—	7+	26.5	4+	89—	41—	42.5	15—	162—	88—
88.5	13—	49—	6+	36.5	0	90—	43—	52.5	22—	148—	91—
98.5	16—	39—	5+	46.5	3—	89—	43—	62.5	28—	129—	91—
808.5	17—	27—	4+	56.5	7—	86—	42—	72.5	35—	105—	89—
18.5	19—	13—	3+	66.5	9—	79—	40—	82.5	39—	79—	83—
28.5	19—	0	1+	76.5	12—	72—	37—	92.5	42—	49—	76—
38.5	20—	14+	0	86.5	14—	62—	33—	102.5	43—	18—	66—
48.5	19—	28+	1—	96.5	14—	52—	27—	12.5	44—	13+	54—
58.5	17—	41+	3—	806.5	16—	41—	21—	22.5	42—	43+	41—
68.5	15—	52+	4—	16.5	16—	30—	15—	32.5	40—	71+	26—
78.5	13—	60+	5—	26.5	16—	20—	8—	42.5	36—	98+	12—
88.5	10—	68+	6—	36.5	16—	10—	1—	52.5	32—	121+	4+
98.5	8—	74+	7—	46.5	15—	0	6+	62.5	27—	140+	18+
908.5	6—	78+	7—	56.5	14—	9+	13+	72.5	20—	156+	33+
18.5	2—	80+	8—	66.5	13—	17+	20+	82.5	14—	167+	46+
28.5	1—	80+	8—	76.5	12—	26+	26+	92.5	8—	174+	58+
38.5	2+	79+	8—	86.5	10—	32+	31+	202.5	2—	177+	68+
48.5	4+	76+	7—	96.5	8—	38+	35+	12.5	4+	176+	76+
58.5	6+	73+	7—	906.5	6—	43+	38+	22.5	10+	171+	82+
68.5	8+	69+	6—	16.5	4—	46+	40+	32.5	17+	162+	86+
78.5	10+	62+	5—	26.5	2—	49+	41+	42.5	22+	148+	87+
88.5	11+	55+	4—	36.5	1+	50+	41+	52.5	28+	132+	86+
98.5	13+	46+	3—	46.5	2+	48+	40+	62.5	32+	111+	82+
7008.5	14+	38+	2—	56.5	5+	47+	37+	72.5	36+	88+	76+
18.5	16+	28+	1—	66.5	8+	42+	33+	82.5	39+	64+	68+
28.5	17+	16+	0	76.5	10+	36+	29+	92.5	41+	35+	57+
38.5	17+	4+	2+	86.5	13+	27+	23+	302.5	42+	7+	45+
48.5	17+	8—	3+	96.5	15+	17+	17+	12.5	41+	23—	31+
58.5	16+	20—	4+	7006.5	17+	6+	10+	22.5	40+	52—	17+
68.5	15+	32—	5+	16.5	18+	7—	3+	32.5	37+	80—	1+
78.5	12+	42—	6+	26.5	17+	20—	5—	52.5	32+	107—	15—
88.5	11+	52—	7+	36.5	17+	35—	12—	52.5	28+	129—	30—
98.5	7+	59—	7+	46.5	16+	48—	19—	62.5	21+	148—	45—
108.5	4+	65—	7+	56.5	15+	60—	26—	72.5	14+	162—	58—
18.5	1+	66—	7+	66.5	13+	70—	31—	82.5	6+	170—	69—

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ÖZET

Hansen pertürbasyon metodu, birinci mertebeden 1949 ve ikinci mertebeden 1961 de olmak üzere, Clemence tarafından Mars'ın hareketine uygulanmıştır. Bu metod Yer'in hareketine şimdiye kadar hiç uygulanmamıştır. Yer'in bugün kullanılan efemerisleri, Astronomical Papers Vol. VI da verilen Newcomb teorisine dayanmaktadır. Bu çalışmamızda, Hansen metodunu Neptün tarafından bozulan Yer'in hareketine uyguladık. Elde ettiğimiz seküler perturbasyonlar Newcomb metodunun verdiği seküler perturbasyonlarla iyi bir uyuşma göstermektedir.

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