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by

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The Mössbauer Effect

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ABSTRACT

The essence of the Mössbauer effect is not γ-ray emission without recoil, but it is γ-ray emission without transferring any nuclear transition energy to the internal energy of the crystalline solid. The problem has been first treated classically by starting with the kinematics of γ-ray emmission for an idealized nucleus-plus-lattice system. Next, using the basic ideas of quantum mechanics, it has shown that the effect is a quantum mechanical phenomenon and can not be explained classically.

1. INTRODUCTION

Mössbauer effect is the phenomenon of recoil-free emission and resonant absorption of nuclear gamma rays emitted from atomic nuclei chemically bound in a crystalline solid. It allowes precise determination of extremely small energy shifts as a result of the Doppler effect modulation of gamma-ray energy. The effect was discovered by Rudolph L. Mössbauer in 1957. The Physics of the Mössbauer effect has already been discussed by many authors [1–11]. The purpose of this paper is to discuss briefly the basic features of this phenomenon as clearly as possible and present it in an easy way to understand.

When a free nucleus in an excited state decays to its ground state with the emission of a γ-ray, then the atom noımally recoils and the energy of the emitting gamma ray will depend on the kinetic energy of the recoiled atom. The laws of conservation of energy and momentum requires that the recoil energy of the nucleus of mass m upon the emission of a gamma ray of frequency υ is

$$(1) \qquad E_r = \frac{(h\nu)^2}{2\ mc^2}$$

and the energy of the γ-ray would be E_0 - E_r where E_0 is the nuclear transition energy.

Because of its considerably high energy, generally, a γ-photon imparts a measurable recoil to the free nucleus during emission, so that part of the excitation energy of the nucleus is lost as recoil energy. Hence any γ-photon emitted has insufficient energy to excite a nucleus of an absorber.

If the atom is tightly bound to the lattice, the entire solid rather than the individual atom recoils and the γ-ray might have the excitation energy E_0 of the nucleus. Thus, we must emphasize here that the Mössbauer effect is not γ-ıay emission without recoil, but rather is γ-ray emission without transfer of energy to internal degrees of freedom of the lattice and the subsequent inverse process which is called recoil-free resonant absorption and the reemission of gamma ray. The following is the classical and quantum mechahanical discussions of γ-ray emission under certain condition giving Mössbauer effect.

2. CLASSICAL DISCUSSION OF MÖSSBAUER EFFECT

Consider the kinematics of γ-ray emission from atomic nuclei bound in a crystalline solid. Let the mass of the emitting nucleus be m_1 and that of the remaining part of the solid be m_2 ($m_2 \gg m_1$). We assume that m_1 and m_2 are bound to each other by a massless spring of force constant k and unstreched length a. It makes an idealized nucleus-plus-lattice system which can be schematized as shown in Fig. (1). This is just a simplified

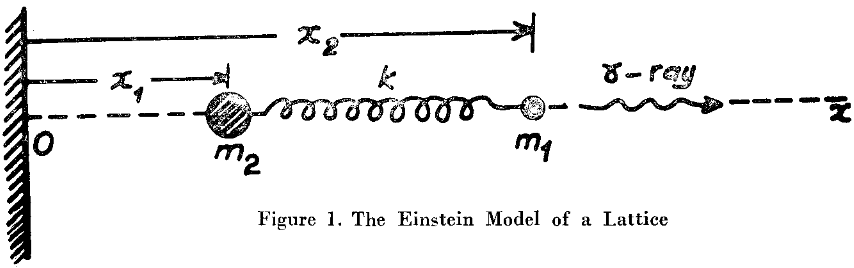

Figure 1. The Einstein Model of a Lattice

Einstein model of a lattice and it is an oscillator of single vibrational mode of frequency $w = (k/\mu)^{1/2}$ where μ is the reduced mass of the system.

Imagine that the nucleus of mass m_1 emits a gamma ray in the transition between the excited state of energy E_o to the ground state in the direction of x-axis. Before emision the system consists of the two masses m_1 and m_2 at rest with the nucleus in energy state E_o. After emission it consists of two masses m_1 and m_2 with velocities $\dot{x}_1$ and $\dot{x}_2$, respectively, with the nucleus in ground state and an emitted gamma photon of frequency υ. The equations of momentum and energy balance are then,

Momentum : $\dfrac{h\upsilon}{c} = m_1\dot{x}_1 + m_2\dot{x}_2$

Energy : $E_o = h\upsilon + \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k (x_2 - x_1 - a)^2$

These equations become more useful when they are expressed in terms of relative and center of mass corrdinates x and u, respectively, defined as

$$x = x_2 - x_1$$

$$u = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Here a little algebra shows that the above equations become

(2) $$M\dot{u} - \frac{h\upsilon}{c} = o$$

(3) $$E_o = h\upsilon + \tfrac{1}{2} M \dot{u}^2 + \tfrac{1}{2} \mu \dot{x}^2 + \tfrac{1}{2} k (x - a)^2$$

where M is the total mass ($M = m_1 + m_2$). From Eqs. (2) and (3) we get the following equation

(4) $$E_o = h\upsilon + \tfrac{1}{2} \frac{(h\upsilon)^2}{Mc^2} + E_{int}$$

where $E_{int} = \frac{1}{2} \mu \dot{x}^2 + \frac{1}{2} k (x-a)^2$

As we see, E_{int} depends only on the relative corrdinate x and called the "internal energy" of the system. A comparison of the

Eqs. (1) and (4) shows that the second term on the right hand side of the Eq. (4) is what we previously called the "recoil energy". The only difference is that the total mass M and not m_1 appears in this term. Since M is enormously large, the recoil energy is negligible and to a good approximation,

$$(5) \qquad h\upsilon \cong E_o - E_{int}$$

The Mössbauer effect begins to emerge in this equation. Here, if it were possible for E_{int} to be zero, we would have

$$(6) \qquad h\upsilon \cong E_o$$

which is just the mathematical statement of the Mössbauer effect. It describes a transition between an excited state of energy E_o to the ground state, in which the total energy released in the transition is all given to the gamma photon. In other words, if E_{int} can be equal to zero under certain condition, or in other language, if no internal lattice vibrations (phonons) are excited, we have the possibility of the Mössbauer effect.

The question is now under which circumstances E_{int} can be equal to zero. In classical mechanics it is quite clear that it can not be so. Upon the emission of gamma ray, the individual atom suffers a sudden momentum impulse and therefore it transfers a finite momentum to the system which make the internal energy of the lattice to be changed. But, from quantum mechanical point of view the story is different and one can transfer a momentum to a nucleus without changing the internal energy of the system [9].

3. QUANTUM MECHANICAL DISCUSSION OF MÖSSBAUER EFFECT

For simplicity, we consider again the Einstein lattice of Fig. (1). Quantum mechanically, the internal energy of this system is quantized and the allowed values are just the oscillator eigenvalues $E_{int} = (n + 1/2)\hbar w$. For the system, say initially in the ground state $(n = o)$, if there is a finite probability to be still in the ground state after gamma ray emission, the Mössbauer effect will be possible. The purpose now is to calculate this probability called "Debye-Waller factor" and show that it is different from zero.

Let P_{oo} be the probability amplitude for the system to remain in this original oscillator ground state after emitting a gamma photon of momentum $p_o = \frac{h\upsilon}{c}$. The probability amplitude for the nucleus to be in the oscillator ground state, when it has momentum p, is just the ground state wave function $\Phi_o(p)$. Since the nucleus was in the ground state before recoil and it gets an additional momentum p_o after the emission of gamma photon, the probability amplitude for the nucleus to have momentum p after recoil is the same as the probability amplitude for the nucleus to have had momentum p-p_o before recoil and it is just $\Phi_o(p-p_o)$. Therefore, the probability amplitude P_{oo} can now be written as [12]

$$(7) \qquad P_{oo} = \int \Phi_o^*(p) \cdot \Phi_o(p-p_o)\, dp$$

The asterisk* on the right hand side stands for complex conjugate.

In order to evaluate the above integral, we introduce the ground state space function $\Psi_o(x)$, of which $\Phi_o(p)$ is the Fourier transform

$$(8) \qquad \Phi_o(p) = \frac{1}{\sqrt{2\pi\bar{h}}} \int \Psi_o(x)\, e^{-ipx/\bar{h}}\, dx$$

If we put this into Eq. (7), we obtain

$$(9) \quad P_{oo} = \frac{1}{2\pi\bar{h}} \iiint \Psi_o^*(x')\, e^{ipx'/\bar{h}}\, \Psi_o(x) e^{-i(p-p_o)x/\bar{h}}\, dp\, dx' dx$$

Substituting the delta function into Eq. (9), we obt in

$$(10) \qquad P_{oo} = \iint \Psi_o^*(x')\, \delta(x-x').\, \Psi_o(x) e^{ip_ox/\bar{h}}\, dx\, dx'$$

Remembering that $\Psi_o(x) = \int \Psi_o(x)\, \delta(x-x')\, dx'$, then Eq. (10) becomes

$$(11) \qquad P_{oo} = \int \Psi_o^*(x)\, \Psi_o(x)\, e^{ip_ox/\bar{h}}\, dx$$

We can now find the probability f for a transition without energy loss which is

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$\bar{h} = \frac{h}{2\pi}$

(12) $$|P_{oo}|^2 = f = \left| \int \Psi_o^*(x)\, \Psi_o(x)\, e^{ipox/\hbar}\, dx \right|^2$$

The ground state space function for our system which is a harmonic oscillatoı is given as

$$\Psi_o(x) = \sqrt{\frac{mw}{\pi \hbar}}\; e^{-\frac{mw}{2\hbar} x^2}$$

Where m is the mass of the nucleus. Substituting this into Eq. (12), we finally obtain

(13) $$f = e^{-\frac{E_r}{\hbar w}}$$

where E_r is the recoil energy of the free nucleus defined by Eq. (1).

As we see, for the Einstein solid the problem is quite simple. If the energy E_r is of the same order as $\hbar w$, the quantized level spacing of the oscillator, the probability for the Mössbauer effect will be substantial. When the recoil energy is small compared to the smallest amount of energy ($E_e = \hbar w_e$) that can be given to the solid the lattice will not be excited and the γ-ray will have the total transition energy.

Eq. (13) shows that, for a given nucleus of mass m and nuclear transition energy E_0, if E_r is large compared to $\hbar w$ then upon the emission of each γ-photon the lattice will be excited and the probability of obtaining the Mössbauer effect will be small.

We must remember that here we worked out a rather idealized problem in which all possible degrees of freedom and all frequencies of the original problem were replaced by a single degree of freedom and a single frequency. In general, one can use the more accurate model of a lattice, such as the Debye model, and obtain a similar result [6]

$$f = e^{-\frac{3}{2}\frac{E_r}{\hbar w_d}}$$

where w_d is the Debye frequency, and $E_d = \hbar w_d$ is the maximum lattice vibration energy.

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ÖZET

Mössbauer olayı aslında γ-ışınlarının geri tepmesiz olarak neşredilmesi olayı değildir; fakat, çekirdek dönüşüm enerjisinden kristalin iç enerjisine hiç enerji transferi olmadan γ-ışınının neşri olayıdır. Einstein kistal modeli esas alınarak, uyarılmış çekirdekten γ-ışınının neşri olayı önce klasik olarak incelenmiş ve kristalin iç enerjisinde bir değişme yapmadan bir γ-photonunun neşrolunamıyacağı görülmüştür. Kuantum mekaniği açısından durum farklıdır ve bazı şartlarda bir γ-ışınının, çekirdek enerjisine eşit bir enerji ile neşrolunması mümkündür.

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