COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME ~~21-A~~ 22 A ANNÉE 1973

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by

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Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

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The Experimental Determination of the Thermal Neutron Diffusion Length in Graphite

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The experimental determination of the thermal neutron diffusion length in graphite was carried out by measuring the distribution of neutron flux along the x, y, and z axis. The neutron flux distribution have been measured with a BF_3 counter using a Ra - Be neutron source. For the calculation of the results the least squares method was chosen in order to obtain the most probable value of the diffusion length. The value of the diffusion length was found to be L = 58 ± 2 cm for the Diffusion Stack and L = 53,46 cm (average) for the Exponential Stack.

INTRODUCTION

The familiar method for the experimental determination of the diffusion length of thermal neutrons in a moderator, such as graphite, consists essentially of placing a rectangular parallelepiped stack of graphite over a uniform thermal - neutron source and then measuring by the foil activation technique the distribution of neutron flux within the graphite diffusion stack (sigma pile method) [1]. The solution of the appropriate form of the diffusion equation, i. e., for the point source of neutrons in an infinite nonmultiplying medium, gives the flux destribution in terms of $k = \sqrt{\Sigma_a / D}$, the reciprocal of diffusion length L, and the extrapolatad dimensions of the stack [2], where Σ_a is the macroscopic absorption cross section, and D the diffusion coefficient of graphite.

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The appropriate solution of the diffusion equation for the present geometry (see below) is

$$\gamma^2_{mn} = k^2 + \left(\frac{m\,\pi}{a}\right)^2 + \left(\frac{n\,\pi}{b}\right)^2 \tag{1}$$

and

$$\Phi\,(z) = \text{const.} \times e^{-\gamma_{mn} z} \tag{2}$$

where γ is the slope of experimental values of the flux Φ against z, the distance from the source, m and n are the odd integers (harmonics), a and b are the extrapolated dimensions of the diffusion stack in the x and y directions, i. e., the physical x - and y - dimensions of the stack, plus twice the extrapolation distance in each case. Equations (1) and (2) form the basis of the experimental measurements of L by this method.

In wiev of the solution of diffusion equation, it is strictly applicable only to monoenergetic neutrons and for the distances greater than two or three mean paths from strong sources. Consequently, in the measurements made beyond about two diffusion lenghts from the thermal source, the flux consists essentially of the fundamentale ($m = 1$, $n = 1$) term.

Apparatus and Experimental Method

I

Diffusion Stack and Measurement

We used basically the same method for the experimental determination of the diffusion length in graphite. The principle changes introduced here are the use of a BF_3 (boron trifluoride) proportional counter in place of the foil activation and the preference of the least squares method to obtain the most probable value of the diffusion length. The BF_3 counter has the advantage of good sensitivity and rapid indication of the flux.

The graphite diffusion stack, whose cross - section shown in Figure 1, has a shape of a square end horizontal parallelepiped of dimensions 40 in. × 40 in. × 87 in. The 500 mc Ra–Be source was inserted at a point S (4 in., 8 in., 0 in.) on the far end of the stack [3]. Hence the neutron source is not at the center of the x, y-plane.

The neutron flux along the x, y and z axis was measured with a BF_3 counter attached to the end of a thick walled aluminium tube bored with transverse holes at 2 in. intervals. This positioning rod, positioned by means of a spring-loaded dowel which locates in the transverse holes, was used in the 1 1/4 in. square measuring holes of the diffusion stack (see Fig. 1).

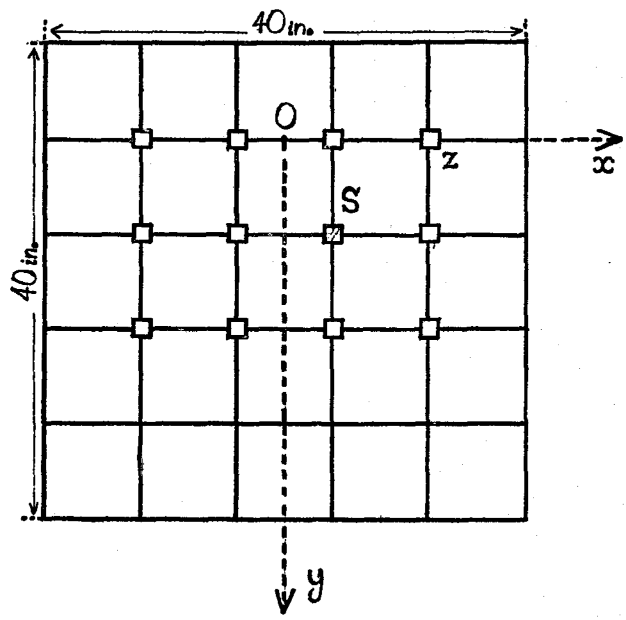

Fig. 1. The end view of the rectangular graphite diffusion stack.

The experimental results are listed in the Tables 1, 2, and 3, and the measured flux variations along z and x axis are plotted as a function of z and x in Figures 2 and 3.

TABLE 1.
Flux Measurements Along the z-axis

Distance from the source plane, in.	Count rate cps	Fast neutron flux measurement (counter covered with a cadmium sheath)
z = 0″	774.4	41.6 c/s
4	772.2	37.5
8	697.8	28.2
12	575.91	18.2
16	442.2	10.8
20	326.8	
24	229.8	
28	155.7	
32	105.6	Background Count = 0.6 c p s
36	70.7	
40	47.4	
44	31.7	
48	21.7	
52	14.6	
56	9.8	
60	6.98	

TABLE 2.

Flux Measurements Along the x-axis

Distance from source plane, in.	x co-ordinate			
	—12″	—4″	+4″	+12″
0″	510.8	885.4	1016.9	758.6 cps
20″	247.7	413.0	456.2	325.9
40″	38.9	61.6	66.9	47.9
60″	6.41	8.71	9.17	6.89

TABLE 3.

Flux variation along a line parallel to the y axis at a distance x = 12 in.

	y =	0″	8″	16″
0″		758.6	1029.9	889.6 cps
20″		325.9	458.9	412.4
40″		47.9	76.3	75.1
60″		6.89	9.84	9.63

Calculation of Experimental Results

The extrapolated length in the x - direction has been found by means of a least squares fit of the measured flux distribution (60″ from the source plane) to $\cos \frac{\pi x}{a}$.

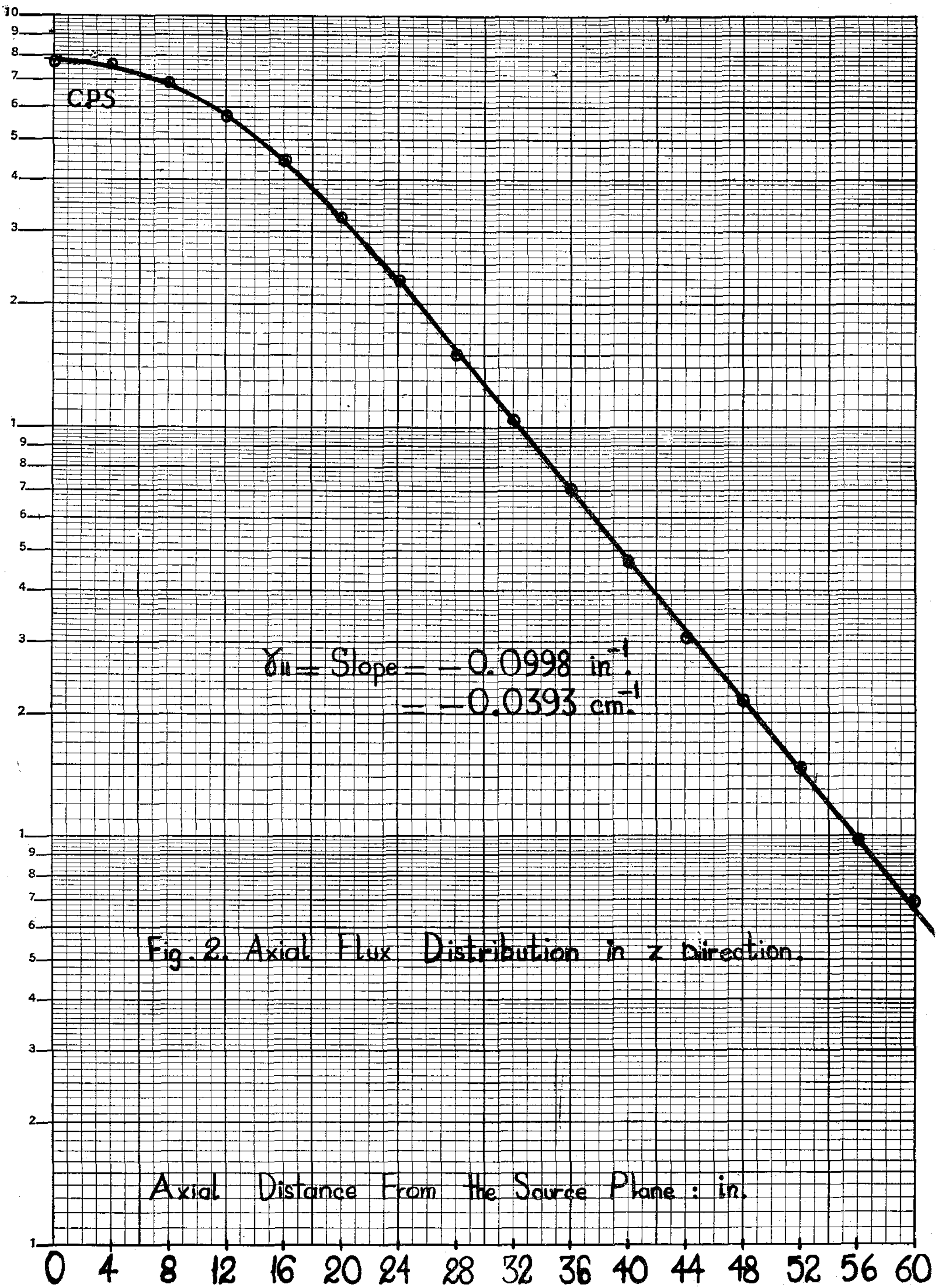

Fig. 2. Axial Flux Distribution in z Direction.

Suppose $\Phi = A \cos \frac{\pi x}{a}$ is the equation of the fitted curve. Let the measured flux be Φ at a point x_i in the stack. The sum of the squares of the deviations of the measured flux from this value is

$$D^2 = \Sigma_i \left(\Phi_i - A \cos \frac{\pi x_i}{a}\right)^2 \tag{3}$$

where A and a are constants requiring optimization. Applying the least squares method conditions:

$$\frac{\partial}{\partial A} \Sigma_i \left(\Phi_i - A \cos \frac{\pi x_i}{a}\right)^2 = 0 \tag{4}$$

$$\frac{\partial}{\partial a} \Sigma_i \left(\Phi_i - A \cos \frac{\pi x_i}{a}\right)^2 = 0 \tag{5}$$

From (4) one gets

$$A = \frac{\Sigma_i \Phi_i \cos \frac{\pi x_i}{a}}{\Sigma_i \cos^2 \frac{\pi x_i}{a}} \tag{6}$$

We chose a value of (a) for the extrapolated length and calculated A from equation (6). Using the values a and A we evaluated D^2 from equation (3), and repeated the same calculations for a range of a's until obtaining the value of (a) at which D^2 reaches a minimum. The results are summarized in Table 4 and D^2 is ploted versus (a) in Figure 4.

TABLE 4.

Optimization of (a) (flux measured 60″ from the source)

a (in.)	A	D^2
40″	9.929	2.052
42	9.748	1.157
43	9.662	0.856
44	9.584	0.632
45	9.508	0.467
48	9.308	0.235
48.5	9.278	0.2255
49	9.249	0.2212 } min.
49.5	9.218	0.2225 } min.
50	9.191	0.2289 } min.
51	9.139	0.2545
54	8.983	0.4069
55	8.951	0.4753
56	8.909	0.5485

Plotting (a) versus D^2 it is found that the minimum occurs at $a = 49.5$ in.

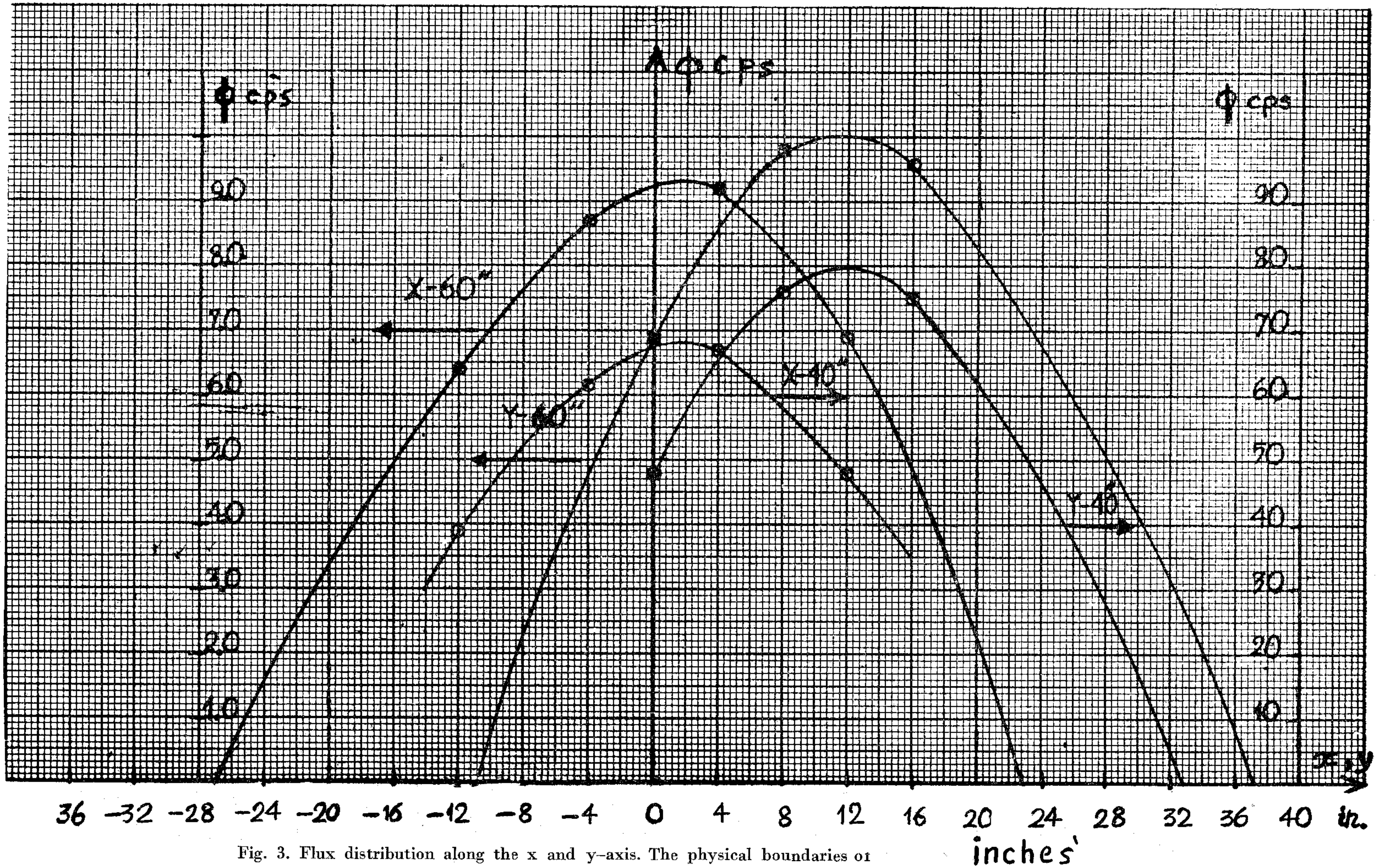

Fig. 3. Flux distribution along the x and y–axis. The physical boundaries of the stack are indicated by vertical dashed lines.

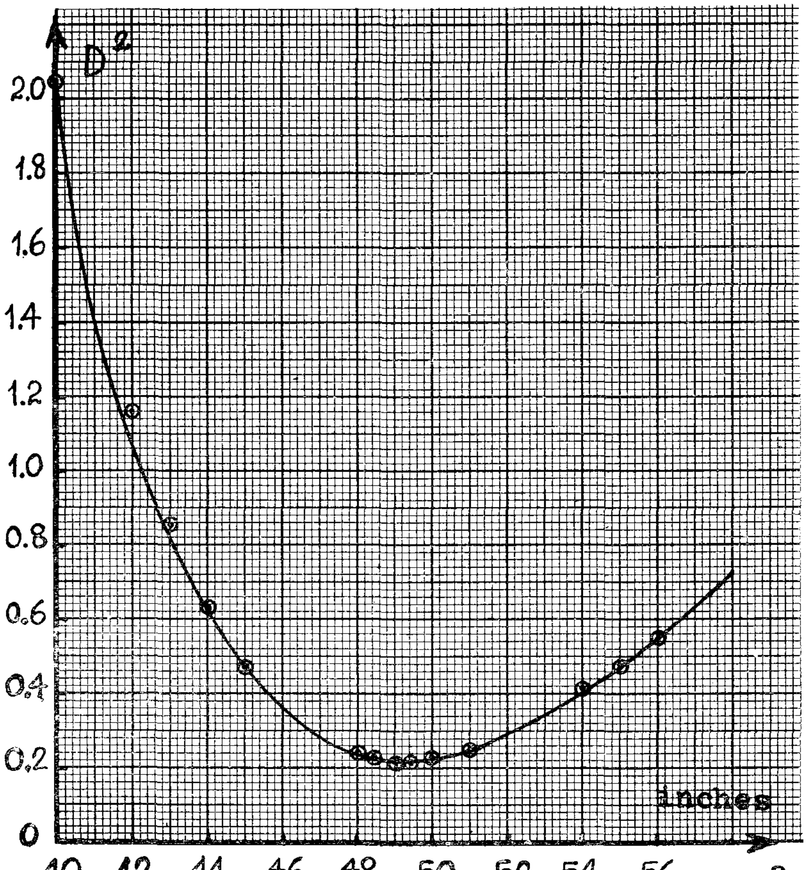

Fig. 4. Optimization of (a)

Plotting the axial flux distribution on a semi-logarithmic paper, the slope at a distance remote from the source is found to be $\gamma_{11} = -0.0998$ in^{-1} $= -0.0393$ cm^{-1} (see Fig. 2). Assuming that the extrapolated length in the y-direction is the same as in the x-direction, i. e., a = b = 49.5 in., then, from equation (1)

$$k^2 = (-0.0998)^2 - 2\left(\frac{\pi}{49.5}\right)^2 = 0.001904 \text{ in}^{-2}.$$

$$\text{The diffusion area} = L^2 = \frac{1}{k^2} = 525.210 \text{ in.}^2$$

$$\text{The diffusion length} = L = 22.92 \text{ in.} = 58.2 \text{ cm.}$$

II

Exponential Stack Without Fuel

The I.C. exponential stack, of which a schematic view is shown in Fig. 5, consists of a pedestal of solid graphite 8′ x 8′ x 2′ (243.84 x 243.84 x 60.96 cm) supported on a machined cast-iron bedplate, Built into the pedestal are four lead blocks, each of which contains a sleeve of metallic beryllium. These are positioned about the nodes of the third harmonics. Slugs of irradiated antimony can be inserted into these beryllium sleeves to generate neutrons of about 30 kev energy. The neutrons are slowed down in the graphite and the top surface of the pedestal acts *as the neutron source* for the graphite lattice - fuel region. This consists of a square lattice of 8″ pitch with fuel chanells of 4 1/4″ diameter. The lattice is made up of blocks 8″ x 8″ x 29″, and

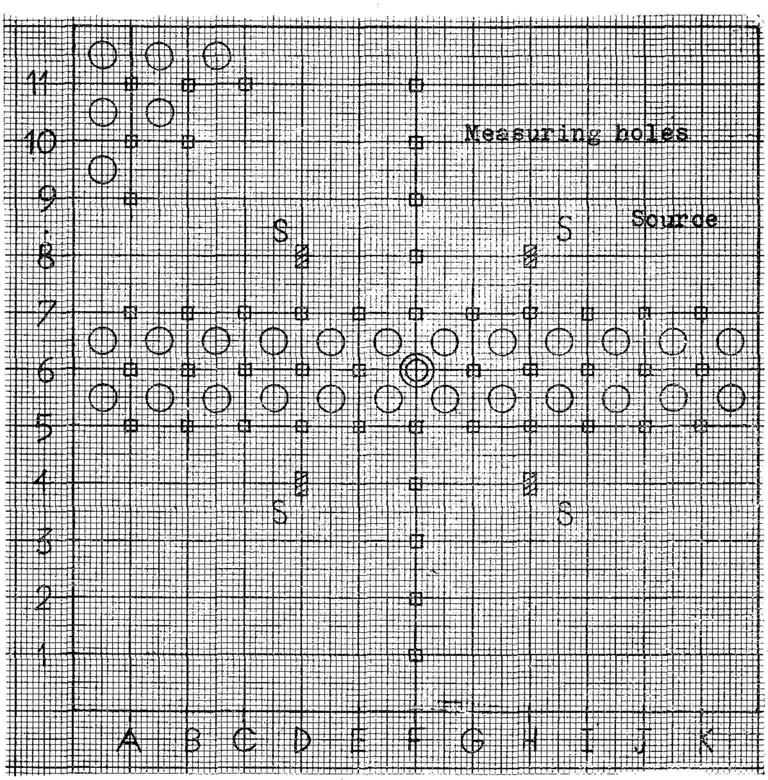

Fig. 5. The top plan view of the Exponential Stack.

the corners of the long sides are cut away, so that when four blocks are placed together there is a 1 1/4″ square hole between them, for flux measurements in the composite lattice. The numbering system used for the measuring holes is shown in Fig. 5, the central measuring hole being F-6.

As in the case of the Diffusion Stack, the positioning rod for locating BF_3 counters consists of a thick walled aluminium tube bored with transvers holes at 2″ intervals. The BF_3 counter is attached to the end of the rod and positioned by means of a spring-loaded dowel which locates in the measuring holes.

Measurements of Neutron Flux

The diffusion length was obtained from measurements of the steady state thermal neutron flux along the three axes of the Exponential Stack. In order to minimize the errors, before taking count rates, we have tested the best operating conditions of the counting equipment and checked the gamma levels around the stack. The background count rates and the fast flux levels are listed in the Table 5. The measured count rates were corrected for dead time and background counts. Taking the total count equal to 10^5, the counting statistics reduced to 0.32 %.

The possible presence of harmonics in the flux distribution was investigated. No significant contribution of harmonics higher than the first could be found.

TABLE 5.

z = Heights above the source	Beckground cps	z	Fast flux cps
0′ 4″	0.188	4 ft	4.122
2′ 0″	0.354	4′ 2″	3.254
3′ 0″	0.360	4′ 4″	3.030
4′ 0″	0.400	4′ 6″	2.832
6′ 0″	0.482	4′ 8″	2.270
		4′ 10″	2.072
		5′ 00″	1.630
		5′ 2″	1.612
		5′ 4″	1.234
		5′ 6″	1.248
		5′ 8″	.....
		5′ 10″	.....
		6′ 00″	.....

Using the exponential assembly as a diffusion stack (unfueled) the flux variations in the x and y directions at heigths 4 ft., 4 ft. 6 ins. and 5 ft above the pedastal have been measured (Figs. 6 and 7). These measurements were made using 11 measuring holes across the stack (F. 1–11 and 6 A-K). The extrapolated dimensions was found by adding calculated extropolation distances to the physical dimensions (a = b = 96 ins) of the stack.

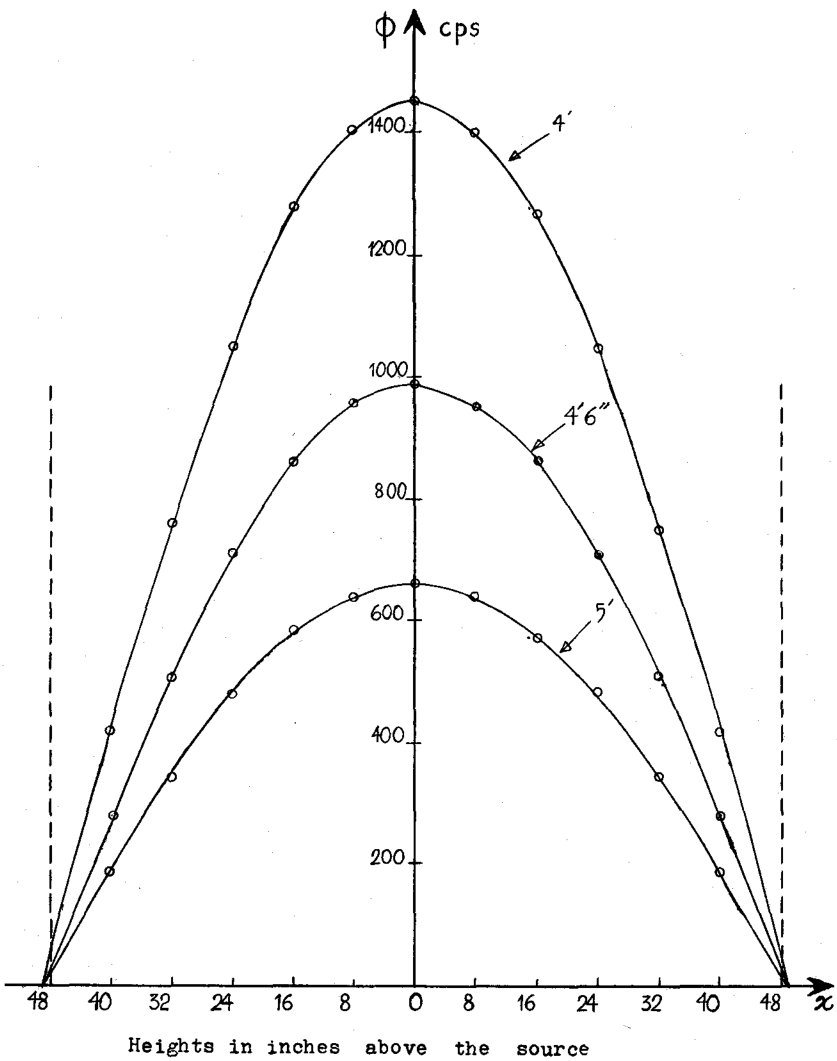

Fig. 6. Flux distributions along the x-axis at the different heights above the source. The physical boundaries of the stack are indicated by vertical dashed lines.

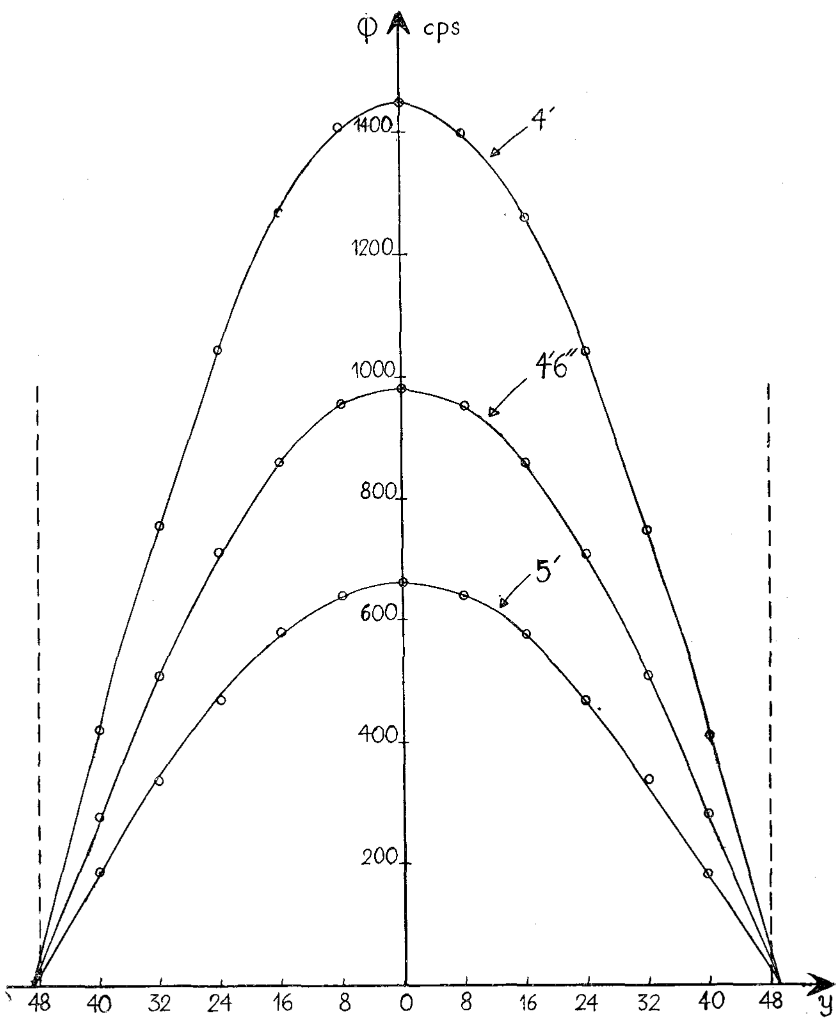

Fig. 7. Flux distributions along the y-axis at the fifferent heights above the source.

The vertical flux distributions along the z - axis in the holes E–6, F–4, F–6 and H–4 have been mesaured and plotted. The average slope was found to be $\gamma_{11} = -0.025874 \text{ cm}^{-1}$ (see Fig. 8).

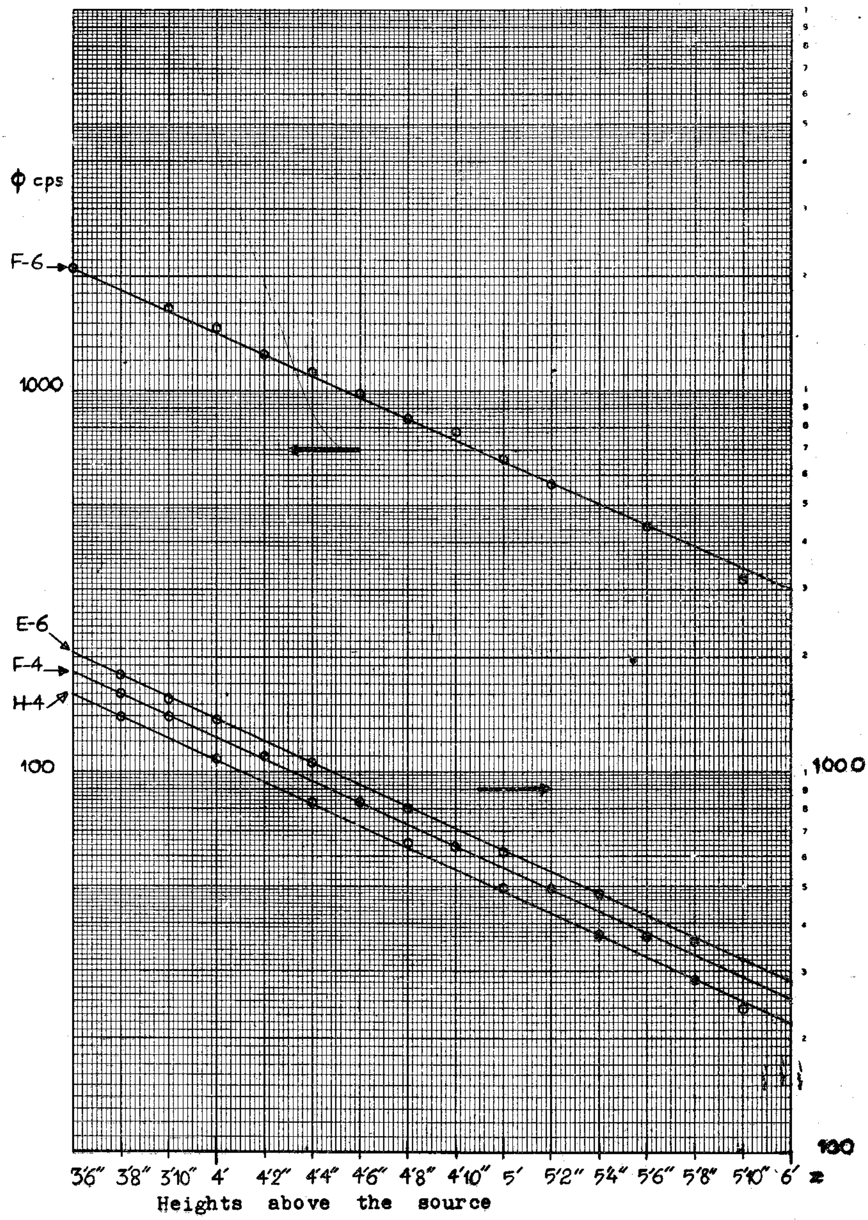

Fig. 8. Axial flux distributions in z direction corresponding to the measuring holes E-6, F-4, F-6 and H-4.

The average linear extrapolation distance, obtained by the graphical method, is

$$d = \frac{4/5'' + 6/5'' + 4.8/5'' + 4/5''}{4} = 0.94 \text{ in.} = 2.39 \text{ cm}$$

and the extropolated dimension of the stack

$$a_{ext.} = b_{ext.} = a_{actual} + 2d = 96'' + 1.88'' = 97.88 \text{ ins.} = 248.62 \text{ cm}$$

and

$$\frac{1}{L^2} = k^2 = \gamma_{11}^2 - 2\left(\frac{\pi}{a}\right)^2 .$$

Therefore the diffusion length in the Exponential Stack is follows:

Measuring holes	Slope γ_{11}	Diffusion length L
E–6	– 0.025828 cm^{-1}	53.62 cm
F–4	– 0.025609 "	54.51 "
F–6	– 0.026308 "	51.79 "
H–4	– 0.025750 "	53.94 "
average	– 0.025874 "	53.45 "

The standard deviation is about $\pm$ 1. A 0.02 % change in γ_{11} gives an approximate chang of 1 cm in L.

The thermal flux measurements fitted by the computer to a cosine function $\left(\cos\frac{\pi x}{a}\right)$ for six runs have given us for the extraplotion distances and the extropolated dimensions of the stack the following values (the least - squares method):

TABLE 6.

Along x-axis		Along y-axis	
extrapolation distance	a total	extropolation distance	b total
2.60 cm	249.04 cm	2.55 cm	248.94 cm
2.30 "	248.44 "	2.50 "	248.84 "
2.30 "	248.44 "	2.55 "	248.94 "
2.40 "	248.64 "	2.60 "	249.04 "
2.45 "	248.74 "	2.65 "	249.14 "
2.35 "	248.54 "	2.65 "	249.14 "
Average	248.64 cm	Average	249.01 cm

Total average a = b = 248.83 cm.

If we take $a = b = 248.83$ cm for the extrapolation dimension of the stack, instead of 248.62 cm, we obtain the value of the diffusion length to be 53.40 cm (corresponding to the average slope $\gamma_{11} = -0.025874 \text{ cm}^{-1}$).

Discussion

In testing the purity of moderators, the thermal diffusion length is the best measure of the purity of the material for reactor use. The diffusion properties of a given material depend on its density. The calculated value must always be confirmed experimentally. Since $\sigma_s = 4.8$ barns and $\sigma_a = 3.2$ mb. for graphite, and its average density obtained by weighing all the individual blocks was found to be $\rho = 1.748 \text{ g/cm}^3$. Thus the calculated diffusion length will be

$$L = \sqrt{\frac{D}{\Sigma_a}} = \frac{A}{\rho \, No \sqrt{\sigma_a \sigma_s (3 - 2/A)}} = 54.57 \text{ cm.}$$

where A is the atomic weight of carbon, ρ its density and No Avogadro's number.

The effects of several assumptions made in the calculation of the diffusion length in the Diffusion Stack must be considered:

a) It was assumed that the flux distribution measured along the x-axis on a plane at the distance of 60 inches from the source was due entirely to thermal neutrons. The epi-cadmium flux was measured at five positions near the source plane (see Table 1) in order to determine the possible contribution of fast neutrons to the total flux. The fast flux is about 5 % of the total. However, this does not provide the proof of the adequate thermalisation.

b) The flux at a plane 60 in. from the source plane was assumed to consist only of the fundamental node corresponding to the condition $m = n = 1$.

c) The same statistical reliablity (1 % standart deviation) was maintained for all measurements of the flux by recording the time taken for at least 10,000 counts to be registered.

d) The intense γ - flux from the Ra - Be source was eliminated from the count by the choice of a suitable bias voltage [6].

Finally, it may be concluded that the final value of the diffusion length (58 cm) - compared with the accurate value of 50.2 cm - is quite resonnable for an experiment of this type.

Although numerious papers have been published on the experimental determination of diffusion lengths, there is, in many instances, little agreement beeween the results of different authors. This is because graphites vary widely in composition and in density.

Acknowledgements. I prepared this work at the Imperial College of Science and Technology, London University, as a OECD fellow. I am indebted to all Nuclear Power Group staff of Mechanical Engineering Department for their help in the preparation of the experiments.

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ÖZET

Termik Nötronların Grafit İçindeki Diffüzyon Uzunluğunun Denel Tâyini

Bu araştırmada grafit içinde (Grafit Difüzyon Bloku ve Eksponensiyel Blok) termik nötron diffüzyon uzunluğu tayin edilerek $L = 58 \pm 2$ cm ve $L = 53.46$ cm (ortalama) bulunmuştur. Ölçmelerde Ra-Be nötron kaynağı kullanılmış ve foil aktifleme tekniği yerine, nötronlar, duyar bir BF_3 sayacı ile detekte edilmiştir. Neticelerin hesap ve analizinde ve özel olarak, ekstrapolasyon uzaklığının göz önüne alınmasında, en küçük kareler metodu tercih edilmiştir. Tecrübelerde kullanılan Grafit Difüzyon Bloku ve Eksponensiyel Blok içinde y, x, z eksenleri boyunca olan nötron akısı değişimleri ölçülmüş ve sonuçlar grafiklerle gösterilmiştir.

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Ankara Üniversitesi Basımevi, 1973