COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES DE L'UNIVERSITÉ D'ANKARA

Série A: Mathématiques, Physique et Astronomie

TOME 23 A ANNÉE 1974

On Cesaro Sums of Divergent Series

by

Ö. ÇAKAR

2

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences
de l'Université d'Ankara

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifiques représentées à la Faculté: Mathématiques pures et appliquées, Astronomie, Physique et Chimie théorique, expérimentale et technique, Géologie, Botanique et Zoologie.

La Revue, à l'exception des tomes I, II, III, comprend trois séries

Série A : Mathématiques, Physique et Astronomie.

Série B : Chimie.

Série C : Sciences naturelles.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adresse: Fen Fakültesi Tabliğler Dergisi, Fen Fakültesi, Ankara, Turquie.

On Cesaro Sums of Divergent Series

Ö. ÇAKAR

Dept. of Mathematics, Faculty of Science
(Ankara University, Ankara)

SUMMARY

Let $\sum_{k=1}^{\infty} a_k$ be an infinite series of real, non-negative numbers and let

$$(\varepsilon) = \{\varepsilon_k\}, \ (k=1,2,\ldots, \ \varepsilon_k = \mp 1)$$

be any sequence of signs.

For a given sequence (ε), we denote the n-th partial sum of the series $\Sigma \, \varepsilon_k$ by

$$s_n(\varepsilon) = \sum_{k=1}^{n} \varepsilon_k \, a_k$$

and the n–th partial C_1 –sum of the series by

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{\nu=1}^{n} s_\nu(\varepsilon) \ .$$

If $\sigma_n(\varepsilon)$ converges then we call

$$\sigma(\varepsilon) = \lim_{n \to \infty} \sigma_n(\varepsilon)$$

a C_1 –attainable point of Σa_k and denote the set of all C_1 –attainable points of Σa_k by SC (a_k).

In this paper we are going to investigate the C_1 –attainable set SC (a_k) of a divergent series Σa_k and give some theorems on that SC $(a_k) = R$ and SC $(a_k) = \varnothing$, where R is the set of real numbers and $\varnothing$ is the empty set.

1. Introduction

It is known that, if a numerical series is conditionally convergent, then it is possible to sum this series to any value by rearranging its terms, [4], [5].

A similar problem has been investigated for divergent series and some interesting results have been obtained by Bagemihl-Erdös, [3]. Also, Erdös-Hanani got some results for the C_1 –attainable set of a divergent series $\Sigma\, a_k$, [1].

In this note we are going to deal with the same type of problems.

2. Notations.

Let $\sum_{k=1}^{\infty} a_k$ be an infinite series of real, non-negative numbers

and let

(2.1) $(\varepsilon) = \{\varepsilon_k\}, \quad (k=1,2,\ldots, \; \varepsilon_k = \mp 1)$

be any sequence of signs.

For a given sequence (ε), we denote the n–th partial sum of the series $\Sigma\, \varepsilon_k a_k$ by

$$s_n(\varepsilon) = \sum_{k=1}^{n} \varepsilon_k a_k$$

and the n–th partial C_1 –sum of the series by

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{\nu=1}^{n} s_\nu(\varepsilon) .$$

If $\sigma_n(\varepsilon)$ converges then we call

$$\sigma(\varepsilon) = \lim_{n\to\infty} \sigma_n(\varepsilon)$$

a C_1 – attainable point of Σa_k and denote the set of all C_1 –attainable points of Σa_k by $SC(a_k)$.

R will denote the set of real numbers and $\varnothing$ will denote the empty set.

3. Theorems For SC $(a_k) = R$.

Let us start giving a theorem which is an immediate consequence of Theorem 1 of Erdös – Hanani [1] and Theorem 3 of Yurtsever, [2] .

T h e o r e m 3.1. Let Σa_k be a series of nonnegative terms havaing a subseries Σa_{n_i} such that

$$\Sigma a_{n_i} = \infty , a_{n_i} \to 0 .$$

If (a_k) is monotone and bounded then SC $(a_k) = R$. (2)

T h e o r e m 3.2. Let $\Sigma a_k = \infty$ be a series of non-negative terms having a subseries Σa_{n_i} such that

$$\Sigma a_{n_i} = \infty , a_{n_i} \to 0 .$$

If, for a definite sequence (ε) ,

a) $$\lim_{k \to \infty} \frac{1}{k+1} \Sigma s_\nu \varepsilon_{\nu+1} \text{ exists,}$$

and

(2) During my stay in University of Lancaster in 1969-71, Prof. I. J. Maddox suggested me that Theorem 3.1. can be imporoved to the following

Theorem 3.1'. Let Σa_k be a series of non-negative terms having a subseries Σa_{n_i} such that

$$\Sigma a_{n_i} = \infty, a_{n_i} \longrightarrow 0 .$$

If $\Sigma |\Delta a_k| = \Sigma |a_k - a_{k+1}| < \infty$, then SC $(a_k) = R$.

Proof. Take $\varepsilon_k = (-1)^k$. Then $\Sigma \varepsilon_k a_k$ is convergent (and se (C,1) summable), for

$$\sum_{k=0}^{n} \varepsilon_k a_k = a_n \left(\sum_{k=0}^{n} \varepsilon_k \right) + \sum_{k=0}^{n-1} \left(\sum_{\mu=0}^{k} \varepsilon_\mu \right) \Delta a_k .$$

Now $\Sigma |\Delta a_k| < \infty$ implies that a_n tends to a limit, l say, as $n \to \infty$. But $a_{n_i} \to 0$ implies that $l = 0$, i.e., $a_n \to 0$.

Hence

$$\sum_{k=0}^{n} \varepsilon_k a_k = o(1) \sum_{k=0}^{n} \varepsilon_k + \sum_{k=0}^{n-1} \left(\sum_{\mu=0}^{k} \varepsilon_\mu \right) \Delta a_k$$

$$= o(1)\, 0\,(1) + \sum_{k=0}^{n-1} 0\,(1)\, \Delta a_k .$$

So the result is immediate.

b) the series $\Sigma s_\nu \, \Delta \, \varepsilon_\nu$ is C_1 – summable, where

$s_\nu = \sum_{\mu=0}^{\nu} a_\mu$, and $\Delta \, \varepsilon_\nu = \varepsilon_\nu - \varepsilon_{\nu+1}$, then SC $(a_k) = R$.

P r o o f. Take the series $\Sigma \, a_k = \infty$ and apply the sequence (ε). According to the Abel partial summation formula, we have

$$(3.1.) \quad \sum_{k=0}^{n} \varepsilon_k \, a_k = \sum_{k=0}^{n} s_k \, \Delta \, \varepsilon_k + s_n \, \varepsilon_{n+1} \quad ,$$

where $s_n = \sum_{\mu=0}^{n} a_\mu$, $s_{-1} = 0$ and $\Delta \, \varepsilon_k = \varepsilon_k - \varepsilon_{k+1}$.

If we put

$$S_j = \sum_{k=0}^{j} \varepsilon_k \, a_k = \sum_{k=0}^{j} s_k \, \Delta \, \varepsilon_k + s_j \, \varepsilon_{j+1} \quad , \quad (j=0,1,2,\ldots),$$

we easily get

$$(3.2) \quad \lim_{j \to \infty} \frac{S_0 + S_1 + \ldots + S_j}{j+1}$$

$$= \lim_{j \to \infty} \frac{1}{j+1} \sum_{k=0}^{j} s_k \, \varepsilon_{k+1} +$$

$$\lim_{j \to \infty} \frac{s \, \Delta \, \varepsilon_0 + \sum_{k=0}^{1} s_k \, \Delta \, \varepsilon_k + \ldots + \sum_{k=0}^{j} s_k \, \Delta \varepsilon_k}{j+1}$$

Since the left-hand side of (3.2) is the C_1 – sum of the series $\Sigma \varepsilon_k a_k$, by Theorem 1 of Erdös-Hanani, [1], the result is straight forward.

T h e o r e m 3.3. Let Σa_k be a series of non-negative terms. If $\Sigma a_k = \infty$ and monotonously $a_k \to 0$, then SC $(a_k) = R$.

P r o o f . Let us write the equality (3.1) in the form of

$$(3.3) \quad \sum_{k=0}^{n} \varepsilon_k \, a_k = \sum_{k=0}^{n} s_k \, \Delta \, a_k + s_n \, a_{n+1} \quad ,$$

where $s_n = \sum_{\mu=0}^{n} \varepsilon_\mu$, $s_{-1} = 0$, and $\Delta a_k = a_k - a_{k+1}$.

Put

$$S_j = \sum_{k=0}^{j} \varepsilon_k a_k .$$

Now, if

a') $\lim_{j \to \infty} \frac{1}{j+1} \sum_{k=0}^{j} s_k a_{k+1}$ exists ,

and

b') the series $\Sigma s_k \Delta a_k$ is C_1 – summable,

then

$$(3.4) \lim_{j \to \infty} \frac{S_0 + S_1 + \ldots + S_j}{j+1}$$

exists.

So, we must show, under the given hyphothesis, that conditions a') and b') are satisfied.

Choose $\{\varepsilon_k\} = (-1)^k$, $(k = 0,1,2, \ldots)$. Then the partial sums s_k 's are bounded, and since $a_k \longrightarrow 0$ monotonously, the series $\Sigma s_k \Delta a_k$ is convergent. (One can easily see that it is absolutely convergent, in fact.) So, condition b') is satisfied. Namely $SC(a_k) \neq \varnothing$. Condition a') is also satisfied because of the Cauchy's Theorem. The limit exists and equal to zero, (Arithmetic Means), [4], [5].

Therefore, according to Theorem 1 of Erdös-Hanani, [1], $SC(a_k) = R$.

4. A Problem of Erdös - Hanani

In this section, we are going to consider a problem due to Erdös-Hanani, (Problem 1, [1]), and show that the best possible result is $C = 1$.

T h e o r e m 4.1. Let Σa_k be a series of nonnegative terms satisfyying $\Sigma a_k = \infty$. If there exists an η_0 with the property that to each η in $0 < \eta \leq \eta_0$ there corresponds an

(4.1) $n_0 = n_0 (\eta)$

such that for every $n > n_0$,

(4.2) $$\sum_{i=1}^{[\eta n / a_n]} a_{n+i} > a_n + \eta ,$$

then $SC(a_k) = R$.

P r o o f. Let σ be any real number. Then, we are going to construct a sequence (2.1) such that

$$\lim_{n \to \infty} \sigma_n (\varepsilon) = \sigma .$$

According to (4.1), for every $\eta = 2^{-i}$, ($i = i_0, i_0 + 1, \ldots$) there exists a number

(4.3) $n_i = n_i (2^{-i})$

such that for every $n > n_i$, (4.2) is satisfied, with $\eta = 2^{-i}$.

Now, choose ε_j arbitrarly for $j = 1, 2, \ldots , n_{i_0}-1$. Then, let us put $n_i = j$ and suppose that

$$\sigma_{j-1} (\varepsilon) = \frac{s_1 (\varepsilon) + \ldots + s_{j-1} (\varepsilon)}{j - 1} \leq \sigma .$$

If $s_{j-1} (\varepsilon) \leq \sigma + 2^{-i}$ we take $\varepsilon_j = +1$ to make $\sigma_j (\varepsilon)$ bigger than σ. But, if $s_{j-1} (\varepsilon) > \sigma + 2^{-i}$, then we choose ε_j so as to make $s_j(\varepsilon)$ as small as possible but not less than $\sigma + 2^{-i}$. Continuing this way, suppose that the final partial sum we reached is $s_{k_1} (\varepsilon)$ and let

$$\sigma_{k_1} (\varepsilon) = \frac{s_1 (\varepsilon) + s_2 (\varepsilon) + \ldots + s_{k_1} (\varepsilon)}{k_1} .$$

Now the means must start decreasing and be $\leq \sigma$. Therefore the partial sums must decrease. Then if

$s_{k_1} (\varepsilon) \geq \sigma - 2^{-i}$, we put $\varepsilon_{k_1+1} = -1$;
but, if $s_{k_1} (\varepsilon) < \sigma - 2^{-i}$, we choose $\varepsilon_{k_1} + 1$ so as to make the left hand side as large as possible but not greater than $\sigma - 2^{-i}$.

Accordingly, we get

$$\sigma_{j_2}(\varepsilon) = \frac{s_1(\varepsilon) + \ldots + s_{k_1}(\varepsilon) + \ldots + s_{j_2}(\varepsilon)}{j_2} \leq \sigma .$$

Then, it follows that the sequence $(\sigma_\nu(\varepsilon))$ attains alternately minimas $\sigma_{j_h}(\varepsilon)$, $(h = 1, 2, \ldots)$ and maximas $\sigma_{k_h}(\varepsilon)$, $(h=1,2,\ldots)$, with $j_1 < k_1 < j_2 < k_2 < \ldots.$ such that

$\sigma_{j_h}(\varepsilon) \leq \sigma$ and $\sigma_{k_h}(\varepsilon) > \sigma$, $(h = 1, 2, \ldots)$.
Therefore the sequence $(\sigma_\upsilon(\varepsilon))$ is monotonically increasing for $j_h \leq \upsilon \leq k_h$ and monotonically decreasing for $k_h \leq \upsilon \leq j_{h+1}$.

To prove the theorem, it is enough to show that the difference between σ and maxima $\sigma_{k_h}(\varepsilon)$ (or, σ and minima $\sigma_{j_h}(\varepsilon)$) tends to zero as $n \to \infty$. So we must show the existence of a number j_0 such that for every $k_h > j_0$

(4.4) $\quad 0 < \sigma_{k_h}(\varepsilon) - \sigma < \eta$

holds.

Let i be an integer such that

(4.5) $\quad 2^{-i} < \eta/6$

and let n_i be the corresponding index fixed by (4.1) :

$n_i = n_i(2^{-i})$.

Further, let h be an integer such that $k_{h-1} > n_i$ and m the greatest index providing $j_h < m \leq k_h$ such that $\varepsilon_m = 1$.

According to our construction, we write

(4.6) $\sigma_{m-1}(\varepsilon) \leq \sigma$.

And if

(4.7) $s_{m-1}(\varepsilon) \leq \sigma + 2^{-i}$

then, for $m \leq j \leq k_h$, we get

(4.8) $\quad \sigma < s_j(\varepsilon) < \sigma + 2^{-i} + 2\, a_m$.

Also, in the case

(4.9) $\quad s_{m-1}(\varepsilon) > \sigma + 2^{-i}$

the relation (4.8) is still valid.

Now, if $s_{m-1}(\varepsilon) > \sigma + 2^{-i}$, then we are going to suppose that

(4.10) $$s_{m-1}(\varepsilon) - (\sigma + 2^{-i}) < 2^{-i}.$$

So, under this assumption, we can put the following

Lemma.

(4.11) $$\sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m .$$

Proof.

1°) Let $s_{m-1}(\varepsilon) \leq \sigma + 2^{-i}$. Since

$$\sigma < s_m(\varepsilon) \leq \sigma + 2^{-i} + a_m ,$$

by (4.8), we can write

$$\sigma < s_m(\varepsilon) - \sum_{j=m+1}^{k_h} a_j \leq \sigma + 2^{-i} + a_m .$$

Therefore, we get

$$\sigma + \sum_{j=m+1}^{k_h} a_j < s_m(\varepsilon) \leq \sigma + 2^{-i} + a_m$$

and $$\sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m .$$

2°) Let $s_{m-1}(\varepsilon) > \sigma + 2^{-i}$. Then

$$s_{m-1}(\varepsilon) = \sigma + 2^{-i} + \alpha ,$$

where $0 < \alpha < 2^{-i}$. Therefore, we get

$$s_{m-1}(\varepsilon) + a_m > \sigma + 2^{-i}$$

$$s_{m-1}(\varepsilon) + a_m - \sum_{j=m+1}^{k_h} a_j > \sigma + 2^{-i}$$

$$\sigma + 2^{-i} + \alpha + a_m - \sum_{j=m+1}^{k_h} a_j > \sigma + 2^{-i}$$

$$\sum_{j=m+1}^{k_h} a_j < \alpha + a_m$$

or

$$\sum_{j=m+1}^{k_h} a_j < 2^{-i} + a_m .$$

This completes the proof of the Lemma.

Now, by the definition of $\sigma_{k_h}(\varepsilon)$, we have

$$\sigma_{k_h}(\varepsilon) = \frac{1}{k_h} \left[(m-1)\, \sigma_{m-1} + \sum_{j=m}^{k_h} s_j(\varepsilon) \right]$$

and by (4.6) and (4.8)

$$(4.12)\quad \sigma_k(\varepsilon) < \sigma + \frac{1}{k_h} (2^{-i} + 2\, a_m)(k_h - m + 1) .$$

If $a_m \leq 2^{-i}$, then we easily get

$$\sigma_{k_h}(\varepsilon) - \sigma < \eta/2 .$$

If $a_m > 2^{-i}$, then obviously $m > n_i$. So. (4.1) and (4.11) give

$$k_h - m < 2^{-i} . \frac{m}{a_m}$$

and, we also have

$$1 < 2^{-i} . \frac{m}{a_m} .$$

Therefore, from (4.12), weget

$$\sigma_{k_h}(\varepsilon) - \sigma < \frac{1}{k_h} (2^{-i} + 2a_m)\ (k_h - m + 1)$$

$$\sigma_{k_h}(\varepsilon) - \sigma < \frac{1}{k_h} . 3\ a_m . 2 . 2^{-i} . \frac{m}{a_m}$$

which implies, by (4.5), that

$$\sigma_{k_h}(\varepsilon) - \sigma < \eta .$$

In a similar way, we can show that the difference between σ and minima $\sigma_{j_h}(\varepsilon)$ tends to zero as $h \to \infty$.

5. A Theorem For $SC(a_k) = \varnothing$.

In this chapter we are going to prove a theorem which gives a sufficient condition for $SC(a_k) = \varnothing$. This teorem will be based on Cauchy's general convergence principle. It is known that, if the sequence

$$\sigma_n(\varepsilon) = \frac{s_1(\varepsilon) + s_2(\varepsilon) + \dots + s_n(\varepsilon)}{n}$$

where $s_n(\varepsilon) = \sum_{\upsilon=1}^{n} \varepsilon_\nu a_\nu,$ is divergent, then the series can not be C_1 — summable. So, what we need is having that, for at least one $k \geq 1$ and fon each n

$$|\sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon)| > \eta$$

where $\eta > 0$.

Take $1 \leq k \leq n$, and write

$$|\sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon)| =$$

$$= \frac{n[s_{n+1}(\varepsilon) + \dots + s_{n+k}(\varepsilon)] - k[s_1(\varepsilon) + \dots + s_n(\varepsilon)]}{n(n+k)}$$

Using

$$s_n(\varepsilon) = \sum_{\nu=1}^{n} \varepsilon_\nu a_\nu$$

we get

$$|\sigma_{n+k}(\varepsilon) - \sigma_n(\varepsilon)| =$$

$$= \left| \frac{k}{n(n+k)} \sum_{\nu=2}^{n+1} (\nu-1)\varepsilon_\nu a_\nu + \frac{1}{n+k} \sum_{\upsilon=1}^{k-1} (k-\upsilon)\varepsilon_{n+1+\nu} a_{n+1+\nu} \right|$$

$$= \left| \frac{k}{n(n+k)} \sum_{\nu=0}^{n+1} (n-\nu) \varepsilon_{n+1-\nu} a_{n+1-\nu} \right.$$

$$+ \frac{1}{n+k} \sum_{\nu=1}^{k-1} (k-\nu)\, \varepsilon_{n+1+\nu}\, a_{n+1+\nu} \quad \Big|$$

$$= \frac{k}{n+k} \Big| \varepsilon_{n+1} a_{n+1} + \sum_{\nu=1}^{n-1} \frac{n-\nu}{n}\, \varepsilon_{n+1-\nu}\, a_{n+1-\nu}$$

$$+ \sum_{\nu=1}^{k-1} \frac{k-\nu}{k}\, \varepsilon_{n+1+\nu}\, a_{n+1+\nu} \quad \Big|$$

$$= \frac{k}{n+k} \Big| \varepsilon_{n+1} a_{n+1} + \sum_{\nu=1}^{k-1} \frac{n-\nu}{n}\, \varepsilon_{n+1-\nu}\, a_{n+1-\nu}$$

$$+ \sum_{\nu=k}^{n-1} \frac{n-\nu}{n}\, \varepsilon_{n+1-\nu}\, a_{n+1-\nu}$$

$$+ \sum_{\nu=k}^{k-1} \frac{k-\nu}{k}\, \varepsilon_{n+1+\nu}\, a_{n+1+\nu} \quad \Big|$$

$$\geq \frac{k}{n+k} \Big[a_{n+1} - \sum_{\nu=1}^{k-1} \frac{n-\nu}{n}\, a_{n+1-\nu} - \sum_{\nu=k}^{n-1} \frac{n-\nu}{n}\, a_{n+1-\nu}$$

$$- \sum_{\nu=1}^{k-1} \frac{k-\nu}{k}\, a_{n+1+\nu} \Big]$$

$$\geq \frac{k}{n+k} \Big[a_{n+1} - \sum_{|\nu|=1}^{k-1} a_{n+1+\nu} - \sum_{\nu=k}^{n-1} a_{n+1-\nu} \Big]$$

$$> \eta \,.$$

So, we can express the following

T h e o r e m 5.1. Let $\Sigma\, a_k$ be a series of nonnegative terms. If there exists a number k $(1 \leq k \leq n)$ and an $\eta > 0$ such that for every n satisfying

$$a_{n+1} - \sum_{|\upsilon|=1}^{k-1} a_{n+1+\nu} > \left(1 + \binom{n}{k}\right) \eta + \sum_{\nu=k}^{n-1} a_{n+1-\nu}$$

REFERENCES

[1] Erdös, P. – Hanani, H., On C_1 – summability of series, Michigan Math. J., 9, (1963), 1–14,

[2] Yurtsever, B., Über die C-Summerbarkeit der unendlichen Reihen, Communication, V, (1953), 1–11,

[3] Bagemihl, F. –Erdös, P., Rearrangements of C_1 –summable series, Acta Math., 92, (1954), 35–53,

[4] Knopp, K., Theory and Applications of Infinite Series, Blackie & Son Limited, 1964,

[5] Hardy, G. H., Divergent Series, Oxford, 1949.

ÖZET

$\sum_{k=1}^{\infty} a_k$ reel ve negatif olmayan terimli bir nümerik sonsuz seri ve

$$(\varepsilon) = \{\varepsilon_k\}, \ (k = 1, 2, \ldots, \ \{\varepsilon_k\} = \pm 1)$$

herhangi bir işaret dizisi olsun.

Verilen bir (ε) dizisi için $\Sigma \varepsilon_k a_k$ serisinin n' inci kısmi toplamını

$$s_n(\varepsilon) = \sum_{k=1}^{n} \varepsilon_k a_k$$

ve n'inci kısmi C_1 –toplamını

$$\sigma_n(\varepsilon) = \frac{1}{n} \sum_{\nu=1}^{n} s_\nu(\varepsilon)$$

ile belirttik ve $\sigma_n(\varepsilon)$'un yakınsak olması halinde

$$\sigma(\varepsilon) = \lim_{n \to \infty} \sigma_n(\varepsilon)$$

'a Σa_k serisinin bir C_1 –erişilir noktası adını verdik. Σa_k nın bütün C_1 –erişilebilir noktalar cümlesini SC (a_k) ile göşterdik.

Bu araştırmamızda ise ıraksak bir Σa_k serisinin bütün C_1 –erişilir noktaları cümlesi olan SC (a_k) cümlesini ele alıp SC $(a_k) = R$ ve SC $(a_k) = \varnothing$ olması hakkında bazı teoremler verdik, burada R reel sayılar cümlesini ve $\varnothing$ ise boş cümleyi ifade etmektedir.

Prix de l'abonnement annuel

Turquie : 15 TL; Étranger: 30 TL.

Prix de ce numéro : 5 TL (pour la vente en Turquie).

Prière de s'adresser pour l'abonnement à : Fen Fakültesi

Dekanlığı Ankara, Turquie.

Ankara Üniversitesi Basımevi–1974