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Matrix Transformations And Generalized Almost Convergence II

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Matrix Transformations And Generalized Almost Convergence II

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ABSTRACT

The purpose of this paper is to investigate some more classes of matrices which will fill up a gap in the existing literature. We have already characterized the $(c_0(p), F_0\mathcal{B}(p))$, $(l(p), F\mathcal{B})$–, and $(M_0(p), F_{\mathcal{B}})$– matrices. In the present paper author characterizes $(w(p), F_{\mathcal{B}})$ – $(w_p, F_{\mathcal{B}})$ –, $(c(p), F_{\mathcal{B}})$–$(l_\infty(p), F_{\mathcal{B}})$ –, and $(\hat{c}(p), F_{\mathcal{B}})$ – matrices.

1. INTRODUCTION

Let l_∞, c and c_0 be the Banach spaces of bounded, convergent and null sequences $x = \{x_k\}$ with the usual norm $\| x \| = \sup_k | x_k |$. A sequence $x \in l_\infty$ is almost convergent [1] if all Banach limits of x coincide. Let $\hat{c}$ denotes the space of almost convergent sequences. If p_k is real such that $p_k > 0$ and $\sup p_k < \infty$, we define (see Maddox [3], Simons [8] and Nanda [6])

$$w(p) = \{x: n^{-1} \sum_{k=1}^{n} |x_k - l|^{p_k} \longrightarrow o \text{ for some } l\}$$

$$l_\infty(p) = \{x: \sup_k | x_k |^{p_k} < \infty\},$$

$$c(p) = \{x: | x_k - l |^{p_k} \longrightarrow o \text{ for some } l\},$$

and

$$\hat{c}(p) = \{x: \lim_k | t_{k,i}(x - l e) |^{p_k} = o \text{ for some } l, \text{ uniformly in } i\}$$

where

$$t_{k,i}(x) = \frac{1}{k+1}\sum_{m=i}^{i+k} x_m$$

when $p_k = p \;\forall\, k$, we have $w(p) = w_p$, $l_\infty\,(p) = l_\infty$,

$c(p) = c$ and $\overset{*}{c}\,(p) = \hat{c}$ respectively.

Quite recently M. Stieglitz [9] generalized almost convergence by defining $F_{\mathcal{B}}$ – converengence in the following manner: Given a matrix sequence $\mathcal{B} = (B_i)$ with $B_i = (b_{nk}\,(i))$, the sequence $x \in l_\infty$ is $F_{\mathcal{B}}$ – convergent to the value Lim $\mathcal{B}$ x, if

$$\lim_n (B_i\, x)_n = \lim_n \sum_{k=0}^{\infty} b_{nk}\,(i)\, x_k = \lim \mathcal{B}\, x \text{ (uniformly in i)}$$

holds. The space $F_{\mathcal{B}}$ of $F_{\mathcal{B}}$ – convergent sequences depends on the fixed choosen matrix $\mathcal{B} = (B_i)$, in case $\mathcal{B}_o = (I)$ it is equal to c and in case $\mathcal{B}_1 = (B_i^{\,(1)})$ it is equal to $\hat{c}$.

We have already examined the classes of $(c_o\,(p),\, F_{o\mathcal{B}}(p))$ –, $(l\,(p),\, F_{\mathcal{B}})$ –, and $(M_o\,(p),\, F_{\mathcal{B}})$ – matrices (see [5]). In this paper, Theorems 2.1 and 2.2 generalize the results of Lascarides and Maddox [2] and Nanda [7]. In Theorems 3.1, 3.2 and 3.3 we determine the matrices $(c\,(p),\, F_{\mathcal{B}})$, $(l_\infty\,(p),\, F_{\mathcal{B}})$ and $(\hat{c}\,(p),\, F_{\mathcal{B}})$ which generalize the results of Stieglitz [9].

2. *We prove the following Theorems*

Theorem 2.1. Let $o < p_k \le 1$, then $A \in (w\,(p),\; F_{\mathcal{B}})$, if and only if

(i) There axist $B > 1$ such that

$$Q_i = \sup_n \sum_{r=o}^{\infty} \max_r\, (2^r\, B^{-1})^{1/p_k}\; |\, c\,(n, k, i)\, | < \infty \;\; (\forall i)$$

(ii) $\lim_n c\,(n, k, i) = \alpha_k$ uniformly in i, k fixed,

(iii) $\lim_n \sum_k c\,(n, k, i) = \alpha$ uniformly in i.

where

$$c\,(n, k, i) = \sum_j b_{nj}\,(i)\, a_{jk}$$

Proof. Necessity. Suppose that $A \in (w(p), F_{\mathcal{B}})$. Since e_k and e are in w (p), (ii) and (iii) must hold. where

$e_k = \} o, o, \ldots, o, 1, o, o, \ldots \}$ and $e = \{1, 1, 1, \ldots\}$.

Now $\sum_k c(n, k, i)\, x_k$ converges for each n and $x \in w(p)$.

Therefore $(c(n, k, i))_k \in w(p)^+$ and

$$\sum_{r=0}^{\infty} \max_r (2^r B^{-1})^{1/p_k} \mid c(n, k, i) \mid < \infty,$$

for each n (see Lascarides and Maddox [2]).

Further, denote $\sigma_{n,i}(x) = T_{n,i}(A x) = \Sigma_k\, c(n, k, i)\, x_k$,

then $\{\sigma_{n,i}\}$ is a sequence of continuous linear functionals on w (p) such that $\lim_n T_{n,i}(A x)$ exists. Therefore by Banach - Steinhaus Theorem [4], (i) holds.

Sufficiency. Suppose that the conditions (i) — (iii) hold. Then $(c(n, k, i))$ and (α_k) are in $w(p)^+$ (see [2]).

Therefore the series $\sum_k c(n,k,i) x_k$ and $\sum_k \alpha_k x_k$ converge for each n and $x \in w(p)$. Put

$f(n, k, i) = c(n, k, i) - \alpha_k$.

Therefore

$$\sum_k c(n, k, i)\, x_k = \sum_k \alpha_k x_k + l \sum_k f(n, k, i) + \sum_k f(n, k, i)(x_k - l)$$

where $l = \lim x_k$. By (ii) we have

$$\lim_n \sum_{k \geq k_o} f(n, k, i)(x_k - l) = 0.$$

Also since

$$\sup_n \sum_{r=o}^{\infty} \max_r (2^r B^{-1})^{1/p_k} \mid f(n, k, i) \mid \leq 2\, Q_1,$$

$$\lim_n \sum_{k \geq k_o} |f(n, k, i) \mid x_k - l \mid = 0.$$

Hence

$$\lim_{n} \sum_{k} c(n, k, i)\, x_k = l\,\alpha + \sum_{k} \alpha_k (x_k - l)$$

and therefore proof is complete

Theorem 2.2 (a). Let $1 \leq p < \infty$, then $A \in (w_p, F_{\mathcal{B}})$ iff

(i) $M = \sup_{n} \sum_{r=o}^{\infty} 2^{r/p}\, T_r^{\,p}(n, i) < \infty$, $(\forall\, i)$

(ii) $\lim_{n} c(n, k, i) = \alpha_k$ uniformly in i, k fixed

(iii) $\lim_{n} \sum_{k} c(n, k, i) = \alpha$ uniformly in i.

where

$$T_r^{\,p}(n, i) = (\sum_{r} |\, c(n, k, i)\, |^q)^{1/q}, \; (p^{-1} + q^{-1} = 1).$$

(the summation is taken over k with $2^r \leq k < 2^{r+1}$).

(b). Let $0 < p < \infty$. Then $A \in (w_p, F_{\mathcal{B}})_{reg}$ if and only if conditions (i), (ii) with $\alpha_k = 0$ and (iii) with $\alpha = 1$ hold.

Proof (a). *Necessity*. Suppose that $A \in (w_p, F_{\mathcal{B}})$. Since e_k and e are in w_p, therefore, (ii) and (iii) must hold. Now define for each n and $r \geq 0$, $g_{r,n}(x) = \sum c(n, k, i)\, x_k$. Sequence $\{g_{r,n}\}$ is of continuous linear functional in w_p

Now

$$|\, g_{r,n}(x)\, | \leq (\sum_{r} |c(n, k, i)\, |^q)^{1/q} (\sum_{r} |\, x_k\, |^P)^{1/P}$$

$$\leq 2^{r/P}\, T_r^{\,P}(n, i)\, \| x \|$$

and

$$\lim_{n} \sum_{r=o}^{1} g_{r,n}(x) = T_{n,i}(Ax) < \infty$$

Therefore by Banach – Steinhaus Theorem there exists K such that

$$|\, T_{n,i}(A\,x)\, | \leq K\, \| x \|$$

Since l is arbitrary and if we define $x \in w_p$ as in Maddox ([4], Theorem 7) we have

$$\sum_{r=o}^{\infty} 2^{r/P}\, T_r^{\,P}(n, i) \leq K$$

Therefore by the same argument as in Theorem (2.1) we see that (i) holds.

Sufficiency. Let us suppose that the conditions (i) – (iii) be satified and $x \in w_p$. Since

$$| T_{n,i} (A x) | \leq \sum_{r=o}^{\infty} \sum_{r} | c(n, k, i) x_k |$$

$$\leq \sum_{r=o}^{\infty} (\sum_{r} | c(n, k, i) |^q)^{1/q} (\sum_{r} | x_k |^p)^{1/p}$$

$$\leq M \| x \| .$$

Therefore $T_{n,i} (A x)$ is absolutely and uniformly convergent for each n. Since

$$\sum_{r=o}^{\infty} 2^{r/P} (\sum_{r} |\alpha_k|^q)^{1/q} < \infty \text{ and } \sum_{r} \alpha_k x_k < \infty .$$

Therefore as in Theorem (2.1), $A \in (w_p, F_{\mathcal{B}})$. Which completes the proof,

Proof of (b) is constructed from the proof of (a).

3. *Some further Results*

Theorem 3.1 (a). $A \in (c(p), F_{\mathcal{B}})$ if and only if

(i) There exists an integer $B > 1$ such that

$$G_i = \sup_n \sum_k | c(n, k, i) | \bar{B}^{\frac{1}{p_k}} < \infty , \qquad (\forall i)$$

(ii) $\lim_n c(n, k, i) = \alpha_k$, uniformly in i, k fixed

(iii) $\lim_n \sum_k c(n, k, i) = \alpha$, uniformly in i,

where

$c(n, k, i) = \sum_j b_{nj}(i) a_{jk}$

(b) $A \in (c_o(p), F_{\mathcal{B}})$ if and only if conditions (i) and (ii) of Theorem (a) holds.

(c) $A \in (c(p), F_{\mathcal{B}})_{reg}$ if and only if conditions (i), (ii) with $\alpha_k = o$ and (iii) with $\alpha = 1$ hold.

Proof (a) *Necessity*. Let $A \in (c(p), F_{\mathcal{B}})$. Define $e = (1\ 1, \ldots)$ and $e_k = (o, o, o, 1, o, \ldots)$. Since e and e_k are in c (p), (ii) and (iii) must hold. Put $\sigma_{ni}(x) = T_{n,i}(A x) = \Sigma_k\, c(n, k, i)\, x_k$. Since $(c(p), F_{\mathcal{B}}) \subset (c_o(p), F_{\mathcal{B}})$, $\{\sigma_{ni}\}$ is a sequence of continuous linear functionals on $c_o(p)$, such that $\lim_n \sigma_{ni}(x)$ exists uniformly in i. Therefore by uniform boundedness principle for $o < \delta < 1$, there exists a constant K such that $\sigma_{ni}(x) \leq K$ for each n and $x \in c(p)$. Let us define $x^r = (x_k^{\,r}) \in c(p)$ by the following:

$$x^r_{\ k} = \begin{cases} \delta^{K/p_k} \operatorname{sgn}(c(n, k, i), & 0 \leq k \leq r; \\ 0 & , r < k. \end{cases}$$

Then, it follows that

$$\sum_{k=0}^{r} |\, c(n, k, i) \,|\, B^{-1/p_k} \leq K$$

for each n and r, where $B = \delta^{-K}$. Therefore (i) holds.

Sufficiency. Suppose that the conditions (i) —— (iii) hold and $x \in c(p)$. Then there exists l such that

$|\, x_k - l\,|^{p_k} \to 0$. Hence for a given $\in > 0$,

there exists an integer k_o such that $\forall\ k_o > k$

$$|\, x_k - l\,|^{p_k/M} \leq \frac{\in}{B(2\,G_i + 1)} < 1$$

and therefore for $k_o > k$

$$B^{1/p_k}\,|\, x_k - l\,| < B^{M/p_k}\,|\, x_k - l\,|$$
$$< \left(\frac{\in}{2\,G_i + 1}\right)^{M/p_k}$$
$$< \frac{\in}{2\,G_i + 1}\ .$$

By (i) and (ii) we have

$$\sum_k \mid c(n, k, i) - \alpha_k \mid B^{-1/p_k} < 2 G_i .$$

Hence

$$\sum_{k > k_o} \mid (c(n, k, i) - \alpha_k)(x_k - l) \mid < \epsilon .$$

Also

$$\lim_n \sum_{k \leq k_o} \mid (c(n, k, i) - \alpha_k)(x_k - l \mid = 0$$

uniformly in i. Therefore combining the above facts we have

$$\lim_n \sum_k c(n, k, i) x_k = l \alpha + \sum_k \alpha_k (x_k - l)$$

uniformly in i. This proves that $A \in (c(p), F_{\mathcal{B}})$.

(b) Since $x \in c_o(p) \Rightarrow l = 0$, therefore the proof is immediate.

(c) First we observe that $\alpha_k = 0$ and $\alpha = 1$, proof follows immediately.

Theorem 3.2. (a). $A \in (l_\infty(p), F_{\mathcal{B}})$ if and only if

(i) $\lim_n c(n, k, i) = \alpha_k$ uniformly in i, k fixed.

(ii) $\sup_n \sum_k \mid c(n, k, i) < \infty \qquad (\forall i)$

(iii) There exists an integer $N > 1$ such that

$\lim_n \sum_k \mid c(n, k, i) - \alpha_k \mid N^{1/p_k} = 0$ uniformly in i.

(b) $A \in (l_\infty(p), F_{o\mathcal{B}})$ iff (i) condition (ii) of Theorem (a) holds, (ii) $\lim_n \sum_k \mid c(n, k, i) \mid N^{1/p_k} = 0$ uniformly in i.

Proof (a). *Necessity*. Suppose that $A \in (l_\infty(p), F_{\mathcal{B}})$. since $e_k \in l_\infty(p)$, (i) must hold. Since $(l_\infty(p), F_{\mathcal{B}})$ $(c, F_{\mathcal{B}})$ (ii) holds. If (iii) is not true then the matrix

$C = (c_{nk}) = (a_{nk} N^{1/p_k}) \notin (l_\infty, F_{\mathcal{B}})$ for some

integer $N > 1$. So that there exists $x \in l_\infty$

such that $B x \notin F_{\mathcal{B}}$. Now $y = (y_k) = (N^{1/p_k} x_k) \in l_\infty (p)$, but $Ay = C x \notin F_{\mathcal{B}}$. This contradicts the fact that $A \in (l_\infty (p), F_{\mathcal{B}})$. Hence (iii) is true.

Suffciency. Suppose that the conditions (i) -- (iii) hold. Choose an integer $N > \max (1, \sup_k | x_k |^{p_k})$. By (ii)

$$| \sum_k (c (n, k, i) - \alpha_k) x_k | < \sum_k | c (n, k, i) - \alpha_k | N^{1/p_k} .$$

By (i) and (iii) we have

$$\lim \sum_k c (n, k, i) x_k = \sum_k \alpha_k x_k$$

uniformly in i. Hence proof is complete.
proof of (b) is obvious if we take $\alpha_k = 0$.

Theorem 3.3 (a) $A \in (\hat{c} (p), F_{\mathcal{B}})$ if and only if

(i) conditions (i), (ii) and (iii) of Theorem (3.1) hold.

(ii) $\lim_n \sum_k | \sum_j b_{nj}^{(i)} (a_{jk} - a_{j,k+1}) - (\alpha_k - \alpha_{k+1}) | B^{1/p_k} = 0$

(b) $A \in (\hat{c} (p), F_{\mathcal{B}})_{reg}$ if and only if conduitions (i), (ii) with $\alpha_k = 0$, (iii) with $\alpha = 1$ and (ii) of (a) hold.

Proof (a) *Necessity*. Let $A \in (\hat{c} (p), F_{\mathcal{B}})$. Now by cirture of the fact $(N(B_i) < \infty$, $A: c (p) \longrightarrow F_{\mathcal{B}}$ and Theorem (3.1) follows all the conditions of (i). To prove condition (ii), let us define a matrix $G = (g_{nk})$ with

$$g_{nk} = \begin{cases} 1, & n = k, \\ -1 & n = k + 1, \\ 0 & \text{otherwise}, \end{cases}$$

and the matrix $\bar{B}_k$ with $\bar{B}_k = (b_{kj}^{(1)} (i))$, $0 \leq i,j < \infty$ we see that it is easy to prove the following conditions:

(iii) $$G : l_\infty (p) \longrightarrow \hat{c}_o (p)$$

(iv) $$G (\overset{-1}{G} x) = x \qquad \text{with} \quad \overset{-1}{G} = (g_{nk}^{(-1)})$$

$$g_{nk}^{(-1)} = \begin{cases} 1 & o \leq k \leq n, \\ 0 & \text{otherwise}. \end{cases}$$

(v) $N[\overset{-1}{G}(I - \bar{B}_k)] = k.$

Let us choose $x \in l_\infty(p)$. Then by (iii), $G\,x \in \hat{c}_o(p)$ and $A(G\,x) = D\,x \in F_{\mathcal{B}}$ i.e $D: l_\infty(p) \longrightarrow F_{\mathcal{B}}$. Thus by Theorem (3.2), contidion (ii) follows immediately.

Sufficiency. Suppose (i) and (ii) holds and $x \in \hat{c}(p)$. We have to show that $A\,x \in F_{\mathcal{B}}$. Since $x \in \hat{c}(p)$ implies

$$| t_{n,i}(x - l\,e) |^{p_k} \longrightarrow 0,\ n \longrightarrow \infty \text{ for some } l, \text{ uniformly in } i.$$

Where

$$t_{n,i}(x) = \frac{1}{n+1}\sum_{k=i}^{i+n} x_k.$$

Hence for a given $\epsilon > 0\ \exists\, k_o \geq 0$ such that $\forall\, k < k_o$

$$| t_{n,i}(x - l\,e) |^{p_k/M} < \frac{\epsilon}{3\,B\,(\sum_k |\alpha_k| + \sum_k |T_{in}(e_k)| + 1)}$$

therefore $B^{1/p_k}\ | t_{n,i}(x - l\,e) | < B^{M/p_k}\ | t_{n,i}(x - l\,e) |$

$$< \frac{\epsilon}{3\,(\sum_k |\alpha_k| + \sum_k |T_{in}(e_k)|)},$$

where

$T_{in}(x) = (B_i(A\,x))_n$. Now we have

$$T_{in}(x) = \Sigma_k (T_{in}(e_k))\,x_k + (T_{in}(e) - \sum_k T_{in}(e_k))\,(\hat{c} - \lim x).$$

By given conditions, we have

$$\lim_n T_{in}(e_k) = \alpha_k \text{ and}$$

$$\lim_n T_{in}(e) = \alpha \text{ uniformly in } i.$$

And hence $\exists\, n_o \geq r$ with

$$\sum_{k=o}^{ko} | \alpha_k - T_{in}(e_k) | < \frac{\epsilon}{3\,(2\,|\,\hat{c} - \lim x\,| + 1)}$$

$$| \alpha - T_{in}(e) | < \frac{\epsilon}{3(|\hat{c} - \lim x| + 1)}$$

which is true for all $i \geq o$ and $n \geq n_o$. Now by Banach – Steinhaus Theorem $L_i \in \hat{c}'(p)$ (continuous dual space of $\hat{c}(p)$), where

$$L_i x = \sum_k (L_i e_k) x_k + (L_i e - L_i e_k)(\hat{c} - \lim x)$$

$$= \sum_k \alpha_k x_k + (\alpha - \sum_k \alpha_k)(\hat{c} - \lim x)$$

$$= L x.$$

Therefore

$$| L x - T_{in}(x) | = | (\alpha - T_{in}(e))(\hat{c} - \lim x) + \sum_k (\alpha_k - T_{in}(e_k))$$

$$(t_{n,i}(x - l e)) |$$

$$\leq | \alpha - T_{in}(e)) \| \hat{c} - \lim x | + 2 | \hat{c} - \lim x | \sum_{k=o}^{k_o} | \alpha_k - T_{in}(e_k) |$$

$$+ \sup_{k_o < k < \infty} | t_{n,i}(x - l e) | \left(\sum_{k=k_o+1}^{\infty} | \alpha_k | + \sum_{k=k_o+1}^{\infty} | T_{in}(e_k) | \right)$$

$$< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} = \epsilon.$$

Hence $A \in (\hat{c}(p), F_{\mathcal{B}})$.

Proof of (b) is immediate if we observe that $\alpha_k = 0$ and $\alpha = 1$ in (a).

Finally the author is grateful to Dr. Z. U. Ahmad for his suggestions and guidance.

ÖZET

Bu çalışmada amacımız, bngüne dek ortaya atılmış olan matris sınıflarındaki bir boşluğu dolduracak matrisler sınıflarını incelemektir. Daha önca $(c_o(p), Fo_{\mathcal{B}}(p))$, $(l(p), F_{\mathcal{B}})$ ve $(M_o(p), F_{\mathcal{B}})$ matrislerinin karekterize etmiştir. Bu araştırmamızda $(W(p), F_{\mathcal{B}})$, $(W_p, F_{\mathcal{B}})$, $(c(p), F_{\mathcal{B}})$, $(l_\infty(p), F_{\mathcal{B}})$ ve $(c(p), F_{\mathcal{B}})$ matrislerini karektorize edeceğiz.

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