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On The Minimal Hypersurfaces

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SUMMARY

In this paper the Euler's theorem and some corollaries for minimal hypersurfaces are obtained. Moreover characterizations for asymptotic curves on the minimal hypersurfaces and surfaces are given. On the other hand a theorem, on the conjugate minimal surfaces, is also given.

Definition 1.1.

Let M and S be a hypersurface of E^n and the shape operator of the hypersurface, respectively $H = \frac{1}{n-1}$ trace S is called the mean curvature function on M.

Definition 1.2.

If the mean curvature function on M is zero, M is called a minimal hypersurface.

Definition 1.3.

The function k_n which is given by

$$k_n: T_M(P) \longrightarrow \mathbb{R}$$
$$V_p \longrightarrow k_n(V_p) = \langle S(V_p), V_p \rangle$$

is called the normal curvature of M in direction V_p where $P \varepsilon M$, $V_p \in T_M(P)$ and $\|V_p\| = 1$ [1].

Theorem 1.1.

Let M be a hypersurface of E^n and

$P \in M$ be a nonumbilic point. Let $X_1, \ldots, X_{n-1}$

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be the principal directions and $k_1, \ldots, k_{n-1}$ be the corresponding principal curvatures. In this case,

$$k_n(V_p) = \sum_{i=1}^{n-1} k_i \cos 2\theta_i$$

where θ_i is the angle between V_p and X_i, $i=1, \ldots, n-1$.

The proof of this theorem can be found in [1].

If M is a surface in E^3 then we have

$$k_n(V_p) = k_1 \operatorname{Cos}^2 \theta_1 + k_2 \sin^2\theta_1.$$

Corollary 1.1.

Let M be a minimal hypersurface in E^n and θ_i, k_i, X_i be as they are in Theorem 1.1. then, we have

$$k_n(V_p) = -\sum_{i=1}^{n-1} k_i \sin^2\theta_i.$$

Proof: By the Theorem 1.1. we have

$$k_n(V_p) = \sum_{i=1}^{n-1} k_i \cos 2\theta_i.$$

Since M is a minimal hypersurface and V_p is a unit tangent vector then we find

$$k_n(V_p) = -\sum_{i=1}^{n-1} k_i \sin^2\theta_i,$$

which completes the proof.

Corollary 1.2.

Let M, θ_i, k_i be as they are in Theorem 1.1. and H be the mean curvature function on M then, we have

$$k_n(V_p) = \frac{n-1}{2} H + \frac{1}{2} \sum_{i=1}^{n-1} k_i \cos^2\theta_i.$$

Proof:

By substituting

$$\cos 2\theta_i = \frac{1}{2} (1 + \cos 2\theta_i)$$

in the Theorem 1.1. we obtain

$$k_n(V_p) = \frac{n-1}{2} H + \frac{1}{2} \sum_{1=i}^{n-1} k_i \cos 2\theta_i$$

which completes the proof.

Corollary 1.3.

If M is a minimal hypersurface of E^n then we have

$$k_n(V_p) = \frac{1}{2} \sum_{1=i}^{n-1} k_i \cos 2\theta_i.$$

Proof: This is obvious from corollary 1.2.

Corollary 1.4.

If M is a minimal surface in E^3 then we can write

$$k_n(V_p) = k_1 \cos 2\theta_1.$$

Proof: This can be easely seen from corollary 1.2.

Definition 1.4.

Let M and S be a hypersurface of E^n and the shape operator of M, respectively. If a curve α on M satisfies the condition

$$k_n(\alpha'(s)) = < S(\alpha'(s)), \alpha'(s) > = o$$

then α is called an asymptotic curve on M.

Theorem 1.2.

Let M be a minimal hypersurface of E^n and $k_1, \ldots, k_{n-1}$ be it's principal curvatures corresponding to the directions $X_1, \ldots, X_{n-1}$, respectively. If $k_i > O$ and $\cos 2\theta_i - \cos 2\theta_1$ have the same sign on M, then a curve on M is asymptotic iff the angles $\theta_2, \ldots, \theta_{n-1}$ are equal to

$\theta_1 + k\pi$, where $\theta_1, \ldots, \theta_{n-1}$ are angles in between the curves with principal directions $X_1, \ldots, X_{n-1}$, respectively.

Proof: Let α be an asymptotic curve on M. In this case, from corollary 1.2. we can write that

$$\sum_{i=1}^{n-1} k_i (\cos 2\theta_i - \cos 2\theta_1) = o.$$

From the hypothesis we find that $\theta_i = \theta_1 + k\pi$, $k \in N$. Conversely let θ_i, $2 \leq i \leq n-1$, be equal to $\theta_1 + k\pi$ then by the corrollary 1.1 we observe that $k_n(\alpha'(s)) = o$.

Theorem 1.3.

Let M be a minimal surface in E^3.

A curve α on M is asymptotic iff the angle θ_1 is equal to $(2k+1)\frac{\pi}{4}$, where θ_1 is an angle between α and the principal direction X_1 on M.

Proof: Let α be an asymptotic curve on M. In the case of Corollary 1.4. we can have

$$\theta = (2k+1)\frac{\pi}{4}, \; k \in \mathbb{N}.$$

Conversely, let $\theta = (2k+1)\frac{\pi}{4}$, then we find $k_n(\alpha'(s)) = O$.

This means that α is an asymptotic curve on M.

Definition 2.1.

Let

$$f: U \subset E^2 \longrightarrow \mathbb{R},$$
$$(u, v) \longrightarrow f(u,v)$$

be given. If f satisfies the equation

$$\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} = O$$

then f is called a harmonic function on U.

Definition 2.2.

Let X be a minimal surface whose parameters are isotermal then x_j – coordinates of X are harmonic functions. If conjugate of x_j is y_j, $1 \leq j \leq 3$ there are some surfaces Y whose coordinates are y_j then $\varphi_j(Z) = x_j + iy_j$ are analytic functions, in this case Y is called conjugate to X.

Theorem 2.1.

Let the surfaces X and Y be conjugate, S and $\bar{S}$ be their shape operators, respectively, then

$$S = \text{ad } \bar{S}.$$

Proof: Since X, Y are conjugate surfaces and $\varphi_j(Z)$ are analytic, then we can write

$$\left.\begin{array}{l} X_u = Y_v \\ X_v = -Y_u \end{array}\right\} \ldots 2.1$$

Using equalities (2.1.) and the linearity of shape operators, we can easely find that

$$S = \text{ad } \bar{S}$$

where ad = adjoint of.

Corollary 2.1.

If K, $\bar{K}$ are the Gaussian curvatures of the conjugate minimal surfaces X and Y, respectively, then

$$K = \bar{K}.$$

Proof: It is obvious from the Theorem 2.1 and the definition of Gaussian curvature.

ÖZET

Bu çalışmada minimal hiperyüzeyler için Euler teoremi ve bazı sonuçlar elde edildi. Ayrıca minimal hiperyüzeyler ve yüzeyler üzerindeki asimptotik eğriler için karakterizasyonlar verildi. Bundan başka eşlenik minimal yüzeyler hakkında da bir teorem ispatlandı.

REFERENCES

1- **Altın A.:** Hiperyüzeyler için Euler teoremi, Doktora tezi, Hacettepe Üniversitesi Temel Bilimler Fakültesi, 1979.

2- **Manfredo P. do Carmo:**
— Diferential Geometry of Curves and Surfaces-Prentice-Hall, inc., New Jersey.