COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES DE L'UNIVERSITÉ D'ANKARA

Série A_1 : Mathématiques

TOME 30 **ANNÉE 1981**

A LA MEMOIRE D'ATATÜRK AU CENTENAIRE DE SA NAISSANCE

The Cousin and Poincaré Problems

by

Cengiz ULUÇAY

4

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

---

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les disciplines scientifiques représentées à la Faculté.

La Revue, jusqu'a 1975 à l'exception des tomes I, II, III, était composée de trois séries:

Série A: Mathématiques, Physique et Astronomie.

Série B: Chimie.

Série C: Sciences naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématiques
Série A_2: Physique
Série A_3: Astronomie
Série B : Chimie
Série C_1: Géologie
Série C_2: Botanique
Série C_3: Zoologie

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté. Elle accepte cependant, dans la mesure de la place disponible, les communications des auteurs étrangers. Les langues allemande, anglaise et française sont admises indifféremment. Les articles devront être accompagnés d'un bref sommaire en langue turque.

Adres: Fen Fakültesi Tebliğler Dergisi Fen Fakültesi, Ankara, Turquie.

DEDICATION TO ATATÜRK'S CENTENNIAL

Holding the torch that was lift by Atatürk in the hope of advancing our Country to a modern level of civilization, we celebrate the one hundredth anniversary of his birth. We know that we can only achieve this level in the fields of science and technology that are the wealth of humanity by being productive and creative. As we thus proceed, we are conscious that, in the words of Atatürk, "the truest guide" is knowledge and science.

As members of the Faculty of Science at the University of Ankara we are making every effort to carry out scientific research, as well as to educate and train technicians, scientists, and graduates at every level. As long as we keep in our minds what Atatürk created for his Country, we can never be satisfied with what we have been able to achieve. Yet, the longing for truth, beauty, and a sense of responsibility toward our fellow human beings that he kindled within us gives us strength to strive for even more basic and meaningful service in the future.

From this year forward, we wish and aspire toward surpassing our past efforts, and with each coming year, to serve in greater measure the field of universal science and our own nation.

The Cousin and Poincaré Problems

Cengiz ULUÇAY

Dept. of Mathematics, Faculty of Science, Ankara University, Ankara.

(Received May 22, 1981; accepted July 15. 1981)

SUMMARY

In this paper we reproduce the proofs of Cousin and Poincaré problems for the structure sheaf A [5] without making explicit use of flabby sheaf theory. We conclude with a Remark in section 3.

For the solutions of these problems we shall merely make direct appeal to a property inherent to A, i.e., sections defined in A can be extended holomorphically to the entire region of definition.

We recall the following

Definitions. Let $G \subset C^n$, be a region (connected open set), and A(G) the ring (C– Algebra) of holomorphic functions on G. Then the set A of all convergent power series (germs) representing the elements of A(G) is called a restricted sheaf over G.

It was proved in [1] that A is coherent as soon as G is a region of holomorphy.

1. The First Cousin Problem. We first recall the following

Definitions. Since G is connected, A(G) is an integral domain, and we can form the field of quotients M(G) whose elements are of the form f/g, f, $g \in A(G)$ with non vanishing g. For any fixed $z \in G$ we can form the quotient field M_z of A_z whose elements are of the form f_z/g_z with $g_z \neq 0$, then

$$M = \bigvee_{z \in G} M_z$$

is a sheaf with the topology generated by the sets

$\{f_z/g_z: z \in U \subset G$ open connected set, $g_z \neq 0$, f, g $\in A(G)\}$
as follows: Let m be the class represented by f/g, f, g $\in$ A(G), then we write $m = f/g$ and $m_z = f_z/g_z$. In the neighborhood U of z, m_z defines

$$m|U = f|U/g|U$$

which in turn defines at every point ζ of U, m_ζ. The union over U of these classes m_ζ defines an open set, and the collection of these sets forms the basis of the topology in question.

The sheaf so defined is called an r-sheaf of germs of meromorphic *functions over G*. *The germs are the elements* m_z *of* M_z *called the stalk of* M. A section over U is defined in the usual way, i.e., it is a continuous mapping $U \to M$ which assigns to each point z of G a meromorphic germ m_z over that point. Moreover, the composition of this mapping with the projection mapping $M \to G$ restricted to U is the identity mapping 1_U.

The sections are called *meromorphic functions*, and the collection of the sections over U is denoted by M(U) or $\Gamma(U, M)$. Thus if $m \in \Gamma(G, M)$, then $m|U \in \Gamma(U, M)$. In the sequel, the notation $m = f/g \in \Gamma(U, M)$ will always mean that f and g are the restrictions to the open set U, of holomorphic functions in A(G), with a non vanishing g.

After these preparations, the first Cousin problem for the structure r-sheaf A can be formulated as follows:

Given a global section of M/A, find a global section of M which is mapped canonically on the given section.

Note that since M_z is the field of quotients of A_z, A_z can be identified with a subset of M_z, and A with a subsheaf of M under the canonical injection $\imath$.

It is clear that the sequence

$$(1) \qquad O \to A \overset{\imath}{\to} M \overset{\pi}{\to} M/A \to O$$

is an exact sequence of sheaves of C- modules, then so is

$$0 \to \Gamma(G, A) \overset{\imath^*}{\to} \Gamma(G, M) \overset{\pi^*}{\to} \Gamma(G, M/A)$$

exact.

Theorem 1.1. If $G \subset C^n$ is a region of holomorphy, then the sequence of sections

$$0 \to \Gamma(G, A) \overset{\imath^*}{\to} (G, M) \overset{\pi^*}{\to} \Gamma(G, M/A) \to 0$$

is exact.

Proof. It will of course suffice to show that π^* is surjective. We shall follow the same pattern of proof as in [2, 3]. Let $\overline{m} \in \Gamma(G, M/A)$ be any given section, and $z \in G$ an arbitrary point. Then $\overline{m}(z) \in M/A$. Since π is surjective there is an element (germ) $\sigma \in M$ such that $\pi(\sigma) = \overline{m}(z)$. Hence there is a neighborhood $U = U(z) \subset G$ and a section $m \in \Gamma(U, M)$ with $\pi o m(z) = \overline{m}(z)$ and $\pi o m = \overline{m}|U$. To summarize, for every $z \in G$, there is an open set $U = U(z) \subset G$ and a section m over U with $\pi o m = \overline{m}|U$.

Consider the collection $U = \{(U, m)\}$ of all pairs (U, m) with $\pi o m = \overline{m}|U$. The collection U has the following additional property: let $(U_1, m_1), (U_2, m_2) \in U$. (i) If $U_1 \cap U_2 = \varnothing$, then there is a section m^* in M over $U^* = U_1 \cup U_2$ whose image under π is $\overline{m}|U^*$, i.e., $(U^*, m^*) \in U$. (ii) Suppose $U_1 \cap U_2 \neq \varnothing$. The sequence of sections

$$0 \to \Gamma(U_1 \cap U_2, A) \to \Gamma(U_1 \cap U_2, M) \to \Gamma(U_1 \cap U_2, M/A)$$

is exact. Since $\pi \circ (m_1 - m_2)|U_1 \cap U_2 = 0$, there is by (1) an $s \in \Gamma(U_1 \cap U_2, A)$ with $\imath o s = m_1 - m_2|U_1 \cap U_2$. By the very definition of A, s can be extended *holomorphically* to a section $s_2 \in \Gamma(U_2, A)$. Now, the section m^* over $U^* = U_1 \cup U_2$ defined by

$$m^*(z) = \begin{cases} m_1(z) & z \in U_1 \\ (\imath o s_2 + m_2)(z) & z \in U_2 \end{cases}$$

lies in $\Gamma(U_1 \cup U_2, M)$, and $\pi o m^* = \overline{m}|U_1 \cup U_2$.

Hence again $(U^*, m^*) \in U$. If we define $(U_1, m_1) \leqq (U_2, m_2)$ to mean $U_1 \subset U_2$ and $m_1 = m_2|U_1$, then a partial ordering is defined in U. Now, consider all chains $(U_\imath, m_\imath)_{\imath \in I}$ in U with the property that either $(U_{\imath_1}, m_{\imath_1}) \leq (U_{\imath_2}, m_{\imath_2})$ or $(U_{\imath_2}, m_{\imath_2}) \leq (U_{\imath_1}, m_{\imath_1})$. Each chain has an upper bound: $U = \bigcup_{\imath \in I} U_\imath$ and $m|U_\imath = m_\imath$ which is an element of U. By Zorn's lemma there is a maximal element $(U_0, m_0) \in U$. In view of the property of U, U_0 cannot be a proper subset of G. For, then there would exist $z^0 \in G$ and a neighborhood of z^0, $U(z^0) \subset G$ so that $(U^*_0 = U_0 \cup U(z^0), m^*_0$ with $m^*_0 | U_0 = m_0) \in U$. Namely, $(U_0, m_0) \leq (U^*_0, m^*_0)$ thus violating the maximality of (U_0, m_0). Since G is a region of holomorphy U_0 cannot contain properly G either. So $U_0 = G$.

As an immediate consequence of theorem 1.1 we can state

Theorem 1.2. If $G \subset C^n$ is a region of holomorphy with the structure r-sheaf A, then the first Cousin problem is always solvable globally.

2. Second Cousin and Poincaré Problems.

a. Čech Cohomology. We recall the following definitions [4].

Cochains of a Covering. Let X be a topological space and $U = (U)_{\iota \in I}$ be an open covering of X.

Let S be a sheaf of abelian groups (or R-modules) over X. If $q \geq 0$ is an integer, and $s = (\iota_0, \ldots, \iota_q)$ is a finite sequence of elements in I, then we set

$$U_s = U_{\iota_0 \ldots \iota_q} = U_{\iota_0} \cap \ldots \cap U_{\iota_q}.$$

Definition 2.1. A q-cochain over U with values in S is a map f which assigns to every sequence $s = (\iota_0, \ldots, \iota_q)$ of q+1 elements in I a section of S:

$$f(s) \in \Gamma(U_s, S)$$

over U_s. Recalling that $\Gamma(U_s, S)$ is an abelian group, (or R-module) then the q-cochains form an abelian group, i.e,. the group product

(2) $$\prod_s (U_s, S)$$

extended over all sequences s of q+1 elements in I.

A q-cochain is called an alternating q-cochain if:

(a) $f(s) = f(\iota_0, \ldots, \iota_q) = 0$ whenever two indices in the sequence $(\iota_0, \ldots, \iota_q)$ are equal, or $U_s = \varnothing$

(b) f(s) is an alternating function of s, i.e., f(s) changes sign if two indices in s are permuted.

The alternating q-cochains form a subgroup of (2). This subgroup is dencted by $C^q(U, S)$.

We define a coboundary operator

$$d = d^q : C^q(U, S) \to C^{q+1}(U, S)$$

by setting

$$(df)(\iota_0, \ldots, \iota_{q+1}) = \sum_{j=0}^{q+1} (-1)^j f(\iota_0, \ldots, \hat{\iota}_j, \ldots, \iota_{q+1}) \,|\, U_{\iota_0 \ldots \iota_{q+1}}$$

where $\hat{\iota}_j$ means that the index ι_j is delated.

It is easy to see that d is a homomorphism with $d^{q+1} \circ d^q = 0$.

Finally, we introduce

$$Z^q (U,S) = \{f: f \in C^q (U,S), d^q f = 0\},$$

$$B^q (U,S) = \{d^q f: f \in C^{q-1} (U,S)\}, C^{-1} = 0$$

the group of q-cocycles and the group of q-coboundaries with values in S respectively. Then $B^q \subset Z^q$. One can therefore define

$$H^q (U,S) = Z^q (U,S) / B^q (U,S)$$

which is called the q-th cohomology group of U with values in S.

The Čech complex is defined by the sequence

$$C^0 (U,S) \xrightarrow{d} C^1 (U,S) \xrightarrow{d} C^2 (U,S) \rightarrow$$

which is exact at every location $q \geq 1$ if and only if $H^q (U,S) = 0$.

If f is a 0-cocycle, then $f(\iota_0) - f(\iota_1) = 0$ in $U_{\iota_0} \cap U_{\iota_1}$ for all ι_0 and ι_1, which means that the sections $f(\iota_0)$ and $f(\iota_1)$ coincide on $U_{\iota_0} \cap U_{\iota_1}$ and so altogether define a single section $f \in \Gamma (X,S)$. Conversely, every section $f \in \Gamma (X, S)$ does so determine a 0-cocycle f. Hence

$$H^0 (U,S) \cong \Gamma (X,S).$$

b. **Second Cousin Problem.** Let X be a n-dimensional complex manifold, R a commutative ring with 1 and S a sheaf of R-modules over X. Finally, let $U = (U_\iota)_{\iota \in I}$ be an open covering of X.

We have the following [2]

Lemma 2.1. If X itself is an alement of the covering U, then $H^q (U,S) = 0$, $q \geq 1$.

Proof. We must show that if $c = c(\iota_0,\ldots,\iota_q) \in Z^q (U,S)$, $q \geq 1$, then there is an $f = f(\iota_0,\ldots,\iota_{q-1}) \in C^{q-1} (U,S)$ such that $df = c$. Now, by hypothesis there is an index $\alpha \in I$ such that $X = U_\alpha \in U$. Let $c \in Z^q (U,S)$, $q \geq 1$. There is a cochain $f \in C^{q-1} (U,S)$ defined by

$$f(\iota_0,\ldots,\iota_{q-1}) = c(\alpha, \iota_0,\ldots,\iota_{q-1}).$$

Since $dc = 0$, we have

$$0 = (dc)(\alpha, \iota_0,\ldots\iota_q) = c(\iota_0,\ldots,\iota_q) - \sum_{j=0}^{q} (-1)^j c(\alpha, \iota_0,\ldots,\hat{\iota}_j,\ldots,\iota_q).$$

Also,

$$(df)(\imath_0,\ldots,\imath_q) = \sum_{j=0}^{q} (-1)^j f(\imath_0,\ldots,\hat{\imath}_j,\ldots,\imath_q)$$

$$= \sum_{j=0}^{q} (-1)^j c(a,\imath_0,\ldots,\hat{\imath}_j,\ldots,\imath_q)$$

$$= c(\imath_0,\ldots,\imath_q).$$

Hence $df = c$. Namely, the Čech cohomology sequence is exact at every location $q \geq 1$, i.e., $H^q(U,S) = 0$, $q \geq 1$.

Lemma 2.2. Let $G \subset C^n$ be a region of holomorphy and $U = (U\imath)_{\imath \in I}$ an arbirary open covering of G. Then $H^q(U, S) = 0$ for all $q \geq 1$.

Proof. Let $c \in Z^q(U,A)$, $q \geq 1$. If $W \subset G$ is an open set, then $c|W \in Z^q(W \cap U,A)$ means $W \cap U = \{W \cap U\imath \neq \varnothing : U\imath \in U\}$ and $(c|W)(\imath_0,\ldots \imath_q) = c(\imath_0,\ldots,\imath_q)\ |W \cap U\imath_0,\ldots,\imath_q$.

Next, let $z^0 \in G$ be arbitrary. Then there is $\imath_0 \in I$ and an open neighborhood $W = W(z^0) \subset U\imath_0$. But then $W \subset W \cap U$. By lemma 2.1, $H^q(W \cap U,A) = 0$, $q \geq 1$. Namely, there is an $f \in C^{q-1}(W \cap U,A)$ such that $df = c|W$. If $V \subset G$ is an open set with the same property, i.e., there is an $f' \in C^{q-1}(V \cap U,A)$ such that $df' = c|V$, then

$$s = (f-f')|W \cap V \in Z^{q-1}(W \cap V \cap U,A).$$

Now, $q = 1$ implies $Z^0(W \cap V \cap U,A) = H^0(W \cap V \cap U,A) = \Gamma(W \cap V,A)$ and so $s \in \Gamma(W \cap V,A)$. But then s can be extended holomorphically to $\hat{s} \in \Gamma(V,A)$. We then set

$$s^* = \begin{cases} f(z) & z \in W \\ f'(z) + \hat{s}(z) & z \in V. \end{cases}$$

Clearly $s^* \in \Gamma(W \cup V,A)$, and since $d\hat{s} = 0$ it follows that $ds^* = c|W \cup V$. Hence

$$H^1((W \cup V) \cap U,A) = 0.$$

If $q > 1$ then we proceed by induction on q. We therefore assume

$$H^{q-1}((W \cup V) \cap U,A) = 0.$$

Accordingly there is an $h \in C^{q-2}(W \cap V \cap U,A)$ such that $dh = s$.

Therefore,

$$h\,(\imath_0,\ldots,\imath_{q-2}) \in \Gamma\,(W \cap V \cap U\imath_0,\ldots,\imath_{q-2}, A)$$

can be extended holomorphically to

$$\hat{h}\,(\imath_0,\ldots,\imath_{q-2}) \in \Gamma\,(V \cap U\imath_0,\ldots,\imath_{q-2}, A).$$

Let

$$f^*\,(\imath_0,\ldots,\imath_{q-1})\,(z) = \begin{cases} f\,(\imath_0,\ldots,\imath_{q-1})\,(z) & z \in W \cap U\imath_0,\ldots,\imath_{q-1} \\ (f' + d\hat{h})\,(\imath_0,\ldots,\imath_{q-1})\,(z) & z \in V \cap U\imath_0,\ldots,\imath_{q-1}. \end{cases}$$

Then $f^* \in C^{q-1}((W \cup V) \cap U, A)$ and $df^* = c\,|W \cup V$. Hence $H^q((W \cup V) \cap U, A) = 0$.

As in theorem 1.1, we may consider the collections $\{(U^*, s^*)\}$, respectively $\{(U^*, f^*)\}$ of all pairs (U^*, s^*), respectively (U^*, f^*), such that $s^* \in \Gamma\,(U^*, A)$, $ds^* = c\,|U^*$, $q = 1$, respectively $f^* \in C^{q-1}(U^* \cap U, A)$, $df^* = c\,|U^*$, $q > 1$. These collections are partially ordered by set inclusion, and every chain has an upper bound which is an element of the collection. By Zorn's lemma there is a maximal element (U_0, s_0), respectively (U_0, f_0), such that $s_0 \in \Gamma\,(U_0, A)$, $ds = c\,|\,U_0$, $q = 1$, respectively $f_0 \in C^q\,(U, A)$, $df_0 = c\,|\,U_0$, $q > 1$. It is clear that an element is maximal only if $U_0 = G$. Hence $c \in B^q\,(U, A)$. Namely, the Čech cohomology sequence is exact at every location $q \geq 1$, i.e.,

$$H^q\,(U, A) = 0,\; q \geq 1.$$

$H^q\,(G, A)$ being the inductive limit of $H^q\,(U, A)$, [4] we may state

Theorem 2.1. If $G \subset C^n$ is a region of holomorphy and A the structure r-sheaf over G, then

$$H^q\,(G, A) = 0,\; q \geq 1.$$

To solve the second Cousin problem for the structure r-sheaf A, let $M^* = M - \{0\}$ be the sheaf of germs of invertible meromorphic functions. M^* contains as a subsheaf the sheaf A^* whose elements consist of those germs m_z of M^* that are invertible holomorphic functions in some neighborhood of z. Thus the germs of A^* are units in A^*. The sheaves A^* and M^* are multiplicative abelian groups. The sections of $\Gamma\,(G, A^*)$ are the nowhere vanishing holomorphic functions on G. The quotient sheaf $D = M^*/A^*$ is called the sheaf of germs of divisors in G, the sections in D being called divisors. The second Cousin problem can then be formulated exactly as the first one: Given a global section of D, find a global section of M^* which is mapped canonically on the given section. By analogy with the solution of the first Cousin problem we may consider the exact sequence

$$1 \to A^* \to M^* \xrightarrow{\pi} D \to O.$$

The induced long exact cohomology sequence is

$$1 \to \Gamma(G,A^*) \to \Gamma(G,M^*) \xrightarrow{\pi^*} \Gamma(G,D) \xrightarrow{\eta} H^1\ (G,A^*) \to H^1\ (G,M^*) \to \ldots.$$

The second Cousin problem will then be solved for all divisors if $\pi^*$is surjective, i,e., $H^1\ (G,A^*) = 0$. However, very little is known about this group. To study the group $H^1(G,A^*)$ we introduce as usual the exact sequence

$$O \to Z \to A \xrightarrow{e} A^* \to 1$$

of sheaves of Z-modules, where Z denotes the constant sheaf of the additive group of integers and

$$e\colon f \to e^{2\pi i f}.$$

The associated long exact cohomology sequence is

$$\ldots \to H^1\ (G,A) \to H^1\ (G,\ A^*) \xrightarrow{\delta} H^2\ (G,Z) \to H^2\ (G,A) \to \ldots.$$

By theorem 2.1 the groups on the left and right being both 0, δ is an isomorphism:

$$H^1\ (G,A^*) \cong H^2\ (G,Z).$$

Combining the maps η and δ, we have

$$c\colon \Gamma\ (G,D) \to H^2\ (G,Z),\ c = \delta \circ \eta.$$

This map is surjective, and the second Cousin problem will be solved for those divisors belonging to the kernel of c. More precisely, the homomorphism c associates to every divisor $d \in \Gamma(G,\ D)$ a 2-dimensional integral cohomology class $c\ (d) \in H^2\ (G,Z)$ called the characteristic class or the *Chern* class of d, and the second Cousin problem is solvable for those divisors whose Chern class vanishes. Therefore

Theorem 2.2 The second Cousin problem for the structure r-sheaf A, can be solved for all divisors if and only if $H^2\ (G,\ Z) = 0$ (or $H^1\ (G,A^*) = 0$).

Recall that the germs of $D = M^*/A^*$ are equivalent classes of germs of meromorphic functions, where germs represented by meromorphic functions m_1, m_2 are equivalent at $z \in G$ if and only if m_1/m_2 is a unit of A^*. Now, let $d = \Gamma(G,D) = H^0(G,D)$ be a divisor on G. This means that there exists a cover $U = (U_i)_{i \varepsilon I}$ of G and meromorphic functions $m_i \in H^0(U_i, M^*)$ with $\pi^*(m_i) = d|U_i$ and $m_i/m_j \in H^0(U_i \cap U_j, A^*)$. Every collection of pairs $\{(U_i,m_i)\}$ for a cover $U = (U_i)_{i \varepsilon I}$ with $m_i \in H^0(U_i, M^*)$ and $m_i/m_j \in H^0(U_i \cap U_j, A^*)$ determines a divisor $d \in \Gamma(G,D)$. One calls $\{(U_i,m_i)\}$ a d-representing Cousin data.

Definition 2.2. A divisor d on G is principal if there exists a meromorphic function m defined on G such that the divisor it defines is equal to d. One writes in this case $d = (m)$.

Thus for any $m \in \Gamma(G,M^*)$, the divisor $\pi^*(m) \in \Gamma(G,D)$ is principal.

Definition 2.3. A divisor d on G is called positive (or integral) and is denoted by $d \geq 0$ whenever there is a d-representing Cousin data $\{(U_i, m_i))\}$ where $m_i \in \Gamma(G, A)$ for all i.

Thus a meromorphic function $m \in \Gamma(G, M^*)$ is holomorphic if and only if $\pi^*(m) = d = (m) \geq 0$.

Note that every divisor can be uniquely written as the difference of two positive divisors, with no common prime omponents.

With these definitions theorem 2.2 takes the form

Theorem 2.3. Let A be the structure r-sheaf of a region of holomorphy $G \subset C^n$. A divisor $d \in \Gamma(G,D)$ is principal if and only if $\eta(d) \in H^1(G,A^*) = 0$ or equivalently if and only if its Chern class c(d) vanishes.

c. **Poincaré Problem.** The Poincaré problems: Given a region G, is every function meromorphic in G a quotient of two functions holomorphic in G ? (and coprime at every point ?), can also be solved for the structure r-sheaf A. However, a positive answer to the question in parenthesis can only be given under the hypothesis $H^2(G, Z) = 0$.

Indeed, let $m \in \Gamma(G, M)$, $m \neq 0$, be a meromorphic function. Then $m \in \Gamma(G, M^*)$ and so $\pi^*(m) = (m) \in \Gamma(G,D)$. We may write uniquely $(m) = d^+ - d^-$ with $0 \leq d^+, d^- \in \Gamma(G,D)$. Now, $H^2(G,Z) = 0$ implies the existence of meromorphic functions $f, g \in \Gamma(G, M^*)$ such that $\pi^*(f) = d^+$, $\pi^*(g) = d^-$. Hence f, g are holomorphic with $\pi^*(f/g) = d^+ - d^-$, and therefore $m = f/g$.

3. Remark. All the results so far obtained are valid of course for a Stein or a Florack manifold which are both regions of holomorphy.

Since every finite dimensional Stein manifold is (i) holomorphically separable and is characterized algebraically by the fact that (ii) ev ery non trivial linear functional is a point functional with finitely generated maximal ideal [3], it follows from our definition of a Florack manifold (it is a manifold satisfying conditions (i), (ii)) that a Stein manifold is a Florack manifold. Conversely, every Florack manifold is a Stein manifold. To see this, we observe the follow ng:

1. Firstly, it is sufficient to show that a Florack manifold is holomorphically convex. We say that a manifold X is holomorphically convex, if (x_i) is a discrete sequence in X (it is a sequence which tends to the ideal boundary point "∞"), then there exists a function f holomorphic on X which is unbounded on (x_i).

2. Secondly, to fix the ideas, let G be a region (open connected set) in C. Then every non trivial linear functional m on A (G) is a point functional m_a, $a \in G$. Namely,

$$m = \begin{cases} m_a \text{ if } a \in G \\ 0 \text{ if } a \notin G, \text{ i. e., } a \in \partial G. \end{cases}$$

The triviality of m implies that G is holomorphically convex. Indeed, let (z_i) be a sequence in G with limit $a \in \partial G$. Then $f(z) = 1/(z-a)$ is holomorphic on G and $(\,|\,f(z_i)\,|\,)$ is unbounded.

Similarly, if G is X, then $m = 0$ implies the existence of a unit in A (X) lying in the maximal ideal and vanishing at "∞", and there by the existence of a function holomorphic on X with pole at "∞". Upon this remark we conclude that

Theorem 3.1. Two Stein manifold of dimension n X,X' are holomorphically equivalent if and only if there is a ring isomorphism between A(X),A(X') that preserves the constant.

Proof, Since a Stein manifold is a region of holomorphy then every non trivial linear functional on A(X) is a point functional with finitely generated fixed maximal ideal. The conclusion follows. [6]

ÖZET

Bu makalede, Cousin ve Poincare problemleri yeniden ele alınmış olup bu kere yumuşak demet teorisi kullanılmadan çözümlenmiştir.

REFERENCES

[1] **C. Uluçay,** Restricted Sheaf Theory, Journal of Fac. Sc. of the K.T.Ü. Vol. III, Fasc. 7, 1980

[2] **H. Grauert – K. Fritzsche,** Several Complex Variables, Springer-Verlag, 1976

[3] **H. Grauert – R. Remmert,** Theory of Stein Spaces, Springer-Verlag, 1979

[4] **J.P. Serre,** Faisceaux Algebriques Coherents, Ann. Math. Vol. 61, pp. 197–278, 1955

[5] **C. Uluçay,** On Coherence Theorems and the Related Cohomology Groups, Journal of the Fac. Sc. of the K.T.Ü. Vol. III, Fasc. 8, 1980

[6] **C. Uluçay,** Maximal Ideals in the Ring of Continuous Complex - Valued Functions of Several Complex Variables, Jour of the Fac. Sc. of the K.T.Ü. Vol. II. Fasc. 2. pp. 17-26, 1979.

Prix de l'abonnement annuel

Turquie: 15 TL; Étranger: 30 TL.
Prix de ce numéro: 5 TL (pour la vente en Turquie).
Prière de s'adresser pour l'abonnement à: Fen Fakültesi Dekanlığı Ankara, Turquie.

Ankara Üniversitesi Basımevi. Ankara – 1982