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.

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by

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1

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On Strongly Regular Dual Summability Methods

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SUMMARY

In this paper we define strongly regularity for a series method and determine necessary and sufficient conditions for a series method to be strongly regular. We also investigate the strongly regularity for dual summability methods.

1. INTRODUCTION

Let A be a summability method given by the sequence -to-sequence transformation (sequence method)

$$t_n = \sum_{k=0}^{\infty} a_{nk} s_k, \quad n = o, 1, 2, \dots \quad . \tag{1}$$

We suppose that, for each n

$$\sum_{k=0}^{\infty} a_{nk}$$

converges; this is a much weaker assumption than the regularity of A. Then we define

$$a'_{nk} = \sum_{i=k}^{\infty} a_{ni}.$$

We also suppose that the sequence (s_k) is formed by taking the partial sums of the series Σx_k; that is to say that

$$s_k = \sum_{v=0}^{k} x_v.$$

Let A′ denote the summability method given by the series -to- sequence transformation (series method).

$$u_n = \sum_{k=0}^{\infty} a'_{nk} x_k, \quad n = 0, 1, 2, \ldots \tag{2}$$

Conditions for the regularity of the methods A, A′ are well known [1].

It is called that the methods A, A′ are dual [2].

2. STRONGLY REGULAR METHODS

The strongly regularity of a sequence method has been defined by Lorentz, G.G., [3]. Similarly, we shall define strongly regularity for series methods.

Definition 2.1. Let Σx_k be a series such that its sequence of partial sums is almost convergent to a value s. If the method A′ sums every series Σx_k of this type, to the same value s then A′ is called strongly regular.

We can give following theorem:

Theorem 2.1. A regular matix A′ $= (a'_{nk})$ is strongly regular if and only if it satisfies the following conditions:

(i) $\lim_k a'_{nk} = o$, for each n

(ii) $\lim_n \sum_{k=0}^{\infty} | \Delta^2 a'_{nk} | = o$

where, $\Delta^2 a'_{nk} = \Delta (a'_{nk} - a'_{n,k+1})$.

Proof. Necessity. Let s_n be nth partial sum of the series Σx_k. Suppose that the sequence (s_n) is almost convergent. First, we shall show that the condition (i) is necessary. Suppose the contrary: Let $\lim_k a'_{nk} \neq o$, for some n. Then we can find a sequence (s_n) such that (s_n) is almost convergent and the following limit

$$\lim_n \sum_{k=0}^{\infty} a'_{nk} (s_k - s_{k-1})$$

does not exist.

Let $x = (x_k) = (1, -1, 1, -1, ...)$. Hence we Obtain the sequence (s_k) as $(s_k) = (1, 0, 1, 0, ...)$. It is easy to see that the sequence (s_k) is almost convergent to $1/2$. In this case,

$$\sum_{k=0}^{\infty} a'_{nk} x_k = \sum_{k=0}^{\infty} (-1)^k a'_{nk}$$

and the series

$$\sum_{k=0}^{\infty} (-1)^k a'_{nk}$$

does not convergent since $\lim_k a'_{nk} \neq o$. Therefore the limit

$$\lim_n \sum_{k=0}^{\infty} a'_{nk} x_k$$

does not exists. Because, for the existence of the above limit, the series

$$\sum_{k=0}^{\infty} a'_{nk} x_k$$

must be convergent for each n, firstly. Thus (i) is necessary.

Let us now show that (ii) is necessary. The sequence (s_n) is bounded since (s_n) is almost convergent. Hence we have $\lim_p a'_{np} s_p = o$. On the other hand, the partial sums of (1) and (2) are connected by the relations

$$\sum_{k=0}^{p} a'_{nk} x_k = \sum_{k=0}^{p-1} (a'_{nk} - a'_{n,k+1}) s_k + a'_{np} s_p. \quad (3)$$

Hence, as $p \rightarrow \infty$ we have

$$\sum_{k=0}^{\infty} a'_{nk} x_k = \sum_{k=0}^{\infty} (a'_{nk} - a'_{n,k+1}) s_k =$$

$$\sum_{k=0}^{\infty} (\Delta a'_{nk}) s_k$$

It is known that A is regular if and only if A' is regular [1]. Moreover it is also known that, if a regular matrix A sums all almost convergent sequences, then

$$\lim_n \sum_{k=0}^{\infty} |a_{nk} - a_{n,k+1}| = 0$$

[3]. Thus we get

$$\lim_n \sum_{k=0}^{\infty} |\Delta a_{nk} - \Delta a'_{n,k+1}| = \lim_n \sum_{k=0}^{\infty} |\Delta^2 a'_{nk}| = 0$$

Sufficiency .Now suppose (i) and (ii) hold and (s_n) is almost convergent to s.

As we mentioned above, it is known that A' is regular if and only if A is regular. Hence, considering the statement (3)
we have

$$\lim_n \sum_{k=0}^{\infty} a'_{nk} x_k = S.$$

Thus the proof is completed.

Theorem 2.2. Let A and A' be two dual summability methods. Then A is strongly regular if and only if A' is strongly regular.

Proof. The condition $\lim_k a'_{nk} = o$, for each n, is evidently satisfied since the series

$$\sum_{k=0}^{\infty} a_{nk}$$

converges for each n. Moreover, it is known that A is regular if and only if A' is regular. On the other hand

$$\lim_n \sum_{k=0}^{\infty} |a_{nk} - a_{n,k+1}| = 0$$

if and only if

$$\lim_n \sum_{k=0}^{\infty} |\Delta^2 a'_{nk}| = o.$$

since $a_{nk} - a_{n,k+1} = \Delta a'_{nk} - \Delta a'_{n,k+1} = \Delta^2 a'_{nk}.$

Thus the proof is completed.

ÖZET

Bu çalışmamızda, seri metodları için kuvvetli regülerliği tarif ettik, sonra bir seri metodunun kuvvetli regüler olması için gerek ve yeter şartları verdik. Ayrıca gösterdik ki dual toplama metodlarından birinin kuvvetli regüler olması için gerek ve yeter şart diğerinin kuvvetli regüler olmasıdır.

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[3] Lorentz, G. G., A Contribution to the theory of Divergent Sequences, Acta Math. 80 (1948), 167-190.