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TESTING FOR STRUCTURAL CHANGE UNDER HETEROSCEDASTICITY: A NOTE AND AN APPLICATION

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ABSTRACT

Testing for structural change or the Chow Test enjoyed numerous applications since its introduction by Chow. There have also been several extentions of this test. One of the more recent extentions is to the case of structures with unequal disturbance variances. This paper extends the computationally simple Lagrange Multiplier test of structural change under heteroscedasticity introduced by Erlat to the case of several structures. The result is then used in testing the Life-Cycle Hypothesis of saving utilizing household expenditure survey data of Ankara.

INTRODUCTION

Testing for structural change or testing for equality between sets of coefficients in two linear regressions has been widely investigated and used in econometric research since the celebrated work of Chow (1960) and the expository note of Fisher (1970). Chow considered the case of two subsamples and also the case where one of the samples does not have sufficient observations for estimation. The generalizations for the multiple subsamples are provided by Dhrymes (1978), Dufour (1982) and Erlat (1985). Dhrymes considered the case where each subsample has sufficient number of observations to allow the separate estimation of the coefficients. Dufour and Erlat considered the case of insufficient number of observations in some subsamples.

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Part of the maintained hypothesis of the Chow test states that variances of the error terms of the separate subsamples are equal to each other. The consequences of violation of this assumption of homoscedasticity is examined by Toyoda (1974) and Schmidt and Sickles (1977). If heteroscedasticity of error terms is significant between subsamples, the level of significance and the power of the test will be effected.

Toyoda derives the approximate distribution and Schmidt and Sickles derive the exact distribution of the test statistic under heteroscedasticity and both investigate the effect of departures from homoscedasticity on the significance level of the test via sampling experiments. Their results indicate that when the sample sizes are equal the true significance level is somewhat larger than the nominal level for large departures from the assumption of homoscedasticity. However, when the sample sizes are unequal the true and the nominal levels of significance diverge by large amounts in either direction.

Jayatissa (1977), Watt (1979), Tsurumi (1984), Rothenberg (1984), Erlat (1984), Honda and Ohtani (1986) and others proposed alternative testing procedures for testing structural change under the maintained hypothesis of unequal disturbance variances for the case of two subsamples.

This paper shows a simple extention of the Erlat's (1984) LM statistic for testing structural change under heteroscedasticity to the case of several subsamples and an application. Section II gives an overview of the alternative test procedures and the related sampling experiments. The extention is shown in section III. The application is provided in section IV. Section V concludes.

AN OVERVIEW OF ALTERNATIVE TESTS

An overview of the alternative tests of structural change in the presence of heteroscedasticity is provided in this section. The extent of the published work in this area continuing up until recently indicates that the proposals have been far from satisfactory. The review in this section omits the works within the framework of Bayesian analysis exemplified by Tsurumi and Sheflin (1985) and others.

The first proposal was by Jayatissa (1977). Jayatissa derives on exact test utilizing Hotelling's T^2 – statistic. It has the advantage of being an exact test. However, as discussed by Honda (1982) and

Tsurumi (1984) it is inefficient, not unique, difficult to compute and has low power. The inefficiency and non–uniqueness results from non –uniqueness of the orthogonal matrix which decomposes the cross–product matrices of the explanatory variables, partitioning of the residuals and an arbitary selection of a minimum number of observations when two sample sizes are unequal.

Watt (1979) introduced a more easily computed Wald (W) statistic. It's asymptotic distribution is derived by Honda (1982). Ohtani and Toyoda (1985a) derived the exact distribution of the Wald test under the condition of similarity of the nature of collinearity among the sample X's, that is, $(X'_2 X_2) = \lambda (X'_1 X_1)$ where λ is a positive scalar.

Sampling experiments are performed by Watt, Honda, and Ohtani and Toyoda to compare the small sample properties of the Jayatissa's test statistic and that of the Watt's Wald statistic. These experiments favor the Wald statistic over Jayatissa's for sample sizes of 30 or larger. While for small sample sizes (less than 30 and 25 respectively for the two subsamples) the conclusions were mixed.* These indicate that in small samples Jayatissa test has low power, and the Wald test has an upward bias in the true size of the test, that is, the true level of significance is larger than the nominal level of significance. These numerical results are confirmed by Honda and Ohtani (1986) who analytically show that the true size of the Wald test has a tendency to exceed its nominal size in finite samples. Watt had proposed the use of *ad hoc* size corrected critical value. The sampling experiments of Watt, Honda, and Ohtani and Toyoda show that the Wald test becomes preferable to Jayatissa's if this size corrected critical value is used.

Goldfeld and Quandt (1978) suggested an asymptotic Chow test. It is an F-criterion conditional on the least squares estimate of the ratio of variances in the two subsamples. It's small sample properties are investigated by Tsurumi and Sheflin (1985) along with another F-criterion conditioned on the posterior mean ratio of the standard deviations of error terms in two subsamples. They found both to perform well and the latter test to be somewhat superior.

The three classical test statistics, the Likelihood Ratio (LR), the Wald (W) and the Lagrange Multiplier (LM) statistics for testing structural change under heteroscedastoicity were derived independently

* Ohtani and Toyoda's (1985a) sampling experiments show that in small samples Jayatissa test is preferable to the Wald test when there is only one regressor. This conclusion reverses as the number of regressors increases.

by Rothenberg (1984) and Erlat (1984). Rothenberg (1984) derived the asymptotic expansion of the LR, W, and the LM statistics for testing structural change under hereroscedasticity and proposed theoretically justified size corrected critical values for each. Ohtani and Toyoda (1985b) examine the small sample properties of the size corrected LR, W and LM statistics and find that the performances of the size corrected W and LM statistics are very good.

Erlat (1984) provides simple expressions for the LR, W and LM statistics for testing structural change under heteroscedasticity in terms of the sum of squared residuals from the restricted and unrestricted estimation of the model under study and proves their asymptotic distribution. Computationally, the simplest of the three statistics is the LM statistic, as it could be obtained by using OLS estimates of the restricted and unrestricted models. He also proposed an LM statistic for the joint test of structural change and unequal variance. This too could be based on the OLS estimates of the restricted and the unrestricted models involved.

Next section shows an extention of the Erlat's LM statistic for testing structural change under heteroscedasticity to the case of several subsamples.

THE MODEL

We consider the following linear regression model:

$$y = X\beta + u \tag{1}$$

Where y is T-vector of observations on the dependent variable, X is TxK matrix of observations on nonstochastic regressors and has rank equal to K. β is K-vector of coefficients and u is T-vector of disturbances. Now, we assume m subsamples generated by different structures of the above model as follows:

$$y_i = X_i \beta_i + u_i \qquad i = 1, \ldots, m.$$

where, y_i are T_i x 1, X_i are T_i x K, u_i are T_i x1 and β_i, are Kx1,

$i = 1, \ldots, m$, and $\sum_{i=1}^{m} T_i = T$.

We assume $T_i > K$, i.e. there are sufficient number of observations in each subsample to carry out the estimation.

These m different equations can be combined into a single equation by making use of the dummy variables:

$$y = Z\delta + u \quad \text{where,} \tag{2}$$

$$Z = (X, D_2, \ldots, D_m),$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_m \end{bmatrix} \qquad D_i = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ X_i \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \qquad \delta = \begin{bmatrix} \beta \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_m \end{bmatrix}$$

so that,

$$y = X\beta + \sum_{i=2}^{m} D_i \alpha_i + u$$

Here, Z is the above defined TxmK matrix of regressors, X is TxK. The D_i, $i = 2, \ldots, m$ are T x K matrices of dummy variables with X_i in the i^{th} position and 0 otherwise. 0 matrices are T_ixK. δ is the mKx1 vector of coefficients.

It can be shown that β is the K-coefficient vector for the first subsample, and the α_i $i = 2, \ldots, m$ are K-coefficient vectors corresponding to the dummy variables reflecting differentials of the (m–1) structures from that of the first structure. That is, $\alpha_i = \beta_i - \beta_1$, $i = 2, \ldots, m$.

The hypothesis to be tested can be formulated as:

$$H_0: \begin{bmatrix} 0 & I_K & 0 & 0 \ldots 0 \\ 0 & I_K & 0 & 0 \ldots 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 \ldots I_K \end{bmatrix} \begin{bmatrix} \beta \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_m \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_m \end{bmatrix} = 0$$

If we assume that $u_i \sim N(0, \sigma^2 I_{Ti})$ then it is well–known that the Chow test of testing for structural change can be carried out within the framework of dummy variables using the following statistic:

$\dfrac{(S_R - S_U) / K}{S_U / (T\text{–}2K)}$ which is distributed as a central-F with K and T–2K degrees of freedom under the null hypothesis. Here, S_R is the sum of squared residuals from the restricted model in (1) and S_U is the sum of squared residuals from the unrestricted model in (2). The maintained hypothesis of this test is $\sigma_i^2 = \sigma^2$, $i = 1, \ldots, m$.

However, sometimes the model is heteroscedastic, i.e., $u_i \sim N(0, \sigma_i^2 I_{Ti})$, $i = 1, \ldots, m$, so that $u \sim n(o, V)$ where, $V = \text{diag}(\sigma_1^2 I_{T1}, \ldots, \sigma_m^2 I_{Tm})$, and I_{Ti} are $T_i \times T_i$ identity matrices.

Let us first look at the LM–statistic for testing structural change under heteroscedasticity. The restricted and the unrestricted models given as in (1) and (2) respectively and the disturbances have the above variance covariance structure under both null and alternative hypothesis.

The general form of the LM–statistic is:

$$LM = \hat{S}_R - \hat{S}_{UR} \qquad (3)$$

where following the notation of Erlat (1984) and of Breusch (1979), $\hat{S}_R$ is the sum of squared residuals of the restricted model, and $\hat{S}_{UR}$ is the sum of squared residuals of the unrestricted model using the restricted model estimates of the disturbance variances.

To apply this to the present case we need to obtain the restricted and the unrestricted estimators of δ, and σ_i^2 $i = 1, \ldots, m$. First note that

$$\hat{\delta}_U = (Z'\hat{V}_U^{-1}Z)^{-1}Z'\hat{V}_U^{-1}y, \quad \text{and}$$

$$\hat{\delta}_{UR} = (Z'\hat{V}_R^{-1}Z)^{-1}Z'\hat{V}_R^{-1}y.$$

Now, we will show that both of these two estimators are identical to the unnestricted OLS estimator of δ given by $\tilde{\delta} = (Z'Z)^{-1} Z'y$.

To show this let us first look at the components of $\hat{\delta}_U$, the result will also follow for $\hat{\delta}_{UR}$. Note that

$$Z = (X, D_2, D_3, \ldots, D_m) = \begin{bmatrix} X_1 & 0 & 0 & \ldots & 0 \\ X_2 & X_2 & 0 & \ldots & 0 \\ X_3 & 0 & X_3 & \ldots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ X_m & 0 & 0 & \ldots & X_m \end{bmatrix}$$

$$(Z'\hat{V}_U^{-1}Z) = \begin{bmatrix} S & S_2 & S_3 & S_4 & \dots & S_m \\ S_2 & S_2 & 0 & 0 & \dots & 0 \\ S_3 & 0 & S_3 & 0 & \dots & 0 \\ S_4 & 0 & 0 & S_4 & \dots & 0 \\ . & . & . & . & \dots & . \\ S_m & 0 & 0 & 0 & \dots & S_m \end{bmatrix}$$

where $S = \sum_{i=1}^{m} S_i$ and $S_i = \sigma_i^{-2} (X_i' X_i)$. The S, S_i and 0's are each K x K so that the result is a KmxKm partioned matrix.* There are nonzero elements in the first row and the first column. There is a diagonal partitioned matrix on the lower right hand side with S_2, ..., S_m on the diagonal. The inverse of this matrix is given by the following:

$$(Z'\hat{V}_U^{-1}Z)^{-1} = \begin{bmatrix} S_1^{-1} & -S_1^{-1} & -S_1^{-1} & \dots -S_1^{-1} \\ -S_1^{-1} & S_1^{-1} + S_2^{-1} & S_1^{-1} & \dots S_1^{-1} \\ -S_1^{-1} & S_1^{-1} & S_1^{-1} + S_3^{-1} & \dots S_1^{-1} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ -S_1^{-1} & S_1^{-1} & S_1^{-1} & \dots S_1^{-1} + S_m^{-1} \end{bmatrix}$$

$$(Z'\hat{V}_U^{-1}y)^{-1} = \begin{bmatrix} \sum_{i=1}^{m} \sigma_i^{-2} X_i' y_i \\ \sigma_2^{-2} X_2' y_2 \\ \sigma_3^{-2} X_3' y_3 \\ . \\ . \\ . \\ \sigma_m^{-2} X_m' y_m \end{bmatrix}$$

* For the two, three and four subsamples cases the matrix is given by respectively:

$$\begin{bmatrix} S & S_2 \\ S_2 & S_2 \end{bmatrix}, \quad \begin{bmatrix} S & S_2 & S_3 \\ S_2 & S_2 & 0 \\ S_3 & 0 & S_3 \end{bmatrix}, \quad \begin{bmatrix} S & S_2 & S_3 & S_4 \\ S_2 & S_2 & 0 & 0 \\ S_3 & 0 & S_3 & 0 \\ S_4 & 0 & 0 & S_4 \end{bmatrix}$$

And finally:

$$\hat{\delta}_U = (Z'\hat{V}_U^{-1}Z)^{-1}Z'\hat{V}_U^{-1}y = \begin{bmatrix} (X'_1X_1)^{-1}X'_1y_1 \\ (X'_2X_2)^{-1}X'_2y_2 - (X'_1X_1)^{-1}X'_1 y_1 \\ (X'_3X_3)^{-1}X'_3y_3 - (X'_1X_1)^{-1}X'_1 y_1 \\ \cdot \\ \cdot \\ \cdot \\ (X'_mX_m)^{-1}X'_my_m - (X'_1 X_1)^{-1}X'_1 y_1 \end{bmatrix} = \begin{bmatrix} \tilde{\beta} \\ \tilde{\alpha}_2 \\ \tilde{\alpha}_3 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\alpha}_m \end{bmatrix}$$

which are OLS estimates of the unrestricted model. Tilda denotes the OLS estimators.

Thus, $\hat{\delta}_U = \tilde{\delta}_U$. The same result will also hold for $\hat{\delta}_{UR}$ with $\hat{V}_R^{-1}$, since the σ_i^2 terms cancel nicely. We thus have, $\hat{\delta}_U = \hat{\delta}_{UR} = \tilde{\delta}_U$. Then, the $\hat{S}_{UR}$ is given by:

$$\hat{S}_{UR} = (y - Z\,\hat{\delta}_{UR})'\ \hat{V}_R^{-1}\ (y - Z\,\hat{\delta}_{UR}) = \sum_{i=1}^{m} \hat{\sigma}_{iR}^{-2}\ \tilde{S}_{iU},$$

This result is obtained since $\hat{\delta}_{UR} = \tilde{\delta}_U$. $\hat{\sigma}^2_{iR}$ will be obtained as below.

Now, we obtain the restricted estimators $\hat{\delta}_R$, and $\hat{\sigma}_{iR}$:

$$\hat{\delta}_R = \begin{bmatrix} \hat{\beta}_R \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} (X'\hat{V}_R^{-1}X)^{-1}X'\ \hat{V}_R^{-1}\ y \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Define $\hat{U}_R = y - x\,\hat{\beta}_R$ and partition $\hat{U}_R$ as $\hat{U}_R = \begin{bmatrix} \hat{U}_{1R} \\ \hat{U}_{2R} \\ \cdot \\ \cdot \\ \cdot \\ \hat{U}_{mR} \end{bmatrix}$

and $\hat{S}_{iR} = \hat{U}'_{iR}\hat{U}_{iR}$. We can show that:

$$\hat{\sigma}_{iR}^2 = \hat{S}_{iR} / T_i \quad i = 1, \ldots, m \tag{4}$$

which could be used in the above expression for $\hat{S}_{UR}$.

Finally, $\hat{S}_R$ is obtained as follows:

$$\hat{S}_R = (y - Z\,\hat{\delta}_R)'\ \hat{V}_R^{-1}\ (y - Z\,\hat{\delta}_R) = (y - X\,\hat{\beta}_R)'\ \hat{V}_R^{-1}(y - X\,\hat{\beta}_R)$$

$$= \sum_{i=1}^{m} \hat{\sigma}_{iR}^{-2}\ \hat{S}_{iR}$$

Substituting for $\hat{\sigma}_{iR}^2 = \hat{S}_{iR} / T_i$ we have:

$$\hat{S}_R = \sum_{i=1}^{m} \frac{T_i}{\hat{S}_{iR}}\ \hat{S}_{iR} = \sum_{i=1}^{m} T_i = T$$

Hence, $\hat{S}_R = T$, $\hat{S}_{UR} = \Sigma\, \hat{\sigma}_{iR}^{-2}\ \tilde{S}_{iU}$, substituting these into the expression for LM in (3) gives the LM statistic for testing structural change under heteroscedasticity as follows:

$$LM = T - \sum_{i=1}^{m} \hat{\sigma}_{iR}^{-2}\ \tilde{S}_{iU}. \tag{5}$$

Erlat (1984) proves for the case of two subsamples that the above LM statistic is asymptotically distributed as χ^2 with K degrees of freedom under the null hypothesis.

This is a rather simple statistic to compute since all of its components can be obtained by using OLS as follows: To obtain $\hat{\sigma}^2_{iR}$, estimate the restricted model by OLS and obtain the residuals. Then, partitioning the vector of residuals according to the subsamples to compute $\hat{\sigma}_{iR}^2$ using the formula in (4). To obtain $\tilde{S}_{iU}$, estimate the restricted model for each of the subsamples and compute their sum of squared residuals. Alternatively, $\hat{S}_{UR}$ can be obtained directly by the OLS estimate of the unrestricted model with the appropoiate weighting of the data by $1 / \hat{\sigma}_{iR}$. This alternative procedure may be easier or more difficult depending on the computational facilities available.

The next section shows an application.

APPLICATION

For the purposes of application of the LM statistic given in (5), for testing structural change under heteroscedasticity we use the data derived from the household expenditure survey of Ankara. The survey was conducted by the State Institute of Statistics in 1965 in Ankara municipality boundaries covering all socio–economic groups. The information about household income, expenditures, demographic and other characteristics were collected by periodic interviews with individual households. The survey period was one month and the sample size was 494 households. Household is defined as the group of people with economic ties living under the same shelter. There are both nucleus and extended family types in the sample. The administrative units in Ankara were grouped into low, middle and high income categories. The systematic multistage stratified sampling method was used. Saving is obtained as the difference between total income and total expenditures.

The Life–Cycle–Hypothesis of saving postulates that household saving is determined by the position of the household head in the life-cycle. Accordingly, saving behavior will vary depending on the age of the household head. We expect to find significant differences in the savings of various groups if we group the households according to the age of the household head. Following functional relationship between per capita saving and per capita income is specified:

$$(S / N)_{ij} = \alpha_j + \beta_j \, (Y / N)_{ij} \quad \text{where,}$$

S = saving, N = household size, Y = income, i = denotes the household, j = denotes the age group of the household head. There are four age categories defined as below. We would like to test for the structural difference in the saving behaviour of these four age groups. Heteroscedasticity is assumed since we have cross–section data (for further on this see below). Following computations are carried out to obtain the LM in (5):

Age	T_i	$\hat{S}_{iR}$	$\hat{\sigma}^2_{iR}$	$\tilde{S}_{iU}$
20–29	44	1 216 695	27 652	651 362
30–39	154	1 073 790	6 974	1 001 310
40–49	104	779 290	7 493	657 410
50–59	106	1 744 809	16 461	1 716 690
60 +	86	1 135 209	13 200	1 066 990
Total	494	5 949 793		5 093 762

$$\tilde{S}_U = \sum_{i=1}^{5} \hat{S}_{iU} = 5\ 093\ 762$$

$$\hat{S}_R = 5\ 949\ 793$$

$$\hat{\sigma}_R^2 = 12\ 044.12$$

$$LM = T - \sum_{i=1}^{5} \hat{\sigma}^{-2}{}_{iR}\ \tilde{S}_{iU} = 494 - 440 = 54$$

The critical value of χ^2 with 2 degrees of freedom are 5.99 and 9.21 at 5 and 1 per cent levels of significance respectively. Thus, we reject the null hypothesis and conclude that the difference in the saving behavior of the four age-groups are significant.

We now test jointly for structural change and heteroscedasticity. The relevant LM statistic is given by Erlat (1984). It is the sum of the LM statistic for the test of structural change under homoscedasticity and the LM statistic for the test of homoscedasticity under the assumption of no structural change. Hence, the two components can be obtained separately. If the null hypothesis is not rejected then no further testing is needed. If the null hypothesis is rejected then the above two components can further be used to identify the source of rejection. Following is the formula:

$$LM = T\left(\frac{\tilde{S}_R - \tilde{S}_U}{\tilde{S}_R}\right) + \frac{1}{2}\sum_{i=1}^{m} T_i^{-1}\left(\frac{\tilde{S}_{iR}}{\tilde{\sigma}_R^2} - T_i\right)^2$$

which is distributed as $\chi_1^2 + \chi_K^2 = \chi^2{}_{K+1}$. This is calculated to be:

$$LM = 71.08 + 114.77 = 185.85$$

χ^2 – critical values:

df	5 %	1 %
1	3.84	6.34
2	5.99	9.21
3	7.82	13.28

Thus we reject the null hypothesis of homoscedasticity and no structural difference. Examination of the components values and the relevant critical values both provided above indicate that both heteroscedasticity and structural differences were the reason for this rejection.

We conclude that the age–groups are heteroscedastic and there are significant differences in the saving behavior of the various age groups defined according to the age of the household head.

CONCLUSION

This paper provides an extention of the Lagrange Multiplier test of structural change under heteroscedasticity introduced by Erlat to the case of several subsamples. This test is computationally rather simple as it is possible to obtain all of its components by ordinary least squares estimates of the restricted and the unrestricted models. An application of this test is provided to testing Life–Cycle Hypothesis of saving. The data used is the result of household expenditure survey conducted in the city of Ankara. The test indicated that there are significant differences in the saving behavior of the households differentiated by the age of the household head. Further testing indicated that both the differences in saving behavior and the heteroscedasticity of the subsamples contributed to this finding.

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REFERENCES

BREUSCH, T.S. 1979. Conflict Among Criteria for Testing Hypothesis: Extentions and Comments, *Econometrica*, 47, 203–207.

CHOW, G.C. 1960. Tests of Equality Between Sets of Coefficients in Two Linear Regressions, *Econometrica*, 28, 591–605.

DHRYMES, P.J. 1978. *Introductory Econometrics*, Springer-Verlag, New York.

DUFOUR, J.M. 1982. Generalized Chow Tests for Structural Change: A Coordinate Free Approach, *International Economic Review*, 23, 565–575.

ERLAT, H. 1983. *Specification Tests in Econometrics*, METU Publications, Ankara, (In Turkish).

ERLAT, H. 1984. Testing For Structural Change Under Heteroscedasticity: A Note, *The Manchester School*, 52, 380–390.

ERLAT, H. 1985. Testing For Structural Change At More Than One Swith Point: Inadequte Degrees of Freedom and Dummy Variables, *The Bulletin of Oxford Institute of Economics And Statistics*, 47, 273–74.

FISHER, F.M. 1970. Tests of Equality Between Sets of Coefficients in Two Linear Regressions: An Expository Note, *Econometrica*. 38, 361–366.

GOLDFELD, S.M. and R.E. QUANDT 1978. Asymptotitc Tests for the Constancy of Regressions in the hetesoscedastic Case, Mimeo. E.R.P. Princeton University.

HONDA, Y. 1982. On Test of Equality Between Sets of Coefficients in Two Linear Regressions when Disturbance Variances are Unequal. *The Manchester School*, 50, 116–125.

HONDA, Y. and K. OHTANI 1986. Modified Wald Tests In Tests of Equality Between Sets of Coefficients in Two Linear Regressions Under Heteroscedasticity, *The Manchester School*, 54, 208–218.

JAYATISSA, W.A. 1977. Test of Equality Between Sets of Coefficients in two Linear Regressions When Disturbance Variances are Unequal, *Econometrica*, 45, 1291–1292.

JUDGE, G.G., W.E. GRIFFITHS, R.C. HILL and T.C. LEE 1980. *The Theory and Practice of Econometrics*, Wiley, New York.

OHTANI, K. and T. TOYODA 1985a. Small Sample Properties of Tests of Equality Between Sets of Coefficients in Two Linear Regressions Under Heteroscedasticity, *International Economic Review*, 26, 37–44.

OHTANI, K. and T. TOYODA 1985b. A Monte Carlo study of the Wald, LM and LR Tests in a Heteroscedastic Linear Model, *Communications in Statistics, Simulation and Computation*, 14, 735–746.

ROTHENBERG, T.J. 1984. Hypothesis Testing in Linear Models When the Error Covariance Matrix is Nonscalar, *Econometrica*, 45, 1293–1298.

SAVING, N.E. 1976. Conflict Among Testing Procedures in a Linear Regression Model with Autoregressive Disturbances, *Econometrica*, 44, 1303–1315.

SCHMIDT, P., and R. SICKLES 1977. Some Further Evidence On the Use of the Chow Test Under Heteroscedasticity, *Econometrica*, 45, 1293–1298.

TSURUMI, H. 1984. On Jayatissa's Test of Constancy of Regressions Under Heteroscedasticity, *Economic Studies Quarterly*, 35, 57–62.

TSURUMI, H., and N. SHEFLIN 1985. Some Tests for the Constancy of Regressions Under Heteroscedasticity, *Journal of Econometrics*, 27, 221–234.

TOYODA, T. 1974. Use of the Chow Test Under Heteroscedasticity, *Econometrica*, 42, 601–608.

WATT, P.A. 1979. Test of Equality Between Sets of Coefficients in Two Linear Regressions when the Disturbance Variances are Unequal: Some Small Sample Properties, *The Manchester School*, 47, 391–396.