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Almost Regularity Of Dual Summability Methods

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Almost Regularity Of Dual Summability Methods

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SUMMARY

In this paper, we have defined almost regularity for a series method and investigated ne-essary and sufficient conditions for a series method to be almost regular. We also examined the almost regularity for dual summability methods.

1. INTRODUCTION

Let l_∞,c,c_o and γ be the linear space of all bounded sequences, convergent sequences, null sequences and convergent series with complex terms, respectively. A sequence $s = (s_n) \in l_\infty$ is almost convergent if and only if

$$\lim_{p\to\infty} \frac{1}{p} \sum_{v=n}^{n+p-1} s_v = b$$

uniformly in n, [4]. This is written as f-lims = b. We denote by F the space of all almost convergent sequences and by F_o the subspace of F consisting of all sequences almost convergent to zero. If a sequence $s = (s_n)$ is convergent and its limit b, then the sequence (s_n) is almost convergant to b, but not convesely in general.

Now let $A = (a_{nk})$ be an infinite matrix with complex terms. If the series

$$(1)\quad t_n = \sum_{k=1}^{\infty} a_{nk}\, s_k$$

converges for $n = 1,2,\dots$, then $A(s) = (t_n)$ is said to be the A-transform of s. Hence, the summability method A is sequence method.

The matrix $A=(a_{nk})$ is called regular if the A–transform of s is convergent to the limit of s for each $s \in c$.

A sequence s is said to be almost A- summable if the A-transform of s is almost convergent.

The matrix $A=(a_{nk})$ is called almost regular if the sequence A(s) almost almost converges to the limit of s for each $s \in c$.

Throughout the paper, the sums will be taken from 1 to ∞.

The following theorem is due to King, J.P., [2].

THEOREM.1.1. The matrix $A=(a_{nk})$ is almost regular if and only if

(i) $\|A\| = \sup_n \sum_k | a_{nk} | < \infty$

(ii) $\text{f-}\lim_n a_{nk} = 0$, for each k

(iii) $\text{f-}\lim \sum_{k=1}^{\infty} a_{nk} = 1$.

The following result may easily be obtained:

COROLLARY. 1.2. A matrix $A = (a_{nk})$ transforms c_o into F_o (i.e., $A \in (c_o, F_o)$) if and only if

(i) $\|A\| = \sup_n \sum_k |a_{nk}| < \infty$

(ii) $\text{f-}\lim a_{nk} = 0$, for each k.

2. DUAL SUMMABILITY METHODS.

Let $A = (a_{nk})$ be a sequence method given by (1). Suppose that, for each n, the series

$$\sum_{k=1}^{\infty} a_{nk}$$

converges; this is a much weaker assumption than the regularity of A. Then we define

$$b_{nk} = \sum_{i=k}^{\infty} a_{ni}.$$

Further, suppose that the sequence (s_k) is defined by

$$s_k = \sum_{i=1}^{k} x_i.$$

Let B denote the summability method given by the series - to - sequence transformation (series method)

$$(2) \quad v_n = \sum_{k=1}^{\infty} b_{nk} x_k, \ (n=1,2,...).$$

The methods A and B are called dual summability methods, [3] It is well-known that A is regular if and only if B is regular, [1].

3. MAIN RESULTS

The strong regularity of a sequence method has been defined by Lorentz, G.G., [4].

Recently, Öztürk, E., [5], have defined strong regularity of a series method and showed that A is strongly regular if and only if B is strongly regular, where A and B are two dual summability methods.

As we mentioned in Section 1, the almost regularity of a sequence method has been introduced by King, J.P., [2].

Similarly, we shall define almost regularity for series method and also investigate almost regularity of dual summability methods.

Let us suppose throughout that the method B is a series method.

DEFINITION. 3.1. Let $\sum_k x_k$ be a convergent series. If the method B almost sums the series $\sum_k x_k$, i.e., if f-lim B (x) exists, then B is said to be almost conservative. Moreover, if f–lim B (x) $= \sum_k x_k$ whenever $x=(x_k) \in \gamma$, then B is called almost regular.

Now we can give the following

THEOREM. 3.2. A matrix $B=(b_{nk})$ is almost regular if and only if

i) $\sup_n \sum_k |\Delta b_{nk}| < \infty$

ii) f–lim $b_{nk} = 1$, for all k,

where $\Delta b_{nk} = b_{nk} - b_{n,k+1}$.

Proof. Let us suppose that B is almost regular. Then, for every $x \in \gamma$

$$B_n(x) = \sum_k b_{nk} x_k$$

converges, for each n, and f–lim $B_n(x) = \sum_k x_k$. If we now put $x=e_k$, $(k=1, 2, \ldots)$, then the necessity of (ii) is trivial, where e_k is a sequence whose k-th component is one and the others are zero.

Now, by Abel's partial summation, we get

$$(3) \quad \sum_{k=1}^{m} b_{nk} x_k = \sum_{k=1}^{m-1} \Delta b_{nk} (s_k - a) + a\, b_{n1} + (s_m - a)\, b_{nm}$$

for each n, where $(x_k) \in \gamma$ and s_m is the m–th partial sum of the series $\sum_k x_k$ and $\sum_k x_k = a$. On the other hand, it is well-known that, if $\sum_k b_{nk} x_k$ converges for each n whenever $x \in \gamma$, then

$$\sup_k |b_{nk}| < \infty, \text{ (for each n).}$$

Hence, letting $m \to \infty$ in (3), we get

$$(4) \quad \sum_k b_{nk} x_k = \sum_k \Delta b_{nk} (s_k - a) + a . b_{n1}.$$

Since f–lim $B(x) = a$ exists and f–lim $b_{n1} = 1$, then we must have

$$\left(\sum_k \Delta b_{nk} (s_k - a) \right) \in F_o.$$

Therefore, one can easily see that (i) is necessary since $(s_k - a) \in c_o$.

Conversely, suppose now that the condition (i) and (ii) hold. From (i), the series $\sum_k (b_{nk} - b_{n,k+1})$ is convergent and $\lim_m b_{nm}$ exists for every n. So the statement (4) holds for each $x \in \gamma$ with $\sum_k x_k = a$. By (i) and (ii), it is easily seen that

$$C = (\Delta b_{nk}) \in (c_o, F_o)$$

since the conditions of Corollary. 1.2 are satisfied. Hence f–lim $B(x) = a$ is obtained. Thus the proof is completed.

THEOREM. 3.3. Let A and B be two dual summability methods. Then A is almost regular if and only if B is almost regular.

Proof. Since

$$b_{nk} = \sum_{i=k}^{\infty} a_{ni},$$

$\Delta b_{nk} = b_{nk} - b_{n,k+1} = a_{nk}$, for all n,k. Hence, $\sup_n \sum_k |a_{nk}| < \infty$ if and only if $\sup_n \sum_k |\Delta b_{nk}| < \infty$. Further, f–lim $\sum_k a_{nk} = 1$ and f–lim $a_{nk} = 0$, for each k, if and only if f–lim$b_{nk} = 1$, (for each k). Now the proof follows from Theorem 1.1 and Theorem 3.2.

ÖZET

Bu çalışmada, bir seri metodunun hemen hemen regülerliğini tanımladık ve bir seri metodunun hemen hemen regüler olması için gerek ve yeter şartları verdik. Ayrıca, dual toplanabilme metotlarının hemen hemen regülerliğini araştırdık.

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