Commun. Fac. Sci. Univ. Ank., Series A_1
V. 35, pp. 67 - 81 (1986)

RELATIONSHIP BETWEEN CENTRAL PROJECTION AND REFLECTING ACTION

MEHMET PALAMUTOĞLU

Faculty of Sciences and Arts, University of Erciyes, Kayseri, Turkey

(Received: 9 January 1986)

ABSTRACT

This paper was presented in the First Saudi Engineering Conference, on May 7.11.1983 in Jeddah, Saudi Arabia. This work is in the area of descriptive geometry and it aims to put forward the relationship between the central and reflected projections of points, lines and planes which take place in space, to obtain practical results through this relation. The projections of elements mentioned above with respect to the system established with this purpose and the practical applications derived from theoretical drawings are pointed out.

INTRODUCTION

This work presents a constructive study of the relationship between the perspective (central projection) of an object on a drawing paper and its reflected projection.

The perspective is obtained by connecting object points to the projection plane by straight lines passing through the projection center [Akın (1967)].

The reflected projection is best visualised by considering the center of projection as a light source and the reflecting plane as a mirror. Light rays passing through object points reflect from the mirror and form an image on the picture (projection) plane [Horninger (1967)].

In this study the object is limited to a plane coinciding with the reflecting plane. It is shown that a relationship of the Perspective Colineation exists between the central and reflected projections for a given object [Horninger (1967)]. The axis of the colineation is the intersection of the reflecting plane with the picture plane. The colineation center is the mirror image of the center of projection with respect to the reflecting plane. Rays of colineation consist of the rays reflected [Horninger (1962)].

This result facilitates finding a reflected projection from the corresponding Central Projection and vice versa. Therefore a complicated or laborious reflected projection can conveniently be obtained from its simpler version to find central projection. It would also help determination of the limits of partial reflected projections whose characteristic points lie out of the picture plane.

In order to obtain the above-mentioned relationship between the two projection, a system has been developed with the name "Central-Reflected Projection" which has been described briefly below. Points, straight lines and plane figures have been studied in this system. Finally, the expected relationship between the two projections and its practical results have been observed and indicated respectively.

CENTRAL-REFLECTED PROJECTION SYSTEM

Let us take a point M in space as the center of perspective, a plane ε which carries the elements like a point A.., a line d.., and a plane $\lambda =$ ABC.. as the reflecting plane and also a plane π which makes an angle α and intersects along with ε as a picture plane. The system thus formed will be called the "Central-Reflected Projection System". When we consider the center of perspective as the source of light in a central illumination, the picture plane as the plane of perspective and reflecting plane as a plane mirror in the lightening, the present search for a relationship between the central illumination and the reflected projection is warranted.

NOTATIONS

The notations used in this paper are shown below with their brief explanations:

M : Center of perspective or illumination

π : Picture plane or plate of perspective

ε : Reflecting plane which makes an angle α with picture plane

M_o			:	Mirror point of M with respect to reflecting plane
x			:	Ground line or the intersection of vertical projection plane with π (3–7)
$e_{,1}$ e_2			:	Horizontal and vertical traces of the plane ε which is perpendicularly taken to the vertical projection plane (3–13)
A..			:	Points to be projected
d..			:	Lines to be projected
λ..			:	Plane figures to be projected
A_m,	d_m,	λ_m	:	Central projection of A, d, λ
A_o,	d_o,	λ_o	:	Reflected projection of A, d, λ
A',	d',	λ'	:	Horizontal projection of A, d, λ (3–7)
A'',	d'',	λ''	:	Vertical ,, ,, ,,
A'_m,	d'_m,	λ'_m	:	Horizontal ,, ,, A_m, d_m, λ_m
A''_m,	d''_m,	λ''_m	:	Vertical ,, ,, ,,
A_o',	d_o',	λ_o'	:	Horizontal ,, ,, A_o, d_o, λ_o
A_o'',	d_o'',	λ_o''	:	Vertical ,, ,, ,,

ASSUMPTIONS

In order to facilitate drawings, let the reflecting plane ε which carries the elements A.., d.., λ be perpendicular to the vertical projection plane and make an angle α with the picture plane. In this case horizontal trace e of ε will be perpandicular to the ground line x; the vertical trace e_2 of ε will coincide with the vertical projection of this plane. The elements cited above will be on the trace $e_2 = \varepsilon''$ in the vertical projection. The straight line d_2 which joins the projection center M to its mirror point M_o and intersects the plane ε at the point I, as d_1, is perpendicular to the trace e_1 in vertical projection, and as d_2, to the trace e_2 in horizontal projection. The projection center M has always been taken under the plane ε in all figures. The mirror point M_o of M will consequently take place above it.

REPRESENTATION OF ELEMENTS

1. POINTS:

While we take into consideration the ε plane as a field of points we classfy these points into two groups:

Points in a General Situation

The projection of any point A situated on the plane ε with regard to the center M on the picture plane π, briefly its perspective, is A_m, and the reflected projection of the same point A with respect to the center M_o is A_o (FIG–1). A_m and A_o have been obtained by joining the point A to M and its mirror point M_o with the perspective and reflecting rays

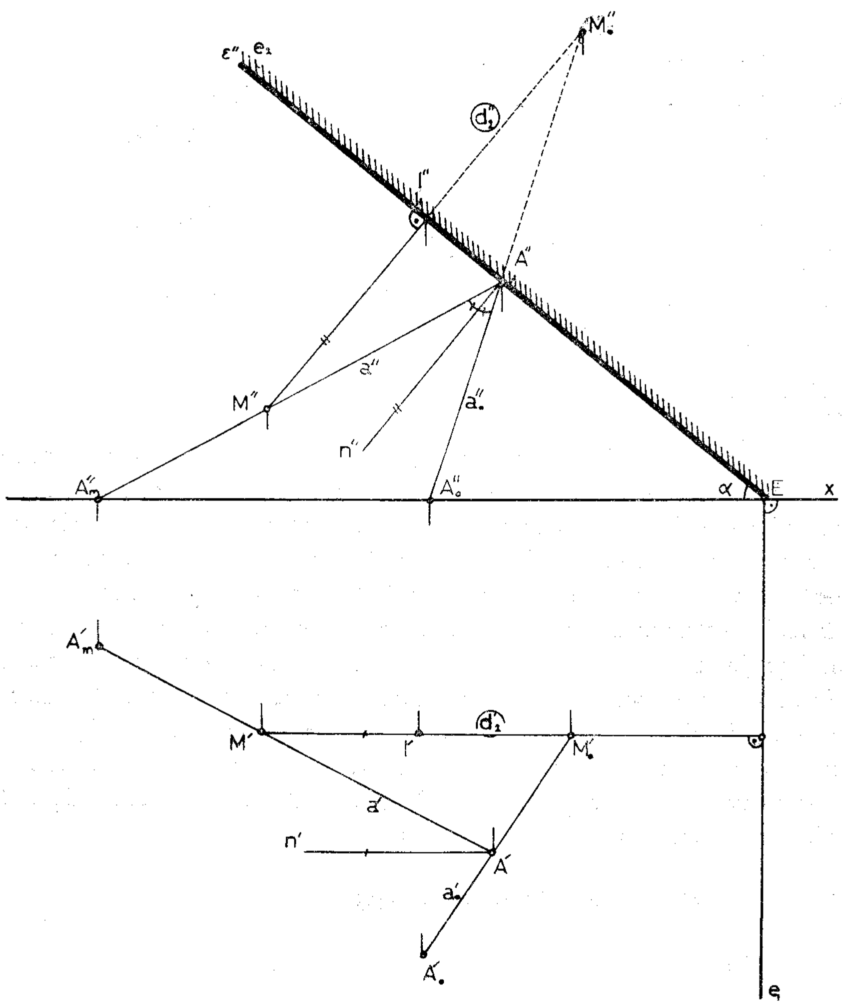

FIG-1

a, a_o, respectively. On the picture plane π the two rays are symmetrical with regard to the plane ε and to the normal n at the point A. An important result can be derived from here:

"At the Central-Reflected Projection System, every point A belonging to the reflecting plane ε can biunivoquely be represented by the pair A_m, A_o on the picture plane π. The projection of the pair in both horizontal and vertical planes can be obtained by joining the related projection of A to the projections of M and M_o with the projections of the rays a, a_o in these planes. The projections of a, a_o intersect at the projection of A in the both planes. In the case of presence of the center M, the planes π and ε, the given point A in the plane ε corresponds to the pair A_m, A_o being taken at random in the plane π".

Points in a Particular Situation

The particular points of the plane ε are described and listed below (FIG–2):

1– Points at infinity or Points of Symmetry

The pair S_m, S_o of every point S on the line at infinity carried by the plane ε consists of the traces of the straight lines s, s_o on the plane π which parallely passes through the points M and M_o to the plane ε and to each other. These lines are symmetrical with regard to the trace e_1. The points of ε at infinity are called "Symmetry Points of the plane ε in the system." the projections S_m, S_o of these points are the central and the reflected Vanishing Points of the rays of s, s_o.

2– Trace Points or Coincidence Points

Every point E on the trace e_1 of the plane ε is represented by the pair E_m, E_o each of which coincides with the other and E. This kind of points are called the Coincidence Points of the plane ε [Palamutoğlu (1973)].

3– Reflecting Disappearance Points

From the pair F_m, F_o of every point F on the line of the plane ε which has the same altitude as M_o, the projection F_m is the proper point of π and the projection F_o is the point of π at infinity. For this reason the points F is called the Reflecting Disappearance Points of the plane ε.

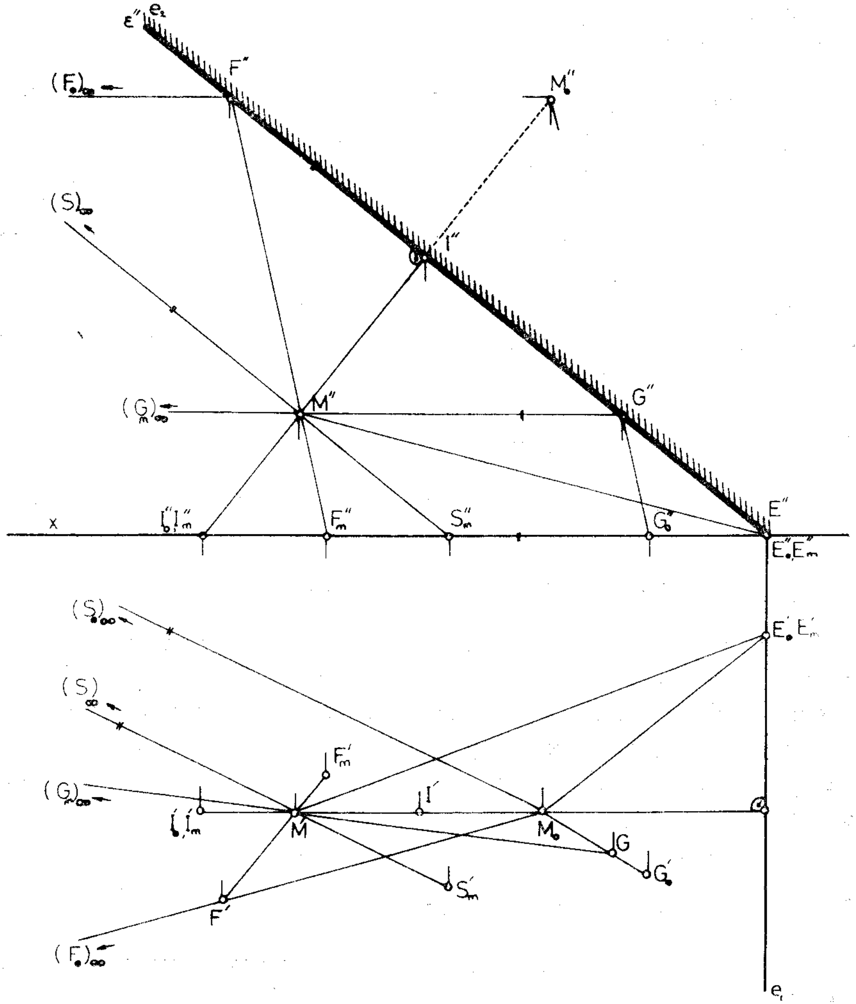

FIG-2

4– Central Disappearance Points

From the pair G_m, G_o of every point G on the line of the plane ε which has the same altitude as M, the projection G_o is the proper point of π; the projection G_m' is the point of at the infinity. For this reason the points G are called the Central Disappearance Points of the plane ε.

5- Incidence Points

The pair I_m, I_o of every point I of the plane ε which has the same altitude as I, consists of the trace of the straight line d_1 as a single point in the plane π. Otherwise, the pair I_m, I coincide at the trace of the line d_1. For this reason the points I are called the Incidence Points of the plane ε.

2. LINES:

While we take into account the plane ε as a straight lines field, there lines are classified into two groups:

Lines in a General Situation

The central projection of any straight line d=AB on the plane ε with respect to the center M is the straight line d_m joining the perspectives A_m, B_m of the points A, B, and the reflected projection of that line is the straight line d_o joining the reflected projections A_o, B_o of the same points (see, FIG–3). Consequently, every straight line in the general situation is biunvoquely known by the pair d_m, d_o in the central reflected projection system. In the case of extention, the pair d_m, d_o intersect each other at the trace point D of d on the trace line e_1. Every pair straight line of the picture plane π whose intersection point takes place on the trace e_1 represents a definite straight line d on the plane ε. A point on one of the element of the pair, the point P_m on the line d_m for example, corresponds to the point P_o on the line d_o. The straight lines p, p_o which join the point P_m to the center M and the point P_o to the mirror point M_o, intersect the straight line d=AB at the point P which the pair P_m, P_o represents. Thus the point P is found on the straight line p that joins the projection P_m to M from one side, on the straight line p_o that joins the projection P_o to M_o from the other side. Hence it is possible to conclude the following:

"Every straight line d of the reflecting plane can be represented by a pair (d_m, d_o) of line that intersects each other on the trace line e_1 of this plane and the trace point D of this line in the central reflected projection system. Every pair (d_m, d_o) of the plane π which intersects at the single point on the trace line e_1, determines a definite straight line d of the plane ε".

Lines in a Particular Situation or Particular Points

Particular straight lines of the reflecting plane ε, and hence the geometrical place of the particular points are treated below:

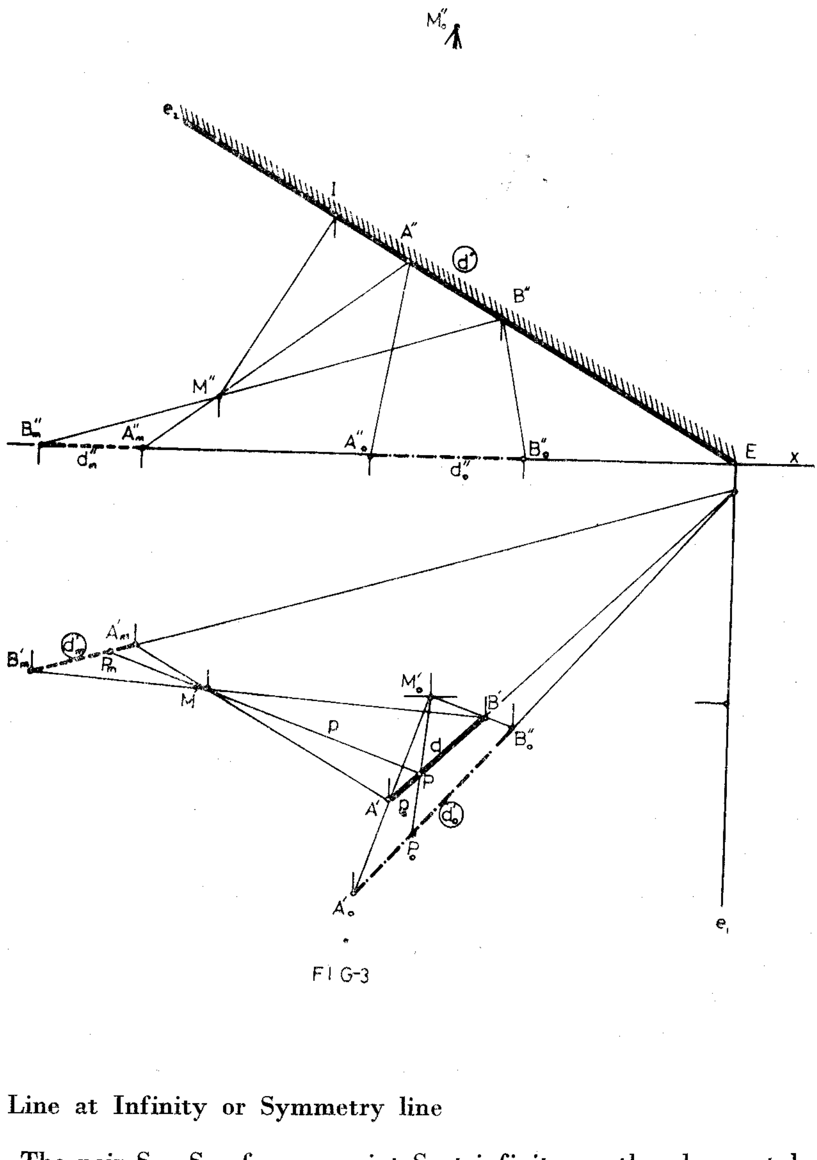

FIG-3

1– Line at Infinity or Symmetry line

The pair S_m, S_o of every point S at infinity on the plane ε takes place in the plane π as a symmetrical pair with regard to the trace e_1 (see 2.1 of the previous subsection). The projections t_m, t_o of the line t at infinity of the plane consist of a symmetrical pair parallel to the trace e_1 consequently. For this reason, the straight line t at infinity is called the symmetry line of the plane ε in system. The projections t_m,

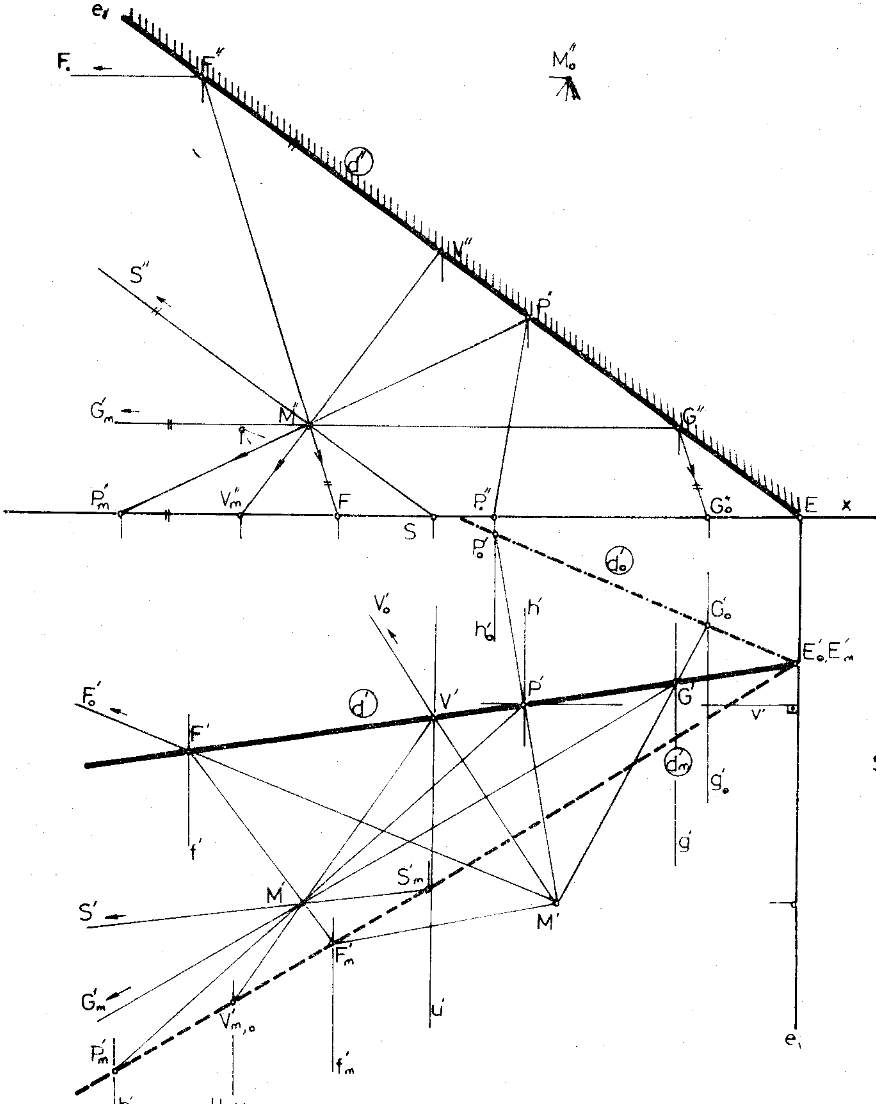

FIG_4

t_0 of this line are the central and reflected Vanishing Traces of the plane ε on the plane π respectively

2– Trace Line or Coincidence Line

The trace line of the plane ε is called the Intersection Line of this plane in the system for carrying the intersection points. Every point E of this line coincides with the pair E_m, E_o belonging to that point. For this reason, the race line e_1 is named the Coincidence Line of the plane ε in the projection system. The coincidence line is the geometrical place of the coincidence points of the reflecting plane ε.

3– Reflecting Disappearance Line

The projection f_m from the pair f_m, f_o belonging to the straight line f passing through the point F at the elevation M_o of the plane ε is a straight line parallel to the trace e_1 from the proper projection F_m of the point F on the plane π. The projection f_o is the straight line at infinity of the plane π. For this reason the said straight line like f is called the Reflected Disappearance Line of the plane ε.

4– Central Disappearance Line

The projection g_o from the pair g_m, g_o belonging to the straight line g passisng through the point G at the elevation M of the plane ε is a straight line parallel to the trace e_1 from the proper projection G_o of the point G on the plane π. The projection g_m is the straight line of the plane π at infinity. For this reason the straight line g is called the Central Disappearance Line of the plane ε.

5– Incidence Line

The pair u_m, u_o of the straight line u at the elevation V of the plane ε represents straight line parallel to the trace e_1 from the the trace of the sıraight line d_1 on the plane π. This means that the pair mentioned will be incident with this line. For this reason the straight line u is called the Incidence Line of the plane ε.

6– Contour Lines

Every straight line h parallel to the plane π through any point P of the plane ε is parallel to tbe trace e_1. The projections h_m, h_o of this line pass parallel to each other and to the trace e_1 through P_m, P_o of the point P respectively. The lines lines like h are called Contour Lines of the plane ε (3–79).

7– Slone Lines

Every straight line v perpendicular to the trace e_1 through any point P of the plane ε is called the Slope Tine of th's plare. The pair v_m, v_o joins the traje of this line on the line e_1 to the projections P_m, P_o of the point P respectively (3–80).

3. PLANE FIGURES

The reflecting plane ε can be considered as a carrier of any plane figures ca tegorically divided into two groups:

Rectilinear Figures

The central projection of any rectilinear figure λ = ABC.. on the plane ε, is the figure λ_m = ABC.. by joining the perspectives A_m, B_m, C_m. of the point A,B,C.., and its reflecting projection is the figure λ_o = $A_oB_oC_o$.. by joining the reflected projection A_o, B_o, C_o.. of the same points (FIG–5). From here it can be stated that every figure in any form such as the triangle λ = ABC in our example is uniquely defined by the pair λ_m, λ_o in the central reflected projection system: The figure

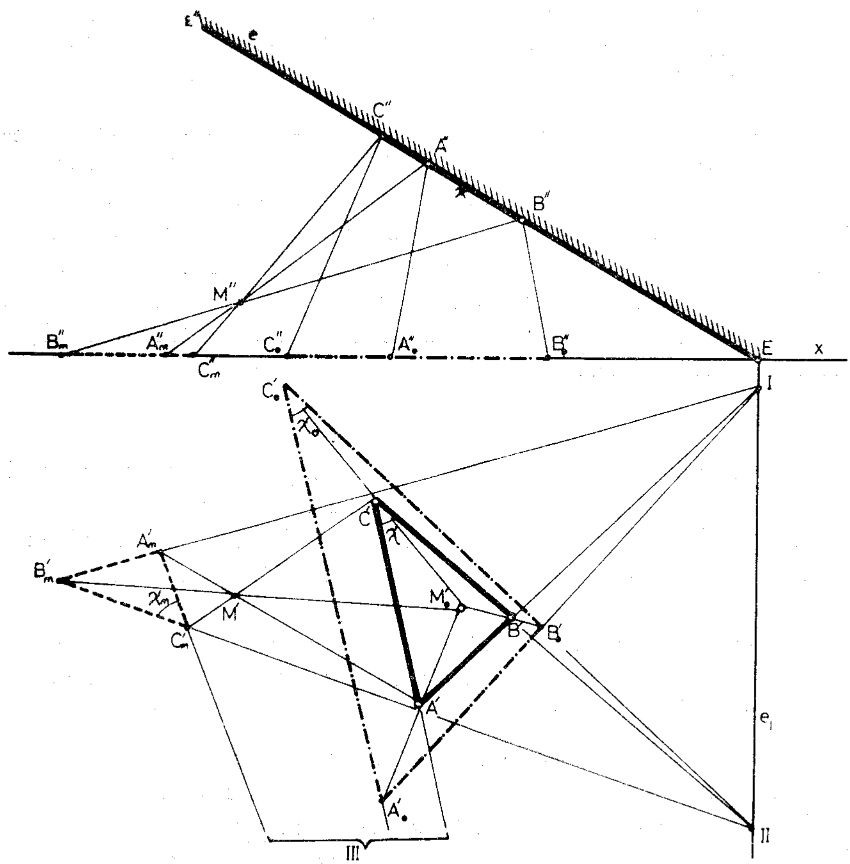

FIG -5

λ_m corresponds to the figure λ in a relationship of the perspective colineation whose center is the point M and the axis is the straight line e_1 (4–195). At the same time the figure λ_o corresponds to the figure λ in another perspective colineation whose center is the point M_o and the axis is the same line e_1. Hence the figures λ_m, λ_o correspond to each other in a third perspective colineation which receives the same trace e_1 as axis. The perspective colineation in space among the figures λ_m, λ_o and λ has been taking place in the case of profile at the vertical projection and in the case of plane at the horizontal projection. Thus the following important relationship between the central and the reflected projection of any plane figure can be deduced:

"The relationship between the central and the reflected projections of any plane figure is the relation of a plane perspective Colineation. The axis of the colineation is the intersection line between the plane carrying the figure and the picture plane. The center of the colineation is the intersection point of the rays joining the corresponding corners"

The picture plane $x = \pi$, the reflecting plane ε which makes an angle α with π, the center of projection M, the triangle $\lambda = ABC$ which is a simplest figure of the rectilinears are given with the vertical and horizontal projections. By means of the relation mensioned above, the central projection A_m, B_m, C_m of the triangle ABC and its reflected projection A_o, B_o, C_o are represented in the vertical and the horizontal projection. Corresponding or colinear edges intersect at the points I, II, III on the trace or axis e_1.

Curvilinear Figures

The central projection of a curvilinear figure, a circle k(0,r) in our example, on the plane ε is the ellipse k_m which forms the intersection of the cone Φ (M,k) with the picture plane π and the reflected projection is the ellipse k_o which forms the intersection of the cone Φ_o (M_o, k) with same plane π (FIG–6). The ellipses k_m, k_o correspond t_o the circle k at a perspective colineation in space. The colineation axis is the trace e_1 and the colineation centers are the points M for k_m, M_o for k_o (see, Rectilinear Curves above).

The horizontal projections of the horizontal diameter AB and the diameter CD perpendicular to it form the major and minor diameters of the projection ellipse k′. The tangents at the points C′, D′ are parallel to the trace e_1. The tangents at the points C'_m, D'_m and C'_o, D'_o belonging to the ellipses k'_m, k'_o are also parallel to the trace e_1. Thus,

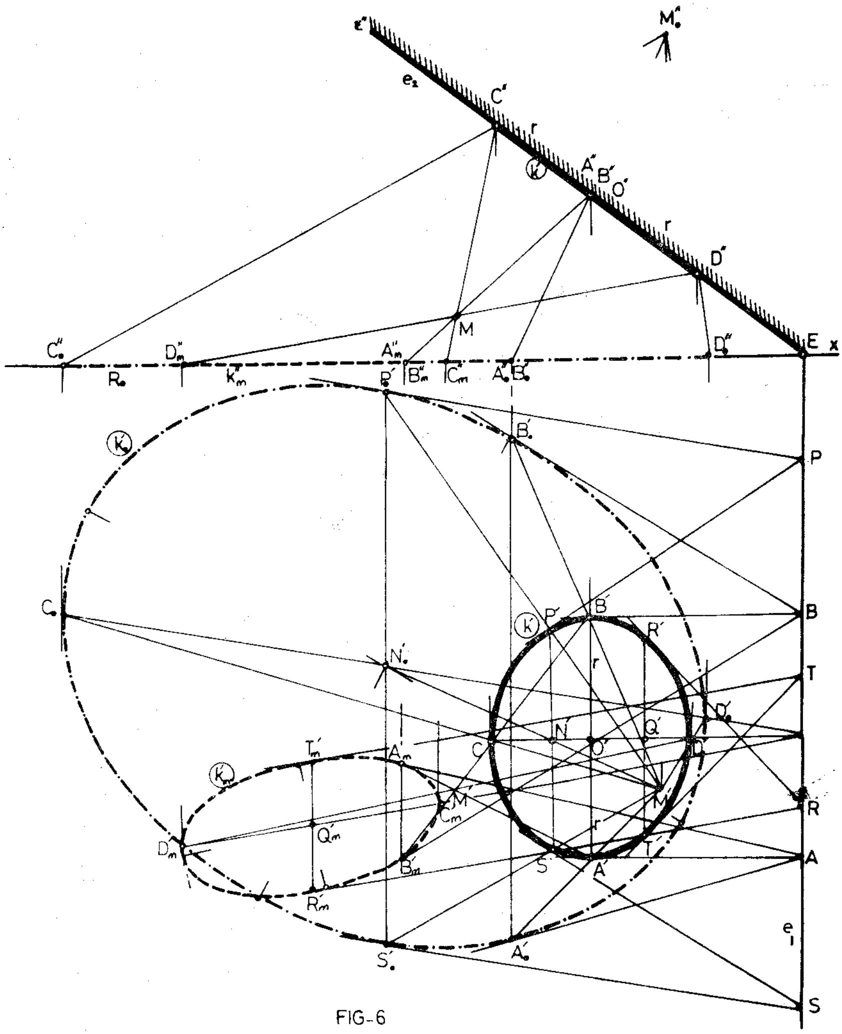

FIG-6

one of the diameters and the center of the ellipses k'_m, k'_o are drawn by indicating the middle points Q_m, N'_m of them. The chords R′T′ and P′S′ are drawn as the conjugate of C′, D′ from the potins Q, N corresponding to these points at the circle k. By means of these, the conjugate diameters R'_m, T'_m and P'_o, S'_o belonging to the former have been obtained. Hence the both conics k'_m, k'_o can be drawn. The tangents at the corresponding points of the conics k'_m, k'_o with regard to k have been intersected at the points P, B, T, Q, A, S on the axis e_1.

If the circle k and the pair belonging to any point on this circle are given, the projections k_m, k_o of this circle can exactly be determined.

The conics k_m, k_o are ellipse, parabola and hyperbola respectively depending on whether the lowest point D of the circle k is over, on or under the plane which is supposed to pass horizontaly from the center M.

In this case, this characteristic point of one of the pair k_m, k_o belonging to the circle k, the conic k_o in our example, take place out of the picture plane, the part remaining in the picture plane can be drawn (FIG–7). The points P, Q, R, S chosen on the arc AB of the circle k are

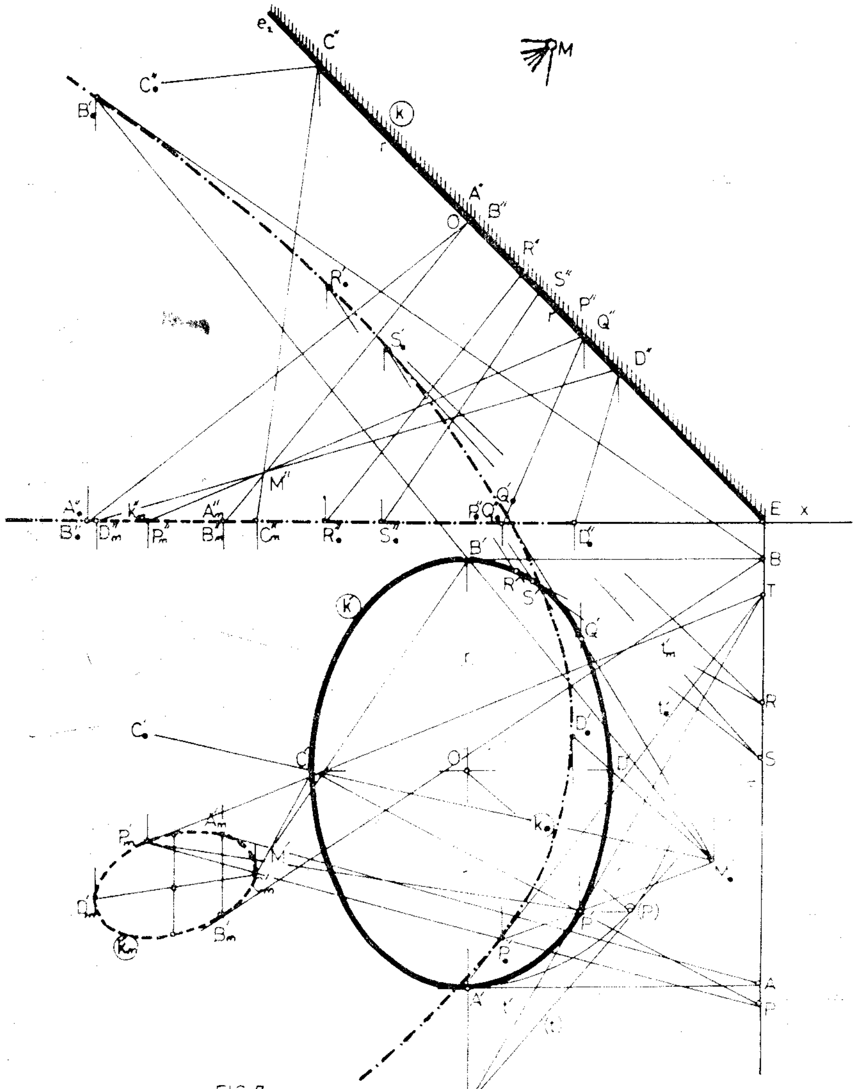

FIG-7

transformed to the conic k_0 thus the partial conic k_0 is completed without the need to know its charateristic elements.

CONCLUSIONS

The central reflected projection system can serve in

(i) Central illumination problems with reflecting,

(ii) Echosounding applications including seismic investigations,

(iii) Drawing of figures whose characteristic points can not be fitted into the plotting medium,

(iv) Determining the limits of shadows related with the curvilinear elements in architecture..., etc.

REFERENCES

AKIN, S., 1967. *Prolems of Descriptive Geometry* (in Turkish), Vol-II, Ari Kitabevi (publisher), İstanbul.

HORNINGER, H., 1967. *Principles of Deseriptive Geometry* (in Turkish), Ari Kitabevi, Istanbul.

HORNINGER, H., 1962. *Deseriptive Geometry Courses* (in Turkish), Technical University of Istanbul.

PALAMUTOĞLU, M., 1973. *Central-Polar Projection System,* (in Turkish), Technical University of Istanbul.

THOMAS, F., 1964. *Engineering Drawing,* Mac-Graw Hill, New York.