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DETERMINATION OF THE BASE SURFACE CONNECTED WITH THE CONGRUENCE GENERATED BY THE INSTANTANEOUS SCREWING AXIS

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ABSTRACT

In this paper, it has been shown that in case the base surface $\vec{x}$ is a WEINGARTEN surface (W-surface) and the congruence $\vec{y}$ generated by the instantaneous screwing axes derived in [3] is normal congruence, the surface $\vec{x}$ may only be a helicoid surface which is a special W-surface.

1. INTRODUCTION

The instantaneous $\vec{G}$ of the moving trihedron which is connected with the point $\vec{x}$ on the base surface $\vec{x}$, which referred to its lines of curvature, for the lines of curvature v = const. is

$$\vec{G} = \vec{g} + \varepsilon \, \vec{g}_0 , \qquad (\varepsilon^2 = 0)$$

$$\vec{g} = - \frac{r\vec{x}_2 \quad q\vec{\xi}}{\sqrt{r^2 + q^2}} , \qquad \vec{g}_0 = \frac{\vec{x}_1 - r\vec{x}_{10} - q\vec{\xi}_0}{\sqrt{r^2 + q^2}}$$

and the congruence $\vec{y}$ generated by $\vec{G}$ is

$$\vec{y} = \vec{r} + t\vec{g} , \qquad (\vec{g}^2 = 1)$$

$$\vec{r} = \vec{x} + \frac{1}{r} \vec{\xi} , \qquad \vec{g} = - \frac{r\vec{x}_2 + q\vec{\xi}}{\sqrt{r^2 + q^2}}$$

[3] and [1].

The condition that $\vec{y}$ is normal, is

$$\bar{q}_1 + \bar{q}^2 = 0 \quad , \quad (\ \bar{q} \neq 0).$$

And it has been shown in [3] that the base surface $\vec{x}$ cannot be developable canal surface, Mulür surface, the surface which have the lines of curvature v = const. and u = const. considering of plane curves, tube-shaped surface or general cylindrical surface.

2. In this work, we will investigate the base surface $\vec{x}$ considering it as a WEINGARTEN surface. In other words, we will assume $\bar{r} = f(r)$, [2] and [4]. We will use the notation $\sqrt{E} = e$, $\sqrt{G} = g$ for the purpose of abbreviation. According to this, the definitions of q and $\bar{q}$ become

$$q = \frac{e_v}{e_g} \quad , \quad \bar{q} = \frac{g_u}{e_g}$$

instead of

$$q = \frac{1}{2} \ \frac{E_v}{E\sqrt{G}} \quad , \quad \bar{q} = \frac{1}{2} \ \frac{G_u}{G\sqrt{E}}$$

3. The condition $\bar{q}_1 + \bar{q}^2 = 0$ may be written as

$$g_u = e.c\,(v)$$

from (2.1).

Here, c(v) may be taken as c(v) = 1 by the transformation

$$(2.2) \qquad \tilde{u} = u, \qquad \tilde{v} = c(v)$$

which does not change the coordinate lines. Therefore, the condition $\bar{q}_1 + q^2 = 0$ may be written as

$$(2.3) \qquad g_u = e.$$

Considering $\bar{r} = f(r)$ and (2.1) in GAUSS-CODAZZI equations, $r_2 = q(\bar{r} - r)$ may be written as

$$\frac{e_v}{e} = \frac{r_v}{f\,(r) - r} \ .$$

Taking $\psi = \psi\,(r)$ arbitrarily, when we take

$$(2.4) \qquad f(r) - r = \frac{\psi}{\psi'}, \quad (\psi' \neq 0)$$

we arrive at $\psi = e\ d(u)$

Here, d(u) may be taken as d(u) = 1 if we make the transformation

$$(2.5) \qquad \tilde{u} = d(u), \quad \tilde{v} = v$$

which does not change the coordinate lines.

From there, we find

$$(2.6) \qquad \psi = e.$$

Considering $\bar{r}_u = f'(r) \cdot r_u$, (2.4) and (2.1), $\bar{r}_1 = \bar{q}(r - \bar{r})$ may be written as

$$\psi' = g . b(v)$$

Here the function b(v) may be taken as b(v) = 1 by the transformation (2.2). Therefore, we find

$$(2.7) \qquad \psi' = g$$

From (2.6), (2.7) and (2.3), we find

$$(2.8) \qquad r_u = \frac{\psi}{\psi''}, \quad (\psi'' \neq 0).$$

Since we may write the equation $-r\bar{r} = q_2 + q^2$ as

$$-r\bar{r} = \left(\frac{e_v}{g}\right)_v \cdot \frac{1}{eg},$$

here considering $\bar{r} = f(r)$, (2.4), (2.3), (2.6), (2.7) and making the necessary abbreviations, we find

$$(2.9) \qquad r^2_v + (r\psi)^2 = a(u).$$

Considering (2.8) and (2.9), we may write

$$(2.10) \qquad \begin{cases} r_v = \sqrt{a(u) - (r\psi)^2} \\ r_u = \dfrac{\psi}{\psi''} \end{cases}$$

From the condition of integrability $r_{vu} = r_{uv}$, we find the equation below:

(2.11) $$a' - 2\,(r\psi)\,(r\psi)' \cdot \frac{\psi}{\psi''} = 2\left(\frac{\psi}{\psi''}\right)' \left|a - (r\psi)^2\right|.$$

The solution of this differential equation is as $F(r,u) = 0$ where r is $r=r(u)$. This case corresponds to $a' \neq 0$. If $a' = 0$, at the end of appropriate calculations, (2.11) becomes

$$\sqrt{a - (r\psi)^2} = -k\,\frac{\psi}{\psi''}$$

where k is a parameter. If we consider (2.10) here we find the differential equation

(2.12) $$r_v + k\, r_u = 0.$$

The solution of the equation is as

(2.13) $$r = r\,(kv - u)\ .$$

From this, we see that the base surface $\vec{x}$ is a helicoid surface. The helicoid surface îs a special WEINGARTEN surface.

Therefore the following theorem may be stated:

2.1. THEOREM

If the base surface $\vec{x}$ of the congruence $\vec{y}$ is a WEINGARTEN surface, in case this congruence is normal congruence, this surface may only be a helicoid surface which is a special W-surface.

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