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ON SOME GENERALIZED SEQUENCE SPACES

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ABSTRACT

In this paper we introduce a general sequence space $\triangle_v(X) = \{x = (x_k): (v_k x_k - v_{k+1} x_{k+1}) \in X\}$, where X is any sequence space. We establish some inclusion relations, topological results, in general case and we characterize the continuous, α-, β- and γ- duals of $\triangle_v(X)$ for various sequence spaces X. The results of this paper, in a particular case, include the corresponding results of KIZMAZ.

1. INTRODUCTION

Let $v = (v_k)$ be any fixed sequence of nonzero complex numbers satisfying

$$\liminf_k |v_k|^{1/k} = r \quad (0 < r \leq \infty). \tag{1.1}$$

Define a function Λ: $C \to C$, where C denotes the set of complex numbers, by

$$\Lambda(z) = \sum_k \frac{z^k}{v_k}. \tag{1.2}$$

(Throughout this paper, $\sum_k$ will mean summation from $k = 1$ to $k = \infty$).

Obviously Λ is an analytic function in the disc $E_r = \{z : |z| \leq r\}$ because of (1.1).

Now let X be any sequence space of complex numbers. Then we define

$$\Delta^\Lambda(X) = \{f: f(z) = \sum_k x_k z^k \text{ such that } \Delta_v(x) \in X\}, \tag{1.3}$$

where $\Delta_v(x) = (\Delta_v(x_k)) = (v_k x_k - v_{k+1} x_{k+1})$.

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We can always get a sequence space

$$\Delta_v(X) = \{x = (x_k) : \Delta_v(x) \in X\}. \tag{1.4}$$

It is easy to show that there exists an algebraic isomorphism between $\Delta^\Lambda(X)$ and $\Delta_v(X)$ in the sense that $f \to x = (x_k)$ is an algebraic isomorphism. Therefore $\Delta_v(X)$ also can be regarded as a set of functions.

Let ι_∞, c and c_0 be the linear spaces of bounded, convergent and null sequences $x = (x_k)$, respectively, normed by

$$\|x\|_\infty = \sup_k |x_k| \qquad (k = 1,2,\ldots).$$

Now we define $\Delta^\Lambda(\iota_\infty)$, $\Delta^\Lambda(c)$ and $\Delta^\Lambda(c_0)$ as follows:

$$\Delta^\Lambda(\iota_\infty) = \{f: f(z) = \sum_k x_k z^k \text{ such that } \sup_k |\Delta_v(x_k)| < \infty\}$$

$$\Delta^\Lambda(c) = \{f: f(z) = \sum_k x_k z^k \text{ such that } \Delta_v(x_k) \to \iota, \text{ for some } \iota, \text{ as } k \to \infty\}$$

$$\Delta^\Lambda(c_0) = \{f: f(z) = \sum_k x_k z^k \text{ such that } \Delta_v(x_k) \to 0 \text{ as } k \to \infty\}.$$

All these classes contain those analytic functions which are analytic in the disc $E_R = \{z: |z| \leq R, \text{ where } R > r\}$.

If f, $f(z) = \sum_k x_k z^k$, belongs to $\Delta^\Lambda(\iota_\infty)$ then the coefficients x_k $(k = 1,2,\ldots)$ satisfy the following conditions:

(i) $\sup_k k^{-1} |v_k x_k| < \infty$,

(ii) $\sup_k |v_k x_k - k(k+1)^{-1} v_{k+1} x_{k+1}| < \infty$.

And conversely if (i) and (ii) hold, then $f \in \Delta^\Lambda(\iota_\infty)$ (see Lemma 2).

Suppose that $f(z) = \sum_k x_k z^k$ and $f \in \Delta^\Lambda(\iota_\infty)$ so that, from (i), we have

$$|x_k|^{1/k} \leq \frac{K^{1/k}\, k^{1/k}}{|v_k|^{1/k}}$$

for some $K > 0$ and for every $k = 1,2,\ldots$. Hence, by (1.1), we obtain

$$\frac{1}{R} = \limsup_k |x_k|^{1/k} \leq \frac{1}{\liminf_k |v_k|^{1/k}} = \frac{1}{r}$$

which implies that $R \geqslant r$. Again if $f \in \Delta^{A}(\iota_{\infty})$, then condition (ii) holds which implies that

$$k^{-2} \ |v_k x_k| \leq K_1$$

for some $K_1 > 0$ and for every $k = 1,2,\ldots$. A similar line of reasoning will lead to $R \geqslant r$. So the class $\Delta^{A}(\iota_{\infty})$ contains those analytic functions which are analytic in the disc E_R.

In a similar way, it can be shown that the classes $\Delta^{A}(c)$ and $\Delta^{A}(c_0)$ contain those analytic functions which are analytic in the disc E_R.

Taking into account the algebraic isomorphism, we view $\Delta^{A}(\iota_{\infty})$, $\Delta^{A}(c)$ and $\Delta^{A}(c_0)$ as sequence spaces $\Delta_v(\iota_{\infty})$, $\Delta_v(c)$ and $\Delta_v(c_0)$, respectively, which are defined as follows:

$$\Delta_v(\iota_{\infty}) = \{x = (x_k) : \sup_k |\Delta_v(x_k)| < \infty\},$$

$$\Delta_v(c) = \{x = (x_k) : \Delta_v(x_k) \to \iota, \text{ for some } \iota \text{ as } k \to \infty\},$$

$$\Delta_v(c_0) = \{x = (x_k) : \Delta_v(x_k) \to 0 \text{ as } k \to \infty\}.$$

If we consider $(v_k) = (1,1,\ldots)$ in (1.4), then $\Delta_v(X)$ becomes $\Delta(X)$, where

$$\Delta(X) = \{x = (x_k) : (x_k - x_{k+1}) \in X\}$$

which was studied by KIZMAZ [2] for $X = \iota_{\infty}$, c and c_0. The results of this paper, in a particular case, include the corresponding results of his.

2. SOME PROPERTIES OF $\Delta_v(X)$

In this section, we give some relations between $\Delta_v(X)$ and X, and we discuss some tpological properties of $\Delta_v(X)$.

Theorem 1: If X is a Banach space normed by $\| \ \|$, then $\Delta_v(X)$ is also a Banach space normed by

$$\|x\|_{\Delta} = |v_1 x_1| + \|\Delta_v(x)\|. \tag{2.1}$$

Proof: Since $(0) \in \Delta_v(X)$, $\Delta_v(X) \neq \varnothing$. Clearly, $\Delta_v(X)$ is a linear space. It is easy to show that $\Delta_v(X)$ is a normed space with norm defined in (2.1).

Now we show that $\Delta_v(X)$ is complete. Let (x^n) be a Cauchy sequence in $\Delta_v(X)$, where $x^n = (x_1^n, x_2^n, \ldots) \in \Delta_v(X)$. Then

$$\|x^m - x^n\|_\Delta \to 0 \text{ as } m,n \to \infty,$$

that is,

$$\|(x_k^m - x_k^n)\|_\Delta \to 0 \text{ as } m,n \to \infty.$$

Hence,

$$|x_1^m - x_1^n| + \|(\Delta_v(x^m) - \Delta_v(x^n))\| \to 0 \text{ as } m,n \to \infty.$$

Therefore $(x_1^1, x_1^2, \ldots)$ and $(\Delta_v(x^1), \Delta_v(x^2), \ldots)$ are Cauchy sequences in C and X respectively. Since C and X are complete, they are convergent. Suppose that $x_1^n \to x_1$ in C and $(\Delta_v(x^n)) \to (z_k)$ in X, as $n \to \infty$.

Let $z_k = \Delta_v(x_k)$ so that $x_k = -v_k^{-1} \sum_{i=1}^{k} z_{i-1}$. Then $(\Delta_v(x^n))$ $= ((\Delta_v(x_k^1)), (\Delta_v(x_k^2)), \ldots)$ converges to $(\Delta_v(x_k))$ in X. Hence,

$$\|x^n - x\|_\Delta \to 0 \text{ as } n \to \infty.$$

Therefore, $\Delta_v(X)$ is complete. Consequently it is a Banach space.

Lemma 1: If $X \subset Y$, then $\Delta_v(X) \subset \Delta_v(Y)$.

Proof: It is trivial.

Theorem 2: Let X be a Banach space and A, a closed subset of X. Then $\Delta_v(A)$ is also closed in $\Delta_v(X)$.

Proof: Since $A \subset X$, $\Delta_v(A) \subset \Delta_v(X)$ by Lemma 1. Now let $x \in \overline{\Delta_v(A)}$, the closure of $\Delta_v(A)$. Then there exists a sequence (x^n) in $\Delta_v(A)$ such that

$$\|x^n - x\|_\Delta \to 0 \text{ as } n \to \infty$$

in $\Delta_v(A)$. Hence,

$$\|(x_k^n) - (x_k)\|_\Delta \to 0 \text{ as } n \to \infty$$

in $\Delta_v(A)$ so that

$$|x_1^n - x_1| + \|(\Delta_v(x_k^n)) - (\Delta_v(x_k))\| \to 0 \text{ as } n \to \infty$$

in A. Thus $\Delta_v(x) \in \bar{A}$ which implies that $x \in \Delta_v(\bar{A})$. Conversely, if $x \in \Delta_v(\bar{A})$, then $x \in \overline{\Delta_v(A)}$. Therefore, $\overline{\Delta_v(A)} = \Delta_v(\bar{A})$ and since A is closed, $\overline{\Delta_v(A)} = \Delta_v(A)$. Consequently, $\Delta_v(A)$ is a closed subset of $\Delta_v(X)$.

Theorem 3: If X is a separable space, then $\Delta_v(X)$ is also a separable space.

Proof: Let X be a separable space. Then there exists a countable subset A of X such that $\bar{A} = X$. Since $\bar{A} = X$, then $\overline{\Delta_v(A)} = \Delta_v(\bar{A}) = \Delta_v(X)$ which can be easilty shown by similar arguments as in the proof of Theorem 2. If we define f: $\Delta_v(A) \to A$ by $f(x) = \Delta_v(x)$ for $x \in \Delta_v(A)$, then it is clear that f is bijective. Therefore $\Delta_v(A)$ is countable, since A is countable. Hence $\Delta_v(A)$ is a countable subset of $\Delta_v(X)$ such that $\overline{\Delta_v(A)} = \Delta_v(X)$. Consequently, $\Delta_v(X)$ is separable.

Theorem 4: In general, $\Delta_v(X)$ need not be a sequence algebra.

Proof: To prove this, we give a counter example. It is well-known that c_0 is sequence algebra. Let $x = y = (\sqrt{k})$. Clearly, $x,y \in \Delta_v(c_0)$, if we choose $(v_k) = (1,1,\ldots)$; but $z \notin \Delta_v(c_0)$, since $\Delta_v(z) = (x_k y_k - x_{k+1} y_{k+1}) = (-1,-1,\ldots)$, where $z = (x_k y_k)$. This completes the proof.

Corollary 1: (i) $\Delta_v(l_\infty)$ is a BK-space with norm defined by

$$\|x\|_\Delta^\infty = |v_1 x_1| + \sup_k |\Delta_v(x_k)|. \tag{2.2}$$

(ii) $\Delta_v(c)$ and $\Delta_v(c_0)$ are separable BK-spaces with the norm as in (2.2).

Proof: Since l_∞, c and c_0 are Banach spaces, then $\Delta_v(l_\infty)$, $\Delta_v(c)$ and $\Delta_v(c_0)$ are also Banach spaces by Theorem 1. Since c and c_0 are separable spaces, then $\Delta_v(c)$ and $\Delta_v(c_0)$ are separable spaces by Theorem 3.

Since $\|x^n - x\|_\Delta^\infty \to 0$, as $n \to \infty$ in $\Delta_v(l_\infty)$, implies that $|x_k^n - x_k| \to 0$, for each $k = 1,2,\ldots$, as $n \to \infty$, it follows that $\Delta_v(l_\infty)$ is also a BK-space, since it is a Banach space with continuous coordinates.

It is easy to show that $\Delta_v(c)$ and $\Delta_v(c_0)$ are BK-spaces.

Assuming $(v_k) = (1,1,\ldots)$ in Corollary 1, we obtain the following results.

Corollary 2: (i) $\Delta(l_\infty)$ is a BK-space with norm

$$\|x\| = |x_1| + \sup_k |x_k - x_{k+1}|. \tag{2.3}$$

(ii) $\Delta(c)$ and $\Delta(c_0)$ are separable BK-spaces with the same norm as in (2.3).

Remark: It may be found in [2] that $\Delta(\iota_\infty)$, $\Delta(c)$ and $\Delta(c_0)$ are BK-spaces.

Now let us define

$$D : \Delta_v(X) \to \Delta_v(X)$$

by $D(x) = y = (0, x_2, x_3, \ldots)$, where X stands for ι_∞, c or c_0. Then D is a bounded linear operator on $\Delta_v(X)$ and $\|D\| = 1$. Further,

$$D[\Delta_v(X)] = \Delta'_v(X) = \{x = (x_k) : x \in \Delta_v(X), x_1 = 0\} \subset \Delta_v(X)$$

is a subspace of $\Delta_v(X)$ and

$$\|x\|_\Delta = \|\Delta_v(x)\|_\infty$$

in $\Delta'_v(X)$. $\Delta'_v(X)$ and X are equivalent as topological spaces [3], since

$$T : \Delta'_v(X) \to X, \text{ defined by, } Tx = y = (\Delta_v(x_k)) \qquad (2.4)$$

is a linear homeomorphism. Also T and T^{-1} are norm preserving and $\|T\| = \|T^{-1}\| = 1$.

Now let $(\Delta'_v(x))^*$ and X^* denote the continuous duals of $\Delta'_v(X)$ and X respectively. Then

$$S : (\Delta'_v(X))^* \to X^*,$$

defined by

$$f_T \to f = f_T \, o T^{-1},$$

is a linear isometry, where X stands for ι_∞, c or c_0. Thus, $(\Delta'_v(X))^*$ is equivalent to X^*. Therefore,

$$(\Delta'_v(\iota_\infty))^* \simeq \iota^*_\infty$$

and

$$(\Delta'_v(c))^* \simeq (\Delta'_v(c_0))^* \simeq \iota_1,$$

since $c^* \simeq c_0^* \simeq \iota_1$, where $\iota_1 = \{x = (x_k) : \sum_k |x_k| < \infty\}$ [1].

3. KÖTHE–TOEPLITZ DUALS OF $\Delta_v(X)$

In this section, we characterize the α-, β- and γ- duals of $\Delta_v(\iota_\infty)$ and $\Delta_v(c)$.

Definition ([1]): Let X be a sequence space and define

(i) $X^\alpha = \{a = (a_k) : \sum_k |a_k x_k| < \infty \text{ for all } x \in X\}$,

(ii) $X^\beta = \{a = (a_k) : \sum_k a_k x_k \text{ converges for all } x \in X\}$,

(iii) $X^\gamma = \{a = (a_k) : \sup_n \left| \sum_{k=1}^{n} a_k x_k \right| < \infty \text{ for all } x \in X\}$.

Then X^α, X^β and X^γ are, respectively, called the α-, β- and γ-dual spaces of X. X^α is also called Köthe-Toeplitz dual space and X^β is also called generalised Köthe-Toeplitz dual space. It is easy to show that $\emptyset \subset X^\alpha \subset X^\beta \subset X^\gamma$. If $X \subset Y$, then $Y^\eta \subset X^\eta$ for $\eta = \alpha, \beta$ or γ.

Lemma 2. The following conditions (1) and (2) are equivalent.

1. $\sup_k |\Delta_v(x_k)| < \infty$,

2. (i) $\sup_k k^{-1} |v_k x_k| < \infty$

   (ii) $\sup_k |v_k x_k - k(k+1)^{-1} v_{k+1} x_{k+1}| < \infty$.

Proof: Suppose that Condition 1 holds. Then

$$|v_1 x_1 - v_{k+1} x_{k+1}| = \left| \sum_{i=1}^{k} \Delta_v(x_i) \right| \leq \sum_{i=1}^{k} |\Delta_v(x_i)| = 0(k)$$

which implies that 2(i) holds. Since

$$|v_k x_k - k(k+1)^{-1} v_{k+1} x_{k+1}| = |k(k+1)^{-1} \Delta_v(x_k) + (k+1)^{-1} v_k x_k| \leq |\Delta_v(x_k)| + (k+1)^{-1} |v_k x_k|,$$

we obtain 2(ii).

Now suppose that Condition 2 holds. Then from the inequality

$$|v_k x_k - k(k+1)^{-1} v_{k+1} x_{k+1}| \geqslant k(k+1)^{-1} |\Delta_v(x_k)| - (k+1)^{-1} |v_k x_k|,$$

we obtain Condition 1.

In the following lemmas, (p_n) will denote a sequence of positive numbers increasing monotonically to infinity.

Lemma 3 ([2]) : (i) If $\sup_n \left| \sum_{k=1}^{n} p_k a_k \right| < \infty$, then

$$\sup_n \left| p_n \sum_{k=n+1}^{\infty} a_k \right| < \infty.$$

(ii) If $\sum_k p_k a_k$ is convergent, then $\lim_n p_n \sum_{k=n+1}^{\infty} a_k = 0$.

Lemma 4: (i) If $\sup_n \left| \sum_{k=1}^{n} p_k v_k^{-1} a_k \right| < \infty$, then

$$\sup_n \left| p_n \sum_{k=n+1}^{\infty} v_k^{-1} a_k \right| < \infty.$$

(ii) $\sum_k k v_k^{-1} a_k$ is convergent, if and only if $\sum_n b_n$ is convergent

with $\lim_n n b_n = 0$, where $b_n = \sum_{k=n+1}^{\infty} v_k^{-1} a_k$.

Proof: (i) If we put $v_k^{-1} a_k$ instead of a_k in Lemma 3(i), the result is immediate.

(ii) If we put $p_n = n$ and choose $v_k^{-1} a_k$ instead of a_k, the result follows from Lemma 3(ii), since

$$\sum_{k=1}^{n} k v_k^{-1} a_k = \sum_{k=1}^{n} k(b_{k-1} - b_k) = \sum_{k=1}^{n} b_{k-1} - n b_n.$$

Now we give the main theorem which characterizes the α-, β- and γ- duals of $\Delta_v(X)$, where X stands for ι_∞ or c.

Theorem 5: Let X stand for ι_∞ or c. Then,

(i) $(\Delta_v(X))^\alpha = \{a = (a_k) : \sum_k k |v_k^{-1} a_k| < \infty\}$,

(ii) $(\Delta_v(X))^\beta = \{a = (a_k) : \sum_k k v_k^{-1} a_k$ converges and

$$\sum_k |b_k| < \infty\},$$

(iii) $(\Delta_v(X))^\gamma = \{a = (a_k): \sup_n |\sum_{k=1}^{n} kv_k^{-1}a_k| < \infty$ and $\sum_k |b_k| < \infty\}$,

where $b_k = \sum_{i=k+1}^{\infty} v_i^{-1}a_i$.

In order to prove Theorem 5, we need the following lemmas.

Lemma 5: (i) $(\Delta'_v(\iota_\infty))^\alpha = G_1$, where

$$G_1 = \{a = (a_k) : \sum_k k|v_k^{-1}a_k| < \infty\},$$

(ii) $(\Delta'_v(\iota_\infty))^\beta = G_2$, where

$$G_2 = \{a = (a_k) : \sum_k kv_k^{-1}a_k \text{ is convergent and } \sum_k |b_k| < \infty\};$$

(iii) $(\Delta'_v(\iota_\infty))^\gamma = G_3$, where

$$G_3 = \{a = (a_k): \sup_n |\sum_{k=1}^{n} kv_k^{-1}a_k| < \infty \text{ and } \sum_k |b_k| < \infty\}.$$

Proof: (i) Let $a \in G_1$ and $x \in \Delta'_v(\iota_\infty)$. Then

$$\sum_k |a_kx_k| = \sum_k k|v_k^{-1}a_k| |k^{-1}| v_kx_k| < \infty,$$

by Lemma 2. Hence, $a \in (\Delta'_v(\iota_\infty))^\alpha$. Now let $a \in (\Delta'_v(\iota_\infty))^\alpha$ which leads to $\sum_k |a_kx_k| < \infty$ for each $x \in \Delta'_v(\iota_\infty)$. Therefore, if we choose

$$x_k = \begin{cases} 0 & \text{if } k = 1 \\ kv_k^{-1} & \text{if } k \geqslant 2 \end{cases} \tag{3.1}$$

then

$$|v_1^{-1}a_1| + \sum_k |a_kx_k| = \sum_k k|v_k^{-1}a_k| < \infty$$

which implies that $a \in G_1$.

(ii) Suppose that $a \in G_2$. If $x \in \Delta'_v(\iota_\infty)$, then by (2.4), there exists one and only one $y = (y_k) \in \iota_\infty$ such that

$$x_k = -v_k^{-1} \sum_{i=1}^{k} y_{i-1} \ (y_0 = 0),$$

and hence

$$\sum_{k=1}^{n} a_k x_k = - \sum_{k=1}^{n} a_k v_k^{-1} \sum_{i=1}^{k} y_{i-1}$$

$$= - \sum_{k=1}^{n} (b_{k-1} - b_k) \sum_{i=1}^{k} y_{i-1} \qquad (3.2)$$

$$= - \sum_{k=1}^{n-1} b_k y_k + b_n \sum_{k=1}^{n-1} y_k.$$

Since, by Lemma 4 (ii), $\sum_k b_k y_k$ is absolutely convergent and $b_n \sum_{k=1}^{n-1} y_k \to 0$ as $n \to \infty$, the series $\sum_k a_k x_k$ is convergent for each $x \in \Delta'_v(\iota_\infty)$, hence $a \in (\Delta'_v(\iota_\infty))^\beta$.

Now let $a \in (\Delta'_v(\iota_\infty))^\beta$, then $\sum_k a_k x_k$ is convergent for each $x \in \Delta'_v(\iota_\infty)$. If we consider the sequence $x = (x_k)$ defined in (3.1), then the series $\sum_k k v_k^{-1} a_k$ converges. This implies that $n b_n \to 0$ as $n \to \infty$ (by Lemma 4(ii)). Again using (3.2) it can be shown that $\sum_k a_k x_k = - \sum_k b_k y_k$ is convergent for all $y \in \iota_\infty$. So we have $\sum_k |b_k| < \infty$ and hence $a \in G_2$.

(iii) It can be proved by the same way as above that $(\Delta'_v(\iota_\infty))^\gamma = G_3$, using Lemma 4(i).

Lemma 6: For $\eta = \alpha$, β or γ, we have

$$(\Delta'_v(\iota_\infty))^\eta = (\Delta'_v(c))^\eta.$$

Proof: We prove for $\eta = \alpha$ only. For $\eta = \beta$ and γ, the proofs are similar.

Since $c \subset \iota_\infty$, then $\Delta'_v(c) \subset \Delta'_v(\iota_\infty)$ and hence $(\Delta'_v(\iota_\infty))^\alpha \subset (\Delta'_v(c))^\alpha$.

Let $a \in (\Delta'_v(c))^\alpha$. Then $\sum_k |a_k x_k| < \infty$ for every $x \in \Delta'_v(c)$. If we consider the sequence $x = (x_k)$ defined in (3.1), then $(x_k) \in \Delta'_v(c)$ and hence $\sum_k k |v_k^{-1} a_k| < \infty$ so that $a \in (\Delta'_v(\iota_\infty))^\alpha$ (by Lemma 5(i)).

This completes the proof.

Lemma 7: For $X = \ell_\infty$ or c, we have

$$(\Delta'_v(X))^\eta = (\Delta_v(X))^\eta$$

where $\eta = \alpha, \beta$ or γ.

Proof: We prove for $\eta = \alpha$ and $X = \ell_\infty$ only. Since $\Delta'_v(\ell_\infty) \subset \Delta_v(\ell_\infty)$, it is clear that

$$(\Delta_v(\ell_\infty))^\alpha \subset (\Delta'_v(\ell_\infty))^\alpha.$$

Let $a \in (\Delta'_v(\ell_\infty))^\alpha$, so that $\sum_k k\,|v_k^{-1}a_k| < \infty$. If $x \in \Delta_v(\ell_\infty)$, then $\sup_k k^{-1}|v_k x_k| < \infty$ (by Lemma 2). Hence,

$$\sum_k |a_k x_k| = \sum_k k\,|v_k^{-1}a_k|\; k^{-1}\,|v_k x_k| < \infty,$$

which implies $a \in (\Delta_v(\ell_\infty))^\alpha$.

The proofs for the other cases are similar.

Now, the proof of Theorem 5 is immediate by Lemma 5,6 and 7.

Assuming $v = (k)$ in Theorem 5, we obtain the following results that give us the α-, β- and γ- duals of the sequence spaces $\Delta_v(\ell_\infty)$ and $\Delta_v(c)$ in terms of some well-known sequence spaces.

Corollary 3: For $X = \ell_\infty$ or c we have

(i) $(\Delta_{(k)}(X))^\alpha = \ell_1$,

(ii) $(\Delta_{(k)}(X))^\beta = \gamma \cap A(k)$,

(iii) $(\Delta_{(k)}(X))^\gamma = m_s \cap A(k)$,

where $\gamma = \{a = (a_k) : \sum_k a_k \text{ converges}\}$, $m_s = \{a = (a_k): \sup_n |\sum_{k=1}^{n} a_k| < \infty\}$ and $A(k) = \{a = (a_k): \sum_k |\sum_{j=k+1}^{\infty} j^{-1}a_j| < \infty\}$,

Putting $v = (1,1,\ldots)$ in Theorem 5, we obtain the following results.

Corollary 4([2]): For $X = \ell_\infty$ or c we have

(i) $(\Delta(X))^{\alpha} = \{a = (a_k) : \sum_k k\,|a_k| < \infty\}$,

(ii) $(\Delta(X))^{\beta} = \{a = (a_k) : \sum_k k a_k$ converges and $\sum_k |b'_k| < \infty\}$,

(iii) $(\Delta(X))^{\gamma} = \{a = (a_k) : \sup_n |\sum_{k=1}^{n} k a_k| < \infty$ and $\sum_k |b'_k| < \infty\}$

where $b'_k = \sum_{i=k+1}^{\infty} a_i$.

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