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SOME APPLICATIONS OF EXPONENTIAL AND LOGISTIC GROWTH MODELS IN BUSINESS AND ECONOMICS

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Abstract

The use of mathematics for predicting and analyzing the future has long been a subject of study due to the fact that it is a measurable source. Correct estimation is vital for the business world and its economic society. In this study, by employing the exponential growth model and the logistic and generalized logistic models, we estimate the size of the population of Turkey, the international investment position and the level of the national income per person in 2025. Furthermore, we examine the changes in sales of houses and the number of mobile phone subscribers in Turkey. We use GeoGebra program to determine the constants and use the computer algebraic system Maple 15 for other computations and graphs.
Keywords: Mathematical modelling, Exponential growth model, Logistic equation, Business, Economics.

ÜSTEL BÜYÜME VE LOJİSTİK BÜYÜME MODELLERİNİN İŞLETME VE İKTİSATTAKİ BAZI UYGULAMALARI

Özet

Geleceğe yönelik öngörü ve analizlerde matematiğin kullanılması, ölçülebilir olması nedeniyle uzun süredir çalışma konusu olmuştur. Doğru tahmin iş hayatı ve ekonomik toplum için önemlidir. Bu çalışmada, üstel büyüme modeli, lojistik model ve genellemelerini kullanılarak 2025 yılı için Türkiye nüfusunun büyüklüğü, kişi başına düşen milli gelir seviyesi ve uluslararası yatırım pozisyonunu tahmin edilmektedir. Ayrıca, 2025 yılına kadar Türkiye'de konut satışlarındaki değişim ve cep telefonu abone sayılarının değişimi incelenmektedir. Denklemlerdeki sabitleri belirlemek için GeoGebra programı ve diğer hesaplamalar ve grafikler için Maple 15 bilgisayar programlama dili kullanılmaktadır.
Anahtar Kelimeler: Matematiksel Modelleme, Üstel Büyüme Modeli, Lojistik Denklem, İşletme, Ekonomi.

Cite

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1. Introduction

The prosperity level of a country is measured by their national income over a certain period. The growth of developing economies represents the direct welfare of a country. Economic crisis and the progress of developments lead decision makers to take measures, such as controlling population planning, improving upon an estimation of population for the future, preparing a development program and developing policies to increase growth velocity. For this reason, quantitative and qualitative forecasts for the future are important. Because of their descriptive and predictive features, mathematical models are used to understand commercial and economical systems.

Thomas Malthus was one of the first demographers who studied population growth [1-2]. His exponential growth model aims to achieve results by fixing variables such as human interaction, climate variables, limited food sources, and natural disaster. Due to the restrictions regarding the Malthus model, Pierre Verhaulst [3] proposed a new model that is known as the logistic differential equation.

Logistic differential equation is frequently used in most scientific fields such as physics, biology, ecology or social sciences [4-18]. Hubbert [4] was the person who first used the logistic equation to model US oil production and other US energy sources. Daim and Harell [5], Höök and Aleklett [6], Harris et all. [7] used logistic growth curve to estimate energy production in the US. Trappey and Wu [8] utilized the Gompertz model, the logistic model and time varying extended logistic model to estimate (the) performance of the sales of 18 market products. McGowann [9] employed a generalized logistic curve be able to predict market development. Miranda and Lima [10] determined human population for some selected Asian countries by using modified Verhulst logistic models. Liu et all. [11] gave an approach to assign the

parameters for analyzing polymerase chain reaction by making use of the logistic growth equation. Auffhammer and Carson [12] studied forecasting China's CO2 emissions using province-level information. Chitnis and Hunt [13] examined the UK household energy and associated with CO2 emissions by applying the structural time series model. Suarez and Menendez [14] prepared a study about forecasting CO2 emission with environmental Kuznets Curves and Logistic Growth Models. Jha and Saha [15] used Gompertz and logistic models to find the best estimation for 3G and 4G Mobile broadband diffusion in India.
For the application of economy and business, the logistic equation model was used by Ramos [16] to predict inflation in the rate of goods, product diffusion, market acceptance, number of mobile phone users, purchasing power of peso and the unemployment rate in the Philippines. Additionally, Kwasnicki [17] published his work for predicting long-term development, consisting of population size in the world, the volume of world output and future structure of global gross domestic product (GDP) on the basis of logistic and exponential curves.
In this paper, with reference to the work done by Ramos and Kwasnicki we calculated the estimated size that the Turkish population is predicted to reach, the level of national income per capita and international investment position in the year 2025. Also, changes in the number of mobile phone subscribers and house sales were analysed. We use both the exponential growth model and logistic growth model with its generalizations in our estimations.

2. Basic Informations for Models and GeoGebra

In this section, we present the mathematical tools which are used for our predictions. Also, we give the exact or the approximate solutions of the models.

2.1. Malthus Growth Model

Malthus growth model which is the pioneer of population models was proposed by Thomas Malthus (1798). This model is described by the following equation

$$\frac{dP}{dt} = kP \qquad (1)$$

where k is the constant growth rate of population P(t). The analytical solution of this equation is

$$P(t) = P_0 e^{kt} \qquad (2)$$

where P_0 is the initial value at $t = 0$.

2.2. The Logistic Equation

The Malthus growth model shows that a population can increase unlimitedly over time. However, every population has a carrying capacity. So, if a population reaches its carrying capacity, there may have been some restrictions that prevented its growth, such as resource shortage, diseases, migration and competition. For this reason, Pierre Verhulst submitted the logistic equation defining growth in this limited area. The logistic differential equation is defined as

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right) \qquad (3)$$

where k is the growth rate and L is the carrying capacity of the population P(t).

Logistic differential equation can be solved analytically by using the method of separation of variables. Hence, the solution of (3) is obtained as

$$P(t) = \frac{L P_0}{P_0 + (L - P_0)e^{-kt}} \qquad (4)$$

where P_0 is the initial value at $t = 0$.

2.3. The Generalized Logistic Equations

In the event (that) the distribution graph of the relative change rate according to P is not linear, we can model the change in P by the generalized logistic differential equation defined as [16-17]

$$\frac{1}{P}\frac{dP}{dt} = k\left(1 - \left(\frac{P}{L}\right)^{\alpha}\right)$$

or

$$\frac{1}{P^{\alpha}}\frac{dP}{dt} = k\left(1 - \left(\frac{P}{L}\right)\right).$$

Throughout this analysis, we used the following models which are the specialized version of the equations mentioned above:

$$\frac{1}{\sqrt{P}}\frac{dP}{dt} = k\left(1 - \left(\frac{P}{L}\right)\right) \qquad (5)$$

$$\frac{1}{P}\frac{dP}{dt} = k\left(1 - \left(\frac{P}{L}\right)^2\right) \qquad (6)$$

$$\frac{1}{P}\frac{dP}{dt} = k\left(1 - \left(\frac{\sqrt{P}}{\sqrt{L}}\right)\right). \qquad (7)$$

The exact solution of the model (5) cannot be calculated analytically in the real space, so the approximate solution obtained by the Euler method is employed. But, model (6) and model (7) have the exact solutions respectively as follows:

$$P(t) = \frac{P_0\, L}{\sqrt{\left({P_0}^2+(L^2-{P_0}^2)\right)e^{-2kt}}} \qquad (8)$$

$$P(t) = \frac{P_0\, L}{\left(\sqrt{P_0}+(\sqrt{L}-\sqrt{P_0})e^{\frac{-kt}{2}}\right)^2}. \qquad (9)$$

To measure the relevance of the use of these models, the data provided the Turkish Statistical Institute were used.

2.4. GeoGebra

GeoGebra is a dynamic math software for all levels of education that combines all geometry, algebra, spreadsheets, graphics, statistics and calculus in one by easy-to-use package [20]. GeoGebra has rapidly expanding usage with millions of users in almost every country.
By processing the data into the GeoGebra, we specify the best approximation line to be represented data. Hence, we can determine the parameters k, L. The parameter k, is the point where the best approximation line intersects the y-axis. The slope of the best approximation line is $\frac{-k}{L}$ and L is calculated from this relation. As a result, by obtaining all parameters, the solution of each model and all approximated values are specified.

2.5. Goodness of Fit Measures

Here, we give the formulas that we use to determine the errors in the predictions. In this way, it can be measured how successful the prediction is. These formulas are given as below.

Mean Square Error (MSE)= $\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n}$

Root Mean Square Error (RMSE)= $\sqrt{\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n}}$

Normalized Root Mean Square Error (NRMSE)= $\frac{RMSE}{y_{max}-y_{min}}$

Mean Absolute Percentage Error(MAPE)= $\left(\sum_{i=1}^{n}\left|\frac{y_i-\hat{y}_i}{y_i}\right|/n\right)100$

3. Materials and Methods

In this part of the study, we introduce economic and business concepts to be dealt with. We also describe how we can establish our model for each problem. When setting the models up, we used data current until 2017. The data of 2018 and 2019 were utilized for verification purposes.

3.1. Turkey's Population Growth

Population growth is one of the major variables for business and economy. From 1935 to 2007, census research was carried out by visiting households over five-year periods by (the) Turkish Statistical Institute (TSI).

Turkey has been recording a census system based on address since 2007 and as a result population statistics have been shared with the public annually. Table 1 shows published census data as well as annual population changes from 2007 to 2017.

When this data is entered into GeoGebra, we can get the regression curve. Thus, the exponential growth model for the population problem is attained as follows:

$$\frac{dP}{dt} = 0.0137579795\, P \qquad (10)$$

$$P_0 = P(0) = 70586256. \qquad (11)$$

Here, 0.0137579795 is the numeric value of parameter k which is the intersection of the best approximation line with the y-axis. If the initial value problem is solved, the solution is obtained as

$$P(t) = 70586256\, e^{0.0137579795t}. \qquad (12)$$

In Table 2, the results obtained from the exponential growth model are demonstrated.

Similarly, for estimation via the logistic equation, by using the distribution of population P to $\frac{1}{P}\frac{dP}{dt}$, the best approximation line is obtained as shown in the Figure 1.

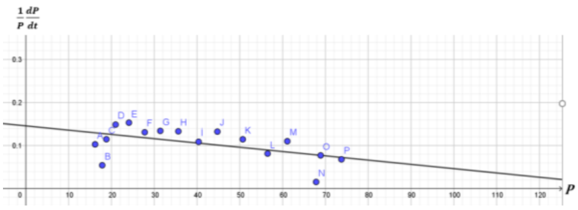

Figure1. The best approximation line graph

Here, k is the point where the best approximation line intersects the y-axis and it is figured out as $k = 0.024928$. The slope of the line in Figure 1 is $-\frac{k}{L} = -\frac{0{,}013030}{86.612.226{,}14}$. From this equation, L can be calculated easily as $L \cong 165693.024$. Therefore, the approximate population can be calculated in the following equation

$$P(t) = \frac{165693.024}{1+1.35\, e^{-0.024928\, t}} \qquad (13)$$

If we also make use of this process for models (5)-(7), we can get the approximate values for the population of Turkey as shown in Table 3. As seen from Table 3, the exponential growth model and models (6)-(7) give more accurate estimations by considering the current data. We predict that the population in 2025 will be around 90 million using the approximations obtained by the models. This prediction is consistent with current data. Goodness of fit measures (GoFs) for annual population data is given in Table 4. For this data set, the best estimation result is obtained from the exponential Growth model in relation to MSE, RMSE and NRMSE criteria. However, if the MAPE criterion is considered, the best estimate will be obtained from Model (7). In Figure 2, we plot the approximations attained by the models except model (5).

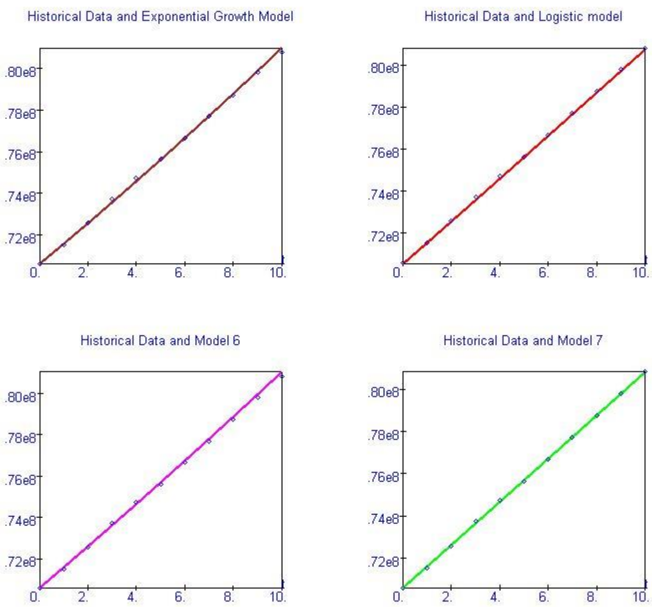

Figure 2. Turkey's population growth for different models.

3.2. Estimation of National Income per Capita in Turkey

The amount of goods and services produced in a country in a certain period indicates any positive or negative growth that the economy reaches from year to year. This case is also a sign of prosperity level. Income derived from the subtraction of indirect taxes from the net monetary value of the eventual goods and services produced in a given period in a country is called its' national income [19].

By using the amounts of national income per capita calculated every year in US dollars since 1999 by Turkish Statistical Institute and by entering this data into GeoGebra we get the approximations obtained from the exponential growth model, the logistic and generalized logistic models respectively:

$$P(t) = 4003\, e^{0.0610795455t} \quad (14)$$

$$P(t) = \frac{12233.58}{1+1.75\, e^{(-0.1731t)}} \quad (15)$$

$$P_{n+1} = P_n + \Delta t\,(14.2175357879)P_n\left(1-\frac{P_n}{12712}\right) \quad (16)$$

$$P(t) = \frac{(4003)11797}{\sqrt{4003^2+(11797^2-4003^2)\, e^{-0.29892634\, t}}} \quad (17)$$

$$P(t) = \frac{(4003)\,13142}{\left(\sqrt{4003}+(\sqrt{13142}-\sqrt{4003})\, e^{\frac{-0.2872303815\, t}{2}}\right)^2} \quad (18)$$

The results of these approximations are submitted into Table 5 and plotted in Figure 3. From these outputs, the model that calculates the highest national income per capita in 2025 is the exponential model and it comes to approximately $ 23,000. The estimation of the exponential model is a national income rate that every economy wants to reach. The logistic model and the models defined by equations (5)-(7) estimate the national income to be approximately $ 12,000. This means the national income will change almost constantly, and this is not a desired situation in any economy. As shown by Table 5, the actual national income per capita has decreased gradually since 2015. Exchange rate fluctuations and economic crises may deviate the national income forecast as seen in 2009 and 2014. We also give goodness of fit measures (GoFs) for the national income per capita data in Table 6. For the data set, the best prediction performance is obtained from Model (6) according to all criteria.

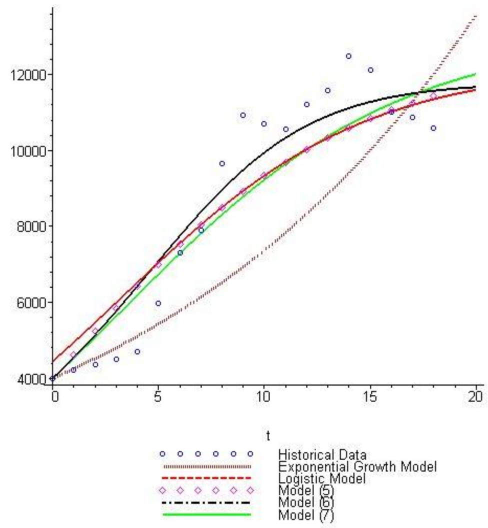

Figure 3. Real and approximate data of national income per capita from 1999 to 2017.

3.3. International Investment Position of Turkey

International investment position refers to statistical data published monthly that shows financial receivables and liabilities between residents in an economy and residents abroad. Additionally, the stock value of financial assets such as gold is held as a source asset at a certain date.

The difference between total financial assets and total financial liabilities is called net international investment position. In other words, the net international investment position is the difference between Turkey's receivables from abroad and debt they have abroad. The resulting values can be a positive or negative. Turkey's international investment position data are shared with the public annually by the Central Bank of the Republic of Turkey. These values are actually negative. Here, we use the data acquired since 1996 in terms of the US dollar.

Approaches of the international investment problem obtained by the exponential growth model, logistic and generalized logistic models are given below, respectively:

$$P(t) = 54767\, e^{0.122572102\, t} \tag{19}$$

$$P(t) = \frac{489506.83}{1+7.937\, e^{-0.285801\, t}} \tag{20}$$

$$P_{n+1} = P_n + \Delta t\, (8.7390262492) P_n \left(1 - \frac{P_n}{428720.4864}\right) \tag{21}$$

$$P(t) = \frac{(54767)416486}{\sqrt{54767^2 + (416486^2 - 54767^2)\, e^{-0.42237914\, t}}} \tag{22}$$

$$P(t) = \frac{(54767)\, 635799}{\left(\sqrt{54767} + (\sqrt{635799} - \sqrt{54767})\, e^{\frac{-0.292199241\, t}{2}}\right)^2} \tag{23}$$

In the years between 1996 and 2019, the actual value of the international investment position and five forecasts are compared in Table 7 in terms of US dollars. According to this table, when Turkey's international investment position was compared with the available data we can say that the exponential growth model was the best forecast until 2005. The graphs of exponential model approach and real data are plotted in Figure 4. The logistic model and its generalizations also make predictions close to the current international investment position data between 2012 and 2017. As a result, the models, except the exponential model prediction of the international investment position will approximately be between 400,000 and 600,000 million dollars, thus economic units should take measures to reduce this. 2018 and 2019 data show that there is a decrease in Turkey's international investment position contrary to the predictions of the models. Because, like national income per capita, the international investment position is affected by the economic crisis and exchange rate fluctuations. Therefore, we can say that the consistency of these estimates will decrease in another possible exchange rate fluctuation. In Table 8, the fit statistics are presented and from this data set we can say that the logistic model provides the best prediction.

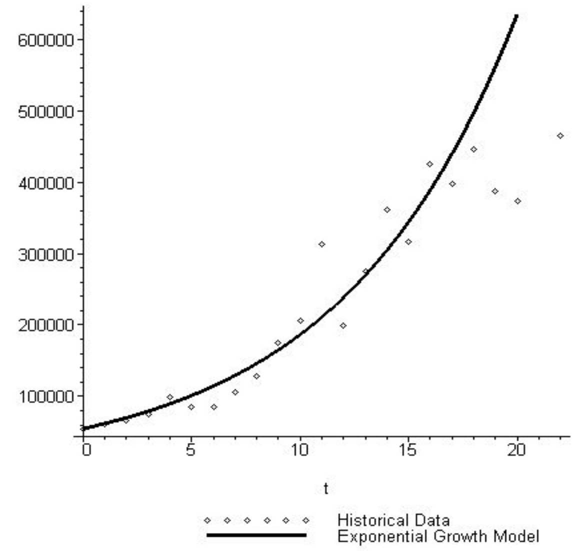

Figure 4. Real and approximate data of international investment position from 1999 to 2017.

3.4. Estimation of Sales of Newly Built Houses in Turkey

The annual number of residential sales made in Turkey are reported on by the TSI. The data declared by this institution includes the sale of the number of new and older houses. In this study, we examined the modeling of new houses that have been sold. In Table 9, we submit the actual house sale numbers in the years 2013-2018 and predictions gained from mathematical models. In Table 10, the calculation of fit measures is tabulated. According to this table, model (7) provides the best prediction when it is compared to other models.

The maximum number of house sales or the carrying capacities predicted by exponential growth model, the logistic model, model (6) and model (7), have already been exceeded when compared to the real data. So, we can say that these models are not appropriate for this problem. From the data, we can see that the model which gives the closest results compared with the actual house sales is sub model 1 given by Equation (5). According to this model, the number of houses to be sold in 2025 is expected to be around 1500000. The rapidly growing construction sector, the increase in house sales for foreigners, especially from the effects of refugees from Syria and other Arab countries, may be some of the reasons for the inaccurate predictions of the other models.

3.5. Mobile Phone Subscribers

With the beginning of mobile phone services in 1994, the second generation of telecommunications processes began in Turkey. While the usage of mobile phones were 81,000 in 1994, this number reached 15 million in 2000 because of the rapid increase in the market of the service. This rapid increase in the number of users is also a natural outcome of technological development. The results obtained from the models which are created by using mobile phone subscriber's data are given in Table 11. Goodness of fit measures (GoFs) for the mobile phone subscriber numbers data can be examined in Table 12.

From these two table, it is seen that the best predictive model for mobile phone subscribers is the model given by Equation (5), which is the generalization of the logistics model. According to this model, it is observed that there will be approximately 8.5 million subscribers in the mobile phone market in 2025.

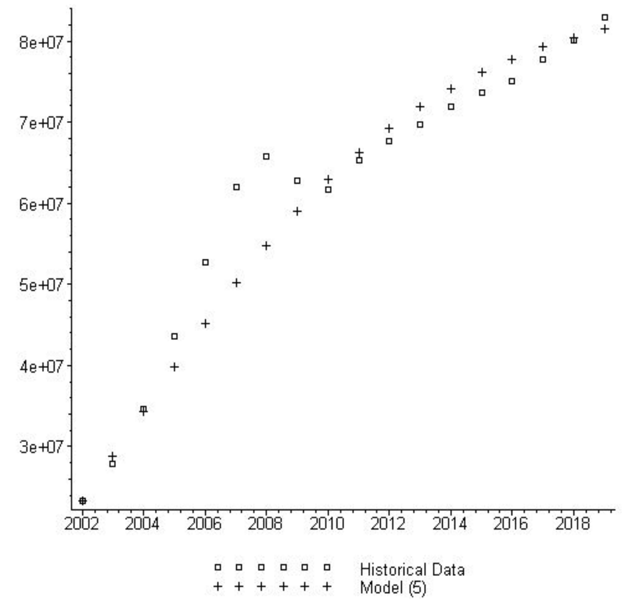

Figure 5. Forecast of the mobile phone subscribers by using generalized logistic equation given by Equation (5).

When Table 7 is examined, then the approximations of the models except model (5), predict fewer mobile phone subscribers than the available data. Moreover, in these estimations, it is understood that the maximum number of mobile phone subscribers L has already been exceeded. The reasons why these models cannot make better predictions can be listed as follows:

1- More mobile phone lines being used,
2- Decreased age of mobile phone users,
3- Expended mobile phone coverage areas.

4. Results and Discussion

In this section models for each research problem, the results and the targets that have been reached in the study, have been interpreted and summarized as below.

The first investigation estimates the population of Turkey in the year 2025. The results obtained with the models established using annual population data are given in Table 3. From this table, it shows that the model which predicts the population best is the logistic model. If the results are taken into account, Turkey's population is expected to be approximately 89 million in 2025.

As the second examination we presents national income per capita. The outcomes attained from the models and data acquired from Turkey Statistical Institutions (TSI) are shown in Table 4. From this table, it can be seen that the national income per capita in 2025 will be approximately 12.000 dollars according to the logistic model. This data shows that predicted national income per capita in 2025 can only reach the levels which were reached in 2013 and 2014. This is only possible if the economic policies to be implemented are consistent. On the other hand, correction of the estimation shows that even if there is no crisis or contraction in the economy, national income will always stay in balance around this number and the upper limit of income that can be obtained with current values have been reached. This situation imposes great responsibilities to policymakers about implementing policies to increase existing capacity.

Our next question is about Turkey's international investment positions. When we analyzed Table 5, it was seen that the exponential growth model gave the best estimation for international investment position of Turkey until 2005. It has been observed that other models predicted closer to the available data after 2012. According to the exponential function, the international investment position for 2025 is evaluated as $ 1915394. This means that if foreign capital and investments increase excessively in Turkey and if new factories are not opened, existing domestic factories may change hands and they may be sold to foreign companies. This indicates that it will be faced with a heavy economic condition. For other models, it is foreseen that the international investment position will be approximately 400000-600000 million dollars.

Following this, our next research topic was on the sales of newly built houses. Table 6 shows both real sales data and numerical values obtained from established models. From this table, we can say that the logistic model, model 6 and model 7 are both unusable since they predict the number of house sales in the future are less than the existing houses that have been sold. Also, we see that the number of house sales estimated by the exponential model does not match the existing new house sales. Among these models, we can say that the model that best matches the existing home sales is the model given by equation (5), which is the generalization of the logistics model. According to model (5), we expect the number of houses which will be sold in 2025 to be approximately 1500000.

Finally, we studied mobile phone subscribers in Turkey. The actual data and the results obtained with the created models are given in Table 7. This table states that the best model for estimating the number of mobile phone subscribers is the sub-model given by Equation (5), which is the generalization of the logistic model. According to model (5), the number of mobile phone subscribers in Turkey in 2025 is foreseen to be about 85 million people. It is seen that other models, except the logistic model, make predictions below the current number of mobile phone subscribers. This is due to the fact that a major portion of users have more than one line because of their requirement to separate business and private life or price campaigns implemented by mobile phone operators. In addition, the age groups that use mobile phones decreases day by day, and as a result the consistency of the models reduce. By determining the number of users who have more than one line and adding data such as the number of subscriptions from the population for 10 years, the above models can give more consistent results.

5. Conclusion

The purpose of the study is to discuss the exponential growth model and the logistic growth models for estimating Turkey's population, national income per capita in Turkey, international investment position, newly built sold houses numbers and mobile phone subscriber numbers. In addition, the size of these variables is expected to predict data for the year 2025.
By analyzing these models with the available data, it is aimed to make more accurate future predictions as well as make them available for private sectors, policy makers and other researchers.
Furthermore, the mentioned models may be used for the estimations of different economic problems such as inflation rate of goods, unemployment, purchasing power of Turkish lira in the future.

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Table1. Turkey's census datas and annual population changes from 2007 to 2017.

Year	Population (P)	$\frac{dP}{dt}$	$\frac{1}{P}\frac{dP}{dt}$
2007	70586256	930.844	0,0131873264
2008	71517100	1.044.212	0,0146008717
2009	72561312	1.161.676	0,0160095782
2010	73722988	1.001.281	0,0135816660
2011	74724269	903.115	0,0120859663
2012	75627384	1.040.480	0,0137579795
2013	76667864	1.028.040	0,0134090080
2014	77695904	1.045.149	0,0134517902
2015	78741053	1.073.818	0,0136373335
2016	79814871	995.654	0,0124745425
2017	80810525		
2018	82003882		
2019	83154997		

Table2. The comparisons of Turkey's population and the estimations achieved by the exponential growth model.

Year t	Actual Population (P)	Approximate Population (Py)	Absolute Error (%)
2013	76667864	70586256	%7.9
2014	77695904	71564091	%7.9
2015	78741053	72555473	%7.8
2016	79814871	73560588	%7.8
2017	80810525	74579627	%7.7
2018	82003882	75612782	%7.8
2019	83154997	76660251	%7.8
2020		77722229	
2021		78798920	
2022		79890526	
2023		80997254	
2024		82119313	
2025		83256916	

Table 3. Comparison of modeling results obtained from annual population data.

Year	Actual Polpulation	Exponential Growth	Logistic Model	Generalized Logistic Equations		
				Model (5)	Model (6)	Model (7)
2007	70586256	70586256	70507670	70586256	70586256	70586256
2008	71517100	71564091	71518051	73100030	71587760	71600738
2009	72561312	72555473	72531878	75652832	72599305	72617425

2010	73722988	73560588	73548852	78244305	73620813	73636064
2011	74724269	74579627	74568673	80874092	74652198	74656399
2012	75627384	75612782	75591033	83541829	75693366	75678174
2013	76667864	76660251	76615626	86247146	76744212	76701135
2014	77695904	77722229	77642138	88989672	77804623	77725029
2015	78741053	78798920	78670258	91769030	78874478	78749601
2016	79814871	79890526	79699669	94584838	79953642	79774599
2017	80810525	80997254	80730054	97436711	81041976	80799772
2018	82003882	82119313	81761095	100324262	82139326	81824870
2019	83154997	83256916	82792472	103247098	83245531	82849646
2020		84410279	83823865	106204822	84360418	83873852
2021		85579619	84854956	109197034	85483804	84897245
2022		86765159	85885425	112223332	86615496	85919583
2023		87967121	86914953	115283310	87755290	86940626
2024		89185735	87943222	118376559	88902972	87960138
2025		90421230	88969918	121502665	90058315	88977884
2026		91673840	89994727	124661213	91221085	89993634
2027		92943803	91017336	127851786	92391032	91007160

Table 4. Goodness of fit measures (GoFs) for annual population data

Models/GoFs		MSE	RMSE	NRMSE	MAPE
Exponential Growth		9.0887e+09	9.5335e+04	0.0076	0.0933
Logistic Model		2.1819e+10	1.4771e+05	0.0118	0.1422
Generalized Logistic Equations	Model (5)	1.3507e+14	1.1622e+07	0.9247	12.3431
	Model (6)	1.2403e+10	1.1137e+05	0.0089	0.1244
	Model (7)	1.1841e+10	1.0882e+05	0.0087	0.0932

Table 5. Comparison of actual data and approximate values attained from forecasting models for national income per capita.

Year	Actual National Income	Exponential Growth Model	Logistic Model	Generalized Logistic Equations		
				Model (5)	Model (6)	Model (7)
1999	4003	4722	4949	4003	4003	4003
2000	4229	5019	5467	4619	4548	4530
2001	4350	5335	5994	5234	5139	5075
2002	4500	5671	6522	5840	5769	5631
2003	4698	6029	7045	6427	6425	6190
2004	5961	6408	7554	6990	7091	6744
2005	7304	6812	8043	7525	7749	7286
2006	7906	7241	8506	8029	8380	7811
2007	9656	7697	8939	8498	8964	8313
2008	10931	8182	9339	8932	9489	8787
2009	10700	8697	9704	9332	9947	9232
2010	10560	9245	10034	9697	10335	9645
2011	11205	9827	10329	10029	10655	10026
2012	11588	10446	10591	10330	10914	10374
2013	12480	11104	10822	10601	11120	10691
2014	12112	11803	11024	10844	11281	10977
2015	11019	12547	11200	11061	11406	11234
2016	10883	13337	11353	11255	11502	11464
2017	10616	14177	11484	11428	11575	11670
2018	9693	15070	11597	11582	11630	11852
2019	9127	16019	11694	11718	11672	12013

2020		17028	11776	11838	11704	12156
2021		18100	11847	11944	11727	12281
2022		19240	11907	12038	11745	12391
2023		20452	11957	12121	11758	12488
2024		21740	12000	12194	11768	12572

Table 6. Goodness of fit measures (GoFs) for the national income per capita data

Models/GoFs		MSE	RMSE	NRMSE	MAPE
Exponential Growth		5.9960e+06	2.4487e+03	0.2889	21.4080
Logistic Model		1.8764e+06	1.3698e+03	0.1616	17.4712
Generalized Logistic Equations	Model (5)	1.6181e+06	1.2720e+03	0.1501	13.1146
	Model (6)	1.1986e+06	1.0948e+03	0.1292	11.7046
	Model (7)	1.7131e+06	1.3088e+03	0.1544	12.7135

Table 7. Comparison of the modeling results obtained for the international investment position.

Year	Actual Data	Exponential Growth Model	Logistic Model	Generalized Logistic Equations		
				Model (5)	Model (6)	Model (7)
1996	54767	54767	49495	54767	54767	54767
1997	60334	61909	60990	111178	67340	67022
1998	65560	69982	74700	179428	82609	81255
1999	75408	79107	90840	247496	100996	97550
2000	98237	89423	109546	305611	122860	115928
2001	85369	101084	130840	349481	148395	136339
2002	85509	114265	154588	379676	177464	158652
2003	105599	129165	180479	399156	209429	182658
2004	128019	146008	208012	411196	243031	208078
2005	174623	165048	236531	418440	276458	234573
2006	205557	186570	265275	422727	307692	261766
2007	313621	210899	293460	425239	335018	289259
2008	199616	238401	320361	426702	357455	316660
2009	275926	269488	345383	427552	374867	343598
2010	361274	304630	368107	428045	387764	369743
2011	316209	344354	388298	428330	396977	394814
2012	426286	389258	405894	428494	403385	418589
2013	397978	440017	420970	428590	407758	440906
2014	446759	497396	433702	428645	410703	461658
2015	387509	562256	444322	428677	412668	480793
2016	372810	635575	453091	428695	413971	498301
2017	465743	718454	460270	428706	414833	514212
2018	371398	812141	466107	428712	415400	528581
2019	352149	918045	470826	428716	415773	541488
2020		1037758	474624	428718	416018	553025
2021		1173083	477669	428719	416179	563294
2022		1326053	480103	428720	416285	572398
2023		1498972	482045	428720	416354	580445
2024		1694438	483590	428720	416400	587534
2025		1915394	484819	428720	416429	593766

Table 8. Goodness of fit measures (GoFs) for the international investment position data

Models/GoFs		MSE	RMSE	NRMSE	MAPE
Exponential Growth		2.9261e+10	1.7106e+05	0.4162	28.5519
Logistic Model		3.5686e+9	5.9738e+04	0.1454	25.6423
Generalized Logistic Equations	Model (5)	2.6287e+10	1.6306e+05	0.3968	101.5155
	Model (6)	4.7520e+9	6.8935e+04	0.1677	33.4607
	Model (7)	5.9477e+9	7.7121e+04	0.1877	30.1297

Table 9. Comparison of modeling results obtained from housing sales data.

Year	Actual housing sale	Exponential Growth Model	Logistic Model	Generalized Logistic Equations		
				Model (5)	Model (6)	Model (7)
2013	325174	325174	325174	325174	325174	325124
2014	695020	436968	744717	684898	706464	753674
2015	1101512	587197	1330360	1202246	1334627	1300442
2016	1532825	789074	1818213	1878696	1881947	1771813
2017	1985800	1060356	2078240	2709612	2092655	2077170
2018	2511983	1424905	2184849	3685835	2142236	2243630
2019		1914785	2223771	4794724	2152243	2326429
2020		2573084	2237370	6021002	2154196	2365797
2021		3457705	2242049	7347509	2154574	2384119
2022		4646458	2243649	8755911	2154647	2392563
2023		6243901	2244196	10227364	2154662	2396436
2024		8390542	2244382	11743113	2154664	2398209
2025		11275195	2244446	13285021	2154665	2399019

Table 10. Goodness of fit measures (GoFs) for the housing sales data

Models/GoFs		MSE	RMSE	NRMSE	MAPE
Exponential Growth		4.8708e+11	6.9791e+05	0.3191	37.0368
Logistic Model		4.1975e+10	2.0488e+05	0.0937	10.7038
Generalized Logistic Equations	Model (5)	3.3862e+11	5.8191e+05	0.2661	19.3909
	Model (6)	5.4082e+10	2.3255e+05	0.1063	10.9477
	Model (7)	3.0082e+10	1.7344e+05	0.0793	9.5650

Table 11. Comparison of modeling results for mobile phone subscriber numbers.

Year	Actual numbers of subscribers	Exponential Growth Model	Logistic Model	Generalized Logistic Equations		
				Model (5)	Model (6)	Model (7)
1994	81276		81276		81276	81276
1995	332716		27045		226676	365269
1996	692779		893761		632169	1496454
1997	1483149	1483149	2888382		1762597	5182285
1998	3382137	1770057	8718177		4904931	13878434
1999	7562972	2112466	22058235		13449851	27340834
2000	14970745	2521113	40733920		33428080	40892492
2001	19502897	3008810	54580031		57805186	50566424
2002	23323118	3590850	60769377	23323118	67010544	56100387
2003	27887535	4285482	62906832	28801333	68542690	58902814
2004	34707549	5114488	63576870	34364864	68747385	60238163
2005	43608965	6103862	63780423	39860576	68773834	60856356
2006	52662709	7284625	63841668	45160845	68777237	61138755

2007	61975807	8693800	63860042	50167482	68777674	61266976
2008	65824110	10375574	63865549	54812130	68777731	61325034
2009	62779554	12382680	63867200	59053965	68777738	61351289
2010	61769635	14778050	63867694	62875725	68777739	61363155
2011	65321745	17636793	63867843	66278934	68777739	61368517
2012	67680547	21048546	63867887	69279074	68777739	61370940
2013	69661108	25120287	63867900	71901165	68777739	61372034
2014	71888416	29979686	63867904	74176021	68777739	61372529
2015	73639261	35779113	63867905	76137284	68777739	61372752
2016	75061699	42700411	63867906	77819204	68777739	61372853
2017	77800170	50960601	63867906	79255078	68777739	61372898
2018	80117999	60818685	63867906	80476240	68777739	61372919
2019	82896108	72583769	63867906	81511466	68777739	61372928
2020		86624752	63867906	82386698	68777739	61372933
2021		103381896	63867906	83124984	68777739	61372934
2022		123380629	63867906	83746565	68777739	61372935
2023		147248022	63867906	84269052	68777739	61372936
2024		175732447	63867906	84707656	68777739	61372936
2025		209727048	63867906	85075430	68777739	61372936

Table 12**.** Goodness of fit measures (GoFs) for the mobile phone subscriber numbers data

Models/GoFs		MSE	RMSE	NRMSE	MAPE
Exponential Growth		1.5454e+15	3.9312e+07	0.6599	65.7179
Logistic Model		2.8680e+14	1.6935e+07	0.2843	31.3663
Generalized Logistic Equations	Model (5)	2.1045e+13	4.5875e+06	0.0770	5.1050
	Model (6)	3.4718e+14	1.8633e+07	0.3128	34.7835
	Model (7)	2.6319e+14	1.6223e+07	0.2723	29.9522