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$\oplus$ - COFINITELY SUPPLEMENTED MODULES

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ABSTRACT

Let R be a ring with identity and M a unitary right R-module. M is called H-cofinitely supplemented if for every cofinite submodule A of M, there exists a direct summand A^1 of M such that $M = A + X$ holds if and only if $M = A^1 + X$, and M is called $\oplus$ -cofinitely suplemented if every cofinite submodule of M has a supplement that is a direct summand of M. In this note we study the structure of these classes of modules.

KEYWORDS : Cofinite submodule, H-cofinitely supplemented module. $\oplus$ -cofinitely suplemented module.

1. INTRODUCTION

In this note all rings are associative with identity and all modules are unital right modules. A submodule N of a module M is called *small*. written $N \langle\langle M$. if $M \neq N + L$ for every proper submodule L of M. Properties of small submodules are given in the $[7, Lemma 4.2]$ and $[10, Proposition 19.3]$. Let M be a module and let N and K be any submodules of M. N is called a *supplement* of K in M if it is minimal with respect to $M = N + K$, equivalently, $M = N + K$ and $N \cap K$ is small in N (see $[7, Lemma 4.5]$). Following $[10]$, an R-module M is called *supplemented* if every submodule of M has a supplement in M and a submodule N of M has *ample supplement* in M if every submodule L such that $M = N + L$ contains a supplement of N in M. The module M is called *amply supplemented* if every submodule of M has ample supplement in M. Following $[7]$. the module M is called

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$\oplus$*-supplemented* if every submodule of *M* has a supplement that is a direct summand of *M*. Clearly $\oplus$-supplemented modules are supplemented. but the converse is false in general. $[7, Lemma A.4(2)]$. Again following $[7]$. a module *M* is called *H-supplemented* if for everysubmodule *A* of *M*. there exists a direct summand A^1 of *M* such that $M = A + X$ holds if and only if $M = A^1 + X$.

Following $[1]$. a submodule *N* of a module *M* is called *cofinite (in M)* if *M/N* is finitely generated, the module *M* is called *cofinitely supplemented* if every cofinite submodule of *M* has a supplement in *M* and *M* is called *amply cofinitely supplemented* if every cofinite submodule of *M* has a ample supplement in *M*. It is clear that, every supplemented modules are cofinitely supplemented and amply supplemented modules are amply cofinitely supplemented. Also finitely generated (amply) cofinitely supplemented modules are (amply) supplemented.

We call module *M H-cofinitely supplemented* if for every cofinite submodule *N* of *M*, there exists a direct summand *K* of *M* such that $M = K \oplus L$, and $M = N + X$ holds if and only if $M = K + X$ and we call *L H-supplement* of *N* in *M*. We call a module *M* a $\oplus$*-cofinitely supplemented* if every cofinite submodule of *M* has a supplement that is a direct summand of *M*. It is clear that $\oplus$-supplemented modules are $\oplus$-cofinitely supplemented, $\oplus$-cofinitely supplemented modules are cofinitely supplemented and H-cofinitely supplemented modules are $\oplus$-cofinitely supplemented. Conversely, finitely generated $\oplus$-supplemented modules are $\oplus$-supplemented. For the other definitions in this note we refer to $[7]$ and $[10]$.

Lemma 1.1 (see $[4.Lemma1.3]$) Let N and L be submodules of a module. M such that N+L has a supplement H in M and $N \cap (H + L)$ has a supplement G in N. Then H+G is a supplement of L in M.

Theorem 1.2 Any finite direct sum of $\oplus$-cofinitely supplemented modules is $\oplus$-cofinitely supplemented.

Proof For the proof, we completely follow the proof of $[4, Theorem 1.4]$. Let M_i be $\oplus$-cofinitely supplemented for each $1 \le i \le n$. Let $M = \oplus_{i=1}^{n} M_i$. To prove that M is $\oplus$-cofinitely supplemented, it is sufficent by induction on n prove this is the case when $n=2$. Let L be any cofinity submodule of M. Then $M=M_1+M_2+L$ so that M_1+M_2+L has a supplement 0 in M. Note that $M_2/(M_2\cap(M_1+L))\cong(M_2+M_1+L)/(M_1+L)=M/(M_1+L)$ so that $M_2\cap(M_1+L)$ is a cofinite submodule of M_2. Since M_2 is $\oplus$-cofinitely supplemented, there exists a supplement H of $M_2\cap(M_1+L)$ in M_2 such that H is a direct summand of M_2. By Lemma 1.1, H is a supplement of M_1+L in M. Note that $M_1/(M_1\cap(L+H))\cong(M_1+L+H)/(L+H)=M/(L+H)$ so that $M_1\cap(L+H)$ is a cofinite submodule of M_1. Since M_1 is $\oplus$-cofinitely supplemented, ther exists a supplement K of $M_1\cap(L+H)$ in M_1 such that K is a direct summand of M_1. By Lemma 1.1, $H+K$ is a supplement of L in M. Since H is a direct summand of M_2 and K is a direct summand of M_1. $H+K=H\oplus K$ is a direct summand of M. Thus $M=M_1\oplus M_2$ is $\oplus$-cofinitely supplemented.

Let M be a module. We consider the following conditions.

$(D1)$ For every submodule N of M, M has a decomposition with $M=M_1\oplus M_2$. $M_1\leq N$ and $M_2\cap N$ is small in M_2.

$(D3)$ If M_1 and M_2 are direct summands of M with $M=M_1+M_2$, then $M_1\cap M_2$ is also a direct summand of M.

Clearly, every $(D1)$ module is $\oplus$-cofinitely supplemented. Hence we have the following Corollary.

Corollary 1.3 Any finite direct sum of modules with $(D1)$ is $\oplus$-cofinitely supplemented.

Proposition 1.4 Let M be a $\oplus$-cofinitely supplemented module. If M is indecomposable then every proper cofinite submodule of M is small in M.

Proof Let N be a proper cofinite submodule of M. Then $M = K \oplus K^1 = N + K$ with $N \cap K$ is small in M. Hence $K = 0$ or $K = M$. If $K = 0$ then $M = N$. If not, then $K = M$ and N is small in M.

A module M is called *loca lif* the sum of all proper submodules of M is also a proper submodule of M.

Proposition 1.5 Let $M = \oplus_{i \in I} M_i$ where each M_i is local. If Rad (M) is small in M then M is $\oplus$-cofinitely supplemented.

Proof For the proof, we use the technic in the proof of $[6, Theorem 2.12]$ Let $M = \oplus_{i \in I} M_i$. Then $M / Rad(M) = \sum_{i \in I} [(M_i + Rad(M)) / Rad(M)]$ and each $[(M_i + Rad(M)) / Rad(M)] \cong M_i / (M_i \cap (Rad(M)))$ is simple. So $M / Rad(M)$ is semisipmle. Let N be a cofinite submodule of M. Than $(N + Rad(M)) / Rad(M)$ is a cofinite submodule and a summand of $M / Rad(M)$. Then $M / (Rad(M) = (N + Rad(M)) / Rad(M)) \oplus [\sum_{i \in J} (M_j + Rad(M)) / Rad(M)]$ for some $J \subset I$, and so $M = N + (\oplus_{i \in J} M_j) + Rad(M)$. Since $Rad(M)$ is small in M, $M = N + \oplus_{j \in J} M_j$ and $N \cap (\oplus_{j \in J} M_j)$ is small in M. Hence $N \cap (\oplus_{j \in J} M_j)$ is small in $\oplus_{j \in J} M_j$. Therefore, M is $\oplus$-cofinitely supplemented.

Lemma 1.6 Let R be any ring and let M be a $\oplus$-cofinitely supplemented R-module. Then every cofinite submodule of the module. $M / Rad(M)$ is a direct summand.

Proof Let $N / Rad\ (M)$ be any cofinite submodule of $M / Rad\ (M)$. Then N is a cofinite submodule of M and by hypothesis there exists a submodule K of M such that $M = N + K = K \oplus K^1$ and $N \cap K$ is small in K. Since $N \cap K$ is also small in M. $N \cap K \leq Rad(M)$. Thus
$M / Rad(M) = (N / Rad(M)) \oplus ((K + Rad(M)) / Rad(M))$ as required.

Theorem 1.7 Let Rad $(M)\langle\langle M$. Then M is H-cofinitely supplemented if and only if every cofinite submodule of M / Rad (M) is direct summand and each cofinite direct summan of M / Rad (M) lifts to a direct summand of M.

Proof $(\Rightarrow)$: By Lemma 1.6, we prove only last statement and let $N/Rad(M)=\overline{N}\leq\overline{M}=M/Rad(M)$ be a cofinite submodule. Then *M/N* is finitely generated and so $M=N+K$ with $N\cap K\leq Rad(M)$ for some $K\leq M$. By assumption, there exists a direct summand L of M such that $M=L\oplus L^1$, for some submodule L^1 of *M*, and $M=N+X$ if and only if $M=L+X$. Hence $M=N+L^1$ and $N\cap L^1$ is small in L^1. It follows that $\overline{M}=\overline{N}\oplus\overline{L^1}$. Now we show $\overline{N}=\overline{L}$. Since *N* is cofinite, *N* + *L* is cofinite and so $\overline{N+L}$ is cofinite in $\overline{M}$. By hypothesis $\overline{M}=\overline{N+L}\oplus\overline{U}$ for some $\overline{U}\leq\overline{M}$. It implies $M=N+L+U$ with $(N+L)\cap U=RadM$. Then $M=N+U=L+U$. By modularity

$N+L=L+((N+L)\cap U)=N+((N+L)\cap U)$.

It follows that $\overline{N}=\overline{L}$ since $(N+L)\cap U=RadM$.

$(\Leftarrow)$: Let *N* be a cofinite submodule of *M*. Then $(N+Rad(M))/Rad(M)=\overline{N}$ is a cofinite submodule of $\overline{M}$. There exists a submodule $\overline{K}$ of $\overline{M}$ such that $\overline{M}=\overline{N}\oplus\overline{K}$ and $\overline{N}=\overline{L}$ for some submodule L of M with $M=L\oplus L^1$. Since $Rad(M)$ is small in *M*, it follows that $M=N+X$ if and only if $M=L+X$.

Lemma 1.8 $(see(1, Lemma 2.7))$ Let R be any ring. The following statements are equivalent for an R-module M.

(1) Every cofinite submodule of M is a direct summand of M.
(2) Every maximal submodule of M is a direct summand of M.
(3) M/Soc(M) does not contain a maximal submodule.

By [1], *Loc [M]* will be the sum of all local submodules of *M* and *Cof (M)* the sum of all cofinitely supplemented submodules of *M* and $Loc(M)\leq Cof(M)$. Therefore, $\oplus-Cof$ *(M)* will denote the sum of all (cofinitely) $\oplus-$ cofinitely supplemented submodules of *M*. Since a $\oplus-$ (cofinitely) supplemented module is a (cofinitely) supplemented module, $Loc(M) \leq \oplus - Cof(M) \leq Cof(M)$. $Loc_1(M)$ will denote the sum of all local submodules which are direct summands of M and $Cof_2(M)$ will denote the sum all $\oplus -$ cofinitely supplemented submodule which are direct summands of M. Clearly, $Loc_1(M) \leq \oplus - Cof_2(M)$.

In case R is the ring Z of rational integers. Then

(1). $Loc_1(M) = Loc(M) = 0$ for every torsionfree R-module M, since local and torsionfree Z-module is zero.
(2) $Cof(M) = \oplus - Cof(M) = \oplus - Cof_2(M) = M$ for every injective R-module M by [10,42.23].
(3) Let M denote the Prüfer p-group $Z(p^{\infty})$ for some prime integer p. Then $Loc_1(M) = 0, \oplus - Cof_2(M) = M$.

Theorem 1.9 Let R be any ring.

(1) M is H-cofinitely supplemented.
(2) Every maximal submodule of M has an H-supplement.
(3) The module $M / Loc_1(M)$ does not contain a maximal submodule.
(4) The module $M / \oplus - Cof_2(M)$ does not contain a maximal submodule.
Then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4)$.

Proof $(1) \Rightarrow (2)$ Clear.
$(2) \Rightarrow (3)$ Let $K / Loc_1(M)$ be a maximal submodule of $M / Loc_1(M)$. Then K is maximal in M. By (2), there exists a direct summand L of M such that $M = K + X$ if and only if $M = L + X$. Then $M = L \oplus L'$ for some submodule L' of M. Hence $M = K + L'$, $K \cap L'$ is small in L' and L' is a local direct summand. Hence $L' \leq Loc_1(M)$. Thus $K / Loc_1(M) = M / Loc_1(M)$. It is a contradiction. $(3) \Rightarrow (4)$ Clear from $Loc_1(M) \leq \oplus - Cof_2(M)$.

Theorem 1.10 Let R be any ring. We consider the following statements for an R-module M.

(1) M is $\oplus -$ cofinitely supplemented.
(2) Every maximal submodule of M has a $\oplus -$ supplement in M.

(3) The module M/Loc(M) does not contain a maximal submodule.
(4) The module M/ $\oplus -$ Cof(M) does not countain a maximal submodule.
(5) The module M/Cof(M) does a countain a maximal submodule.
Then $(1)\Rightarrow(2)\Rightarrow(3)\Rightarrow(4)\Rightarrow(5)$.

Proof $(1)\Rightarrow(2)$ Clear

$(2)\Rightarrow(3)$ Let *K* be a maximal submodule of *M*. Show that *Loc(M)* is not a submodule of *K*. Buy (2), there exists a submodule *L* of *M* such that $M = K + L = L \oplus L'$ and $K \cap L$ is small in *L*.. Note that $L/(K \cap L) \cong (L+K)/ = M/K$, so

that $K \cap L$ is a maximal submodule of *L*. Therefore *L* is a local submodule of *M*. It follows that $Loc(M)$ is not a submodule of *K*.

$(3)\Rightarrow(4)$ Clear since $Loc(M) \leq \oplus - Cof(M)$.

$(4)\Rightarrow(5)$ Since $\oplus - Cof(M) \leq Cof(M)$, clear.

Let *M* be module and *N* a submodule of *M*. Following $[3]$.*N* is called *closed* in *M* if *N/K* is smaal in *M/K* implies $N = K$ for all submodules *K* of *M* contained in *N*. Let *A* and *B* submodules of the module *M* such that $B \leq A$. If *B* is coclosed in *M* and *A/B* is small in *M/B* then *B* is called an *s-closure* of *A* in *M*.

Proposition 1.11 Let M a module such that every submodule of M has a cofinite s-closure in M. Then the following statements are equivalent.
(1) M is $\oplus -$ cofinitely supplemented.
(2) For any cofinite coclosed submodule L of M, there exist a direct summand K of M such that K is a supplement of Lin M.

Proof $(1)\Rightarrow(2)$ is obvious.

$(2)\Rightarrow(1)$ Let *A* be any cofinite submodule of *M*. Since *A* has cofinite s- closure in *M*, there exists a cofinite coclosed submodule *B* of *M* such that $B \leq A$ and *A/B* is small in *M/B*. By (2), there exists a direct summand *K* of *M* such that $M = B + K$ and $B \cap K$ is small in *K*. Note that $M = A + K$.We assume that $K' \langle K$ and $M = A + K'$. Then $M \neq B + K'$ and so $M \neq A + K'$ because A/B is small in M/B. Thus *K* is a supplement of *A* in *M*.

2.COMPLETELY $\oplus$ – COFINITELY SUPPLEMENTED MODULES

In [4], a module M is called completely $\oplus$ – supplemented if every direct summand of M is $\oplus$ – supplemented. In this vein we call a module M *completely* $\oplus$ – *cofinitely supplemented* if every direct summand of M is $\oplus$ – cofinitely supplemented.

Example 2.1 *i.* Every completely $\oplus$ – supplemented module is $\oplus$ – cofinitely supplemented.

ii. Every completely $\oplus$ – cofinitely supplemented module is $\oplus$ – cofinitely supplemented.

Example 2.2 Let R be a discrete valuation ring with field of fractions K. P be the unique maximal ideal of R and let $P = Ra$ for some element $a \in P$. Then $M = (K/R) \oplus (R/P)$ is completely $\oplus$ – supplemented and so M is completely $\oplus$ – cofinitely supplemented. But M is not $(D1)$ and M satisfies $(D3)$. (see.[4.Example 2.1])

Lemma 2.3 Let M be an indecomposable module. Then M is hollow if and only if M is completely $\oplus$ – cofinitely supplemented.

Proof Clear.

Proposition 2.4 Let $M = N \oplus K$ such that N and K have local endomorphism rings. Then Mis completely $\oplus$ – cofinitely supplemented if and only if N and K are hollow modules.

Proof $:\Rightarrow$ From Lemma 2.3

$\Leftarrow$: Let L be a direct summand of M. If $L = M$ then L is $\oplus$ – cofinitely supplemented by Corollary 1.3. We assume that $L \neq M$. Then either $L \cong N$ or $L \cong K$ by Krull- Schmidt- Azumaya Theorem [2, Corollary 12.7]. In either case L *is* $\oplus$ – cofinitely supplemented. Then M is completely $\oplus$ – cofinitely supplemented.

Example 2.5 Let a be any integar and M denote the Z- module $(Z/a^iZ)\oplus(Z/a^jZ)$, $(i,j\in N)$. By $[10,31.14]$, Z/a^iZ and Z/a^jZ have local endemorphism rings. Then M *is* completely $\oplus$ – cofinitely supplemented by Proposition 2.4.

The module M has finite Goldie dimension if M does not contain an infinite direct sum of non-zero submodulers. It is well- know that a module. M has finite Goldie dimension if and only if there exists a positive integer n and uniform submodules $U_i, (1\le i\le n)$, of M such that $U_1\oplus\ldots\oplus U_n$ is essential in M (See[2]).

Theorem 2.6 Let M be a non-zero module with finite Goldie dimension. Then the following statements are equivalent.
(1) Every direct summand pf M is a finite direct sum of hollow modules.
(2) M is completely $\oplus$ – cofinitely supplemented module.

Proof $(1)\Rightarrow(2)$ From Corollary 1.3.
$(2)\Rightarrow(1)$ Let N be a direct summand of M. N has a decomposition $N=A_1\oplus\ldots\oplus A_n$, where each A_i is indecomposable for $1\le i\le n$ for some finite integer $1\le n$ because N has finite Goldie dimension. By Lemma 2.3. each A_i is hollow.

Recall that a module M is linearly compact if for every index set I. elements m_i in M and submodules $N_i(i\in I)$ such that the cosets m_i+N_i satisfy the finite intersection property, $\cap_I(m_i+N_i)$ is non-empty (See, namely [9] or [10]).

Corallary 2.7 Let M be a linearly compact module. Then the following statements are equivalent.
(1) Every direct summand of M is a finite direct sum of hollow modules.
(2) M is completely $\oplus$ – cofinitely supplemented module.

Proof By [9, Proposition 3.4] and Theorem 2.6.

We know that, a module M is said to have *finite exchange property* if for any finite index set I, whenever $M\oplus N=\oplus_I L_i$ for modules N and L_i, then $M\oplus N=M\oplus(\oplus_I K_i)$ for submodules $K_i\le L_i$ (See, namely [7]).

Lemma 2.8 Assume that M_i is completely $\oplus-$cofinitely supplemented and has the finite exchange property for each i=1,…,n. Then $\oplus_{i=1}^{n} M_i$ is completely $\oplus-$cofinitely supplemented.

Proof By Theorem 1.2, $\oplus_{i=1}^{n} M_i$ is $\oplus-$cofinitely supplemented. Let N_1 be a direct summand of $\oplus_{i=1}^{n} M_i$ and suppose that $\oplus_{i=1}^{n} M_i = N_1 \oplus N_2$ for some submodule N_2 of *M*. By [7, Lemma 3.20], $\oplus_{i=1}^{n} M_i$ and N_2 have the finite exchange property. So $N_1 \oplus N_2 = \oplus_{i=1}^{n} M_i = N_2 \oplus (\oplus_I K_i)$ where K_i is a direct summand of M_i. By hypothesis, every K_i is $\oplus-$cofinitely supplemented and so $\oplus_{i=1}^{n} K_i$ is $\oplus-$cofinitely supplemented. Hence every direct summand of $\oplus_{i=1}^{n} M_i$ is $\oplus-$cofinitely supplemented. Thus $\oplus_{i=1}^{n} M_i$ is completely $\oplus-$cofinitely supplemented.

Let *M* be a module. *M* has summand sum property if the sum of any two direct summands of *M*. is direct summand of *M* nad denoted by *SSP*.

Theorem 2.9 *Let M be an R- module. Assume that M is* $\oplus-$*cofinitely supplemented and N is a submodule of M. If for every direct summand K of M.* $(N+K)/N$ *is direct summand of M/N, then M/N is* $\oplus-$*cofinitely supplemented. In particular, if M has SSP, then M is completely* $\oplus-$*cofinitely supplemented.*

Proof Any cofinite submodule of *M/N* has the form *T/N* where *T* is a cofinite submodule of *M* and $N \subseteq T$. Since *M* is $\oplus-$cofinitely supplemented.there exists a direct summand. M_1 of M such that $M = T + M_1$ and $T \cap M_1$ is small in M_1.Now $M/N = T/N + (M_1+N)/N$. By hypothesis, $(M_1+N)/N$ is a direct summand of M/N. Since $T/N \cap ((M_1+N)/N) = (T \cap (M_1+N))/N = (N + (M_1 \cap T))/N$ and $T \cap M_1$ is small in M_1, $(N+(M_1 \cap T))/N$ is small in $(M_1+N)/N$ and so $T/N \cap (M_1+N)/N$ is small in $(M_1+N)/N$. Hence M/N is $\oplus-$cofinitely supplemented.

Assume that has SSP. Let N be a direct summand of M.Then $M = N \oplus N'$ for some submodule N' of M. Assume that L is a direct summand of M.Since N has SSP, $L+N'$ is a direct summand of M, that is, $M = (L+N') \oplus K$ for some submodule K of M. Hence $M/N' = ((L+N')/N') \oplus ((K+N')/N')$. Therefore M/N' is $\oplus-$cofinitely supplemented and so N is $\oplus-$cofinitely supplemented.

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ÖZET

R birimli bir halka ve M birimli sağ $R-$modül olsun. M/N sonlu üreteçli olan M nin N alt modülüne M de ***eş sonlu alt modül*** denir. M nin her eş sonlu A alt modülü için, $M = A + X$ sağlanır ancak ve ancak $M = A' + X$ olacak şekilde M nin bir A' dik toplananı varsa M modülüne $H-$***eş sonlu tümlenmiş modül***, ve M nin her eş sonlu alt modülü M nin bir dik toplananı olan M de bir tümlek alt modüle sahip ise, M modülüne $\oplus-$***eş sonlu tümlenmiş modül*** denir. Bu çalışmada, modüllerin bu sınıfları incelendi.

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