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Review Article

Mathematical Modeling of Food Processing Operations: A Basic Understanding and Overview

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ABSTRACT

Modeling is the core of food processing supported by many approaches and governed by heat, mass, and momentum transfer equations. The objective of this paper is to mainly discuss and introduce mathematical modeling of some food processes. Food processing is unique from other material processing, as it includes complex multiphase transport and change in material properties during processing. It poses a great challenge in food process engineering. Now a day's, consumers are taking more precautions before eating something. The way of food processing effectively impacts food quality. Most of the conventional industries use thermal processes like pasteurization, sterilization, and freezing. In recent years the main aim has been to improve these conventional processing technologies. Characterization of temperature distribution is done by mathematical modeling during processing, so this review paper aims to introduce mathematical modeling as a potential tool for the food processing industry. The mathematical models discussed in this article captures the essential features of a complex object or process based on a theoretical understanding of the phenomena and available measurements.

RESEARCH ARTICLE

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INTRODUCTION

Food processing is a transformation of the raw product into consumer-ready product by introducing several treatments like thermal (pasteurization, sterilization, drying, cooking etc.), mechanical (crushing, churning, or homogenization etc.) or combination (e.g., extrusion), and several other processes. Food materials are quite complex and non-homogeneous in nature. So, food processing is done to transfer raw ingredients into food or transfer food products into another form. Physico-structural properties and transport phenomenon is involved readily in food processing. Food constantly changes whether food is processed or not because it absorbs moisture (when kept in humidified conditions) or loses (when kept in dehumidified conditions or high temperatures). During processing like thermal treatment (pasteurization, sterilization, or cooling process), along with product temperature, other properties like biochemical (colour, flavour, nutrition etc.), physical (shrinkage, texture, bulk density etc.), chemical (denaturation, Maillard reaction, oxidation etc.) also change. Other dependent properties of food like thermal and electrical conductivity, specific heat, effective moisture diffusivity, permeability, and viscosity are a function of moisture content, temperature, and composition. So, during processing, all the properties mentioned above change, and the degree of changes depend on the extent of treatment. So, before processing food, optimization is done to have a more efficient process, with minimum losses. While optimizing any process, there needs to be detailed information about the characteristics and problems of food processing operations. Major food processing problems are covered in the following contexts: (a) While giving a thermal treatment or in a cooling process, the temperature change and biochemical changes like changes in nutrient profile, color, and texture have equal weightage to be understood. (b) When the food has continuous gain (from humid surroundings) or moisture loss (heating especially due to evaporation). (c) The basic properties like density, thermal conductivity, diffusivity, specific heat, permeability etc., are functions of composition, moisture content, and temperature, which keeps on changing, and thus their structure and changing behaviour needs to be understood thoroughly (d) The shrinkage (loss of moisture) or swelling (gain of moisture as a function of changing hygroscopicity can bring continuous changes in the food structure. (e) Irregular spaces are yet another problem. (f) Sometimes, multiple processes like temperature, moisture content, and phase change are often coupled with each other.

The prediction of different models has been developed based on the environmental conditions and the type of treatment offered to the corresponding primary food product. For example, suppose the prediction is done on the basis of microbial growth. In that case, it is extremely important to understand the responses of different microbes to the factors responsible for their growth. Thus, in the case of microbiological context, a model will be a mathematical expression that can integrate the survival, growth, and other biochemical processes under provided conditions (Prabbhakar *et al.*, 2019). Similarly, for other food processes, like evaporation of milk to form skimmed milk powder or let's say conversion of barley into beverage, all will include an expression which will describe the process, considering all parameters in the similar conditions at which the experiment is to be or has been done. As per Balsa-Canto *et al.* (2016), there are six major areas of division of drivers in innovations according to consumers survey, which can be classified as health value, pleasure, safety, ethics, convenience, and physical.

The competitiveness has thus increased many folds due to the increasing consumer demand which has added pressure on the companies to continuously modify with the market change. These goals can be efficiently and effectively achieved by optimization of models and computer-aided simulation. In recent years, several researchers have studied to understand the mechanism of food processing by conducting several experiments and using mathematical models.

Models have a vast scope. The relationships between the input and output processes make any model suitable to analyse the physical processes occurring in food. We term it as "research objective" in modeling. To say, if we consider the modeling of a sterilization process, the input parameter will be steam temperature, and output parameter will be bacterial death; thus, the impact of input on output parameters will be understood. After the revolution came in hardware and software parts and recent emerging computing techniques, the use of the model to design processes, and checking the "what if" situations arose. Hence, taking the above example, one can easily check the effect of the process variable (temperature) on the quality of sterilization undergoing. Future developments for a better change will introduce an optimization software that will bring the complete temperature profile in front of the researcher for best sterilization quality. This further increases the scope of modeling. Thus, the major drivers for advances in modeling are product processes and equipment design. There have been emerging technologies of food processing in today's date, including pulsed electric field, ultrasonic processing, non-ionizing radiations, etc. All these processes are characterized by some features. The research and development team generally have big challenges in front of them during competencies for developing an appropriate modeling approach due to (a) more and more complex food operations, (b) oversimplifying the food engineering phenomenon, and (c) generating compliance to food safety standards.

Along with process development, model modification and development are also taking place. Hence, making a selection of a proper model for a particular process is also important. For example, for understanding heat and mass transfer in drying, frying, baking, cooking, roasting, toasting, etc., selecting a model is important. Most researchers use a diffusion-based or single-phase model for heat and mass transfer for the above-mentioned process. These models are very simplified and do not consider the fundamentals of the physics of food processing (Khan *et al.*, 2018). In contrast to these models, the multiphase model is the best one, as it considers all phases' (liquid water, vapor, air) transport mechanisms separately. So, this model gives a better insight view of the transport mechanism. Mercier *et al.* (2014), Gulati *et al.* (2015), Kumar *et al.* (2016), Gulati *et al.* (2016) used the multiphase model to explain food processing in a more explicit way.

Models have numerous applications in food industries, whether it is logistic, quality, designing, or R&D, costing, energy consumption, processing. Friis *et al.* (2002) have shown CFD as a qualitative tool for designing hygienic process equipment using a 3-D flow model. Using this model, we can know a set of hydrodynamic parameters that play a crucial role in cleaning the closed process system; by using this, we can design complex equipment parts that will be very easy to clean. Current and novel applications of logistic optimization by industries are important and necessary because factors such as perishability, safety, variability, and flat payoff costs, because of its biological nature, the food supply chain is affected by increasing demand, pollution, water availability, and increasing costs and complexity. Modeling and simulation play an important role in scheduling operations, assessing sustainability, reducing waste, designing transportation networks, and planning the infrastructure of food supply chains. (García-Flores *et al.*, 2015), along with this modeling, also plays an important role in total quality management (TQM). Quality costing is the first step for preparing a case for the TQM initiative. Moreover, a realistic estimation of quality costs is an essential element of any TQM initiative. However, very few organizations use this method as it is very costly.

Baking is one of complex food processing where several phenomena occur simultaneously, which is chemical, biochemical, and physical phenomena such as gelatinization and formation of the porous structure. These changes affect the final quality of the product. These qualities also depend on flour type, fermentation time, protein content, and type of additives etc. Mohammadi *et al.*, (2020) has developed a mathematical model to predict crust temperature and weight loss of toast bread at different oven temperatures and time.

Zugarramurdi *et al.* (2007) have developed a mathematical model for the calculation of the costs associated with a specific quality level due to HACCP-based system implementation. The confidence development and validation of models is still a challenge even though there have been endless efforts in building rigorous models and simulation of numerical. In this paper, we will describe how model is formulated, with their actual application in food industries, and show up to what extent models wok like a physical prototype and how it is reconciled with experimental data.

MODELING APPROACH

Model is a mathematical analogue of a physical process. In modeling, different processes are expressed in the form of equations (mathematical) which are based on fundamental physics and have the general form of partial differential equations. Computational way of process simulation is achieved by the solution set of such partial differential equations. Model is generally used for understanding of physical process by using relationship between input and output parameters or used for designing or in checking the 'what if' scenarios, which helps in avoiding laborious and time-consuming experiments. Trystram (2012) has explained the main step necessary for modeling food or food processing. Modeling is widely used as a tool for optimum design, to understand process, prediction, used in different sector-wise innovation like automotive and aerospace industries, while this tool's contribution is unutilized in the food sector. The advantages of modeling as explained by Datta *et al.*, (2007) are many like providing insights into the process deeper as in the case of physics-based models which was not possible while experiments, then reducing the total number of experiments to few, which reduces time and expenses, optimization of processes, the prediction-based working (what if scenarios), automating complete process with control capability.

In modern science, all simple problems are solved easily. What remain problems are complex ones, which cannot be solved by simple experiments and deductions. To understand these problems, we have to decompose them into their constituting processes, and each of these processes has to be studied, analyzed, and modeled separately. In food processing, a mathematical model is used for the approximate representation of processes in mathematical terms. As processes are very complex and difficult to understand, it will be impossible for mathematical models to provide actual representation of a whole process. In formulating a mathematical model as shown in Figure 1, we only consider major input and output; negligible inputs and outputs are not considered. Still, mathematical modeling is an effective tool to evaluate the effect of process parameters on the outcome of certain processes as it is a rapid and expensive method. Mathematical modeling is only done after getting data in tabular or graphical form.

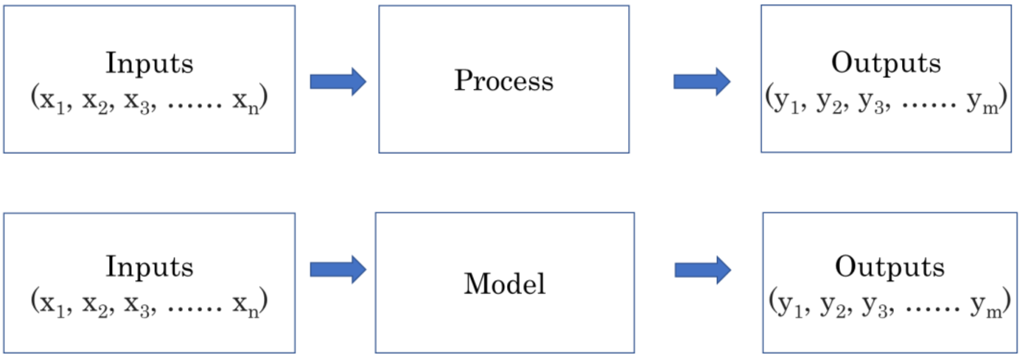

Figure 1. Comparison input/out for process and model.

The knowledge of fundamental physical mechanisms governing processes is a requirement in physical-based mathematical modeling. Integrated knowledge of engineering, chemistry, and microbiology is required to understand the multidisciplinary character of food processing. Erdogdu *et al.* (2017) have mentioned essential steps for mathematical modeling. Mathematical models may be classified into the In-observation model and physical-based models based on the selection of initial points while designing a model. In-observational models have the data from experiments which is the base for developing the model. This type of model is entirely empirical. The empirical model comes from introspection or observation (both). There is usually no theoretical background for them; these models are generally used to classify and characterize data, know generalized behaviour from the given measurements, and learn something from these observations. An empirical model is only applicable within the range of data during the experiment.

In contrast to observational models, the starting point for the physical-based models is universal laws of physics that determine the physics and the related equations. The very fundamental laws, such as Newton's laws of motion, define the models which are based on fundamental physics. These models have been validated against the data set generated during experiments, but the data doesn't exist before the existence of the model itself. Physics-based models are generally overweighed against others in the way that they are more trustable and calculations are more precise (since the initial point is universal conservation laws); also, the parameters can be easily measured with the help of available techniques. Among the various physical models, the most popular are continuum (macroscale) models (Datta, 2008). Many of the models based on observation and physical models are reported by Datta *et al.* (2007). Datta, 2008 has also mentioned that just like in other areas, analytical approaches were the foremost for the start of physics-based models, the first one being the work in sterilization and dehydration. Sterilization was also the base for the start of numerical modeling. Until the late 1980s, the analytical model was used for most of the problems. The mid-1990s became a revolution in the field of modeling of food processing as by then, there were so many improvements in numerical methods assisted with computer technology which was boomed by the introduction of Computational fluid dynamics (CFD) software making modeling very easy and user-friendly. Mathematical modeling may be categorized as per Figure 2.

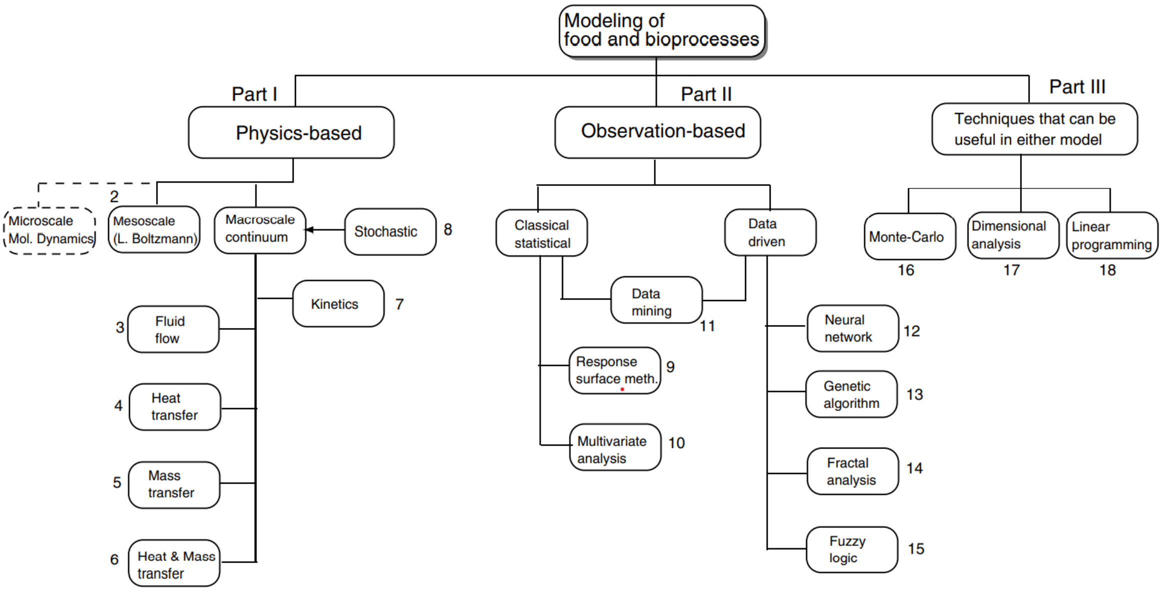

Figure 2. Different methods of modeling with some examples (Datta, 2008).

MODELING OF FOOD AND FOOD PROCESS

Keeping the base as existing theory and measurements, the main aim of modeling is to capture the essential mechanisms during the processing of food. The main steps are given below. Some of them being the structural design of the model: variable to be chosen, nature of the relationship, the ability of calculations are validated by test of identification of design of the structure.

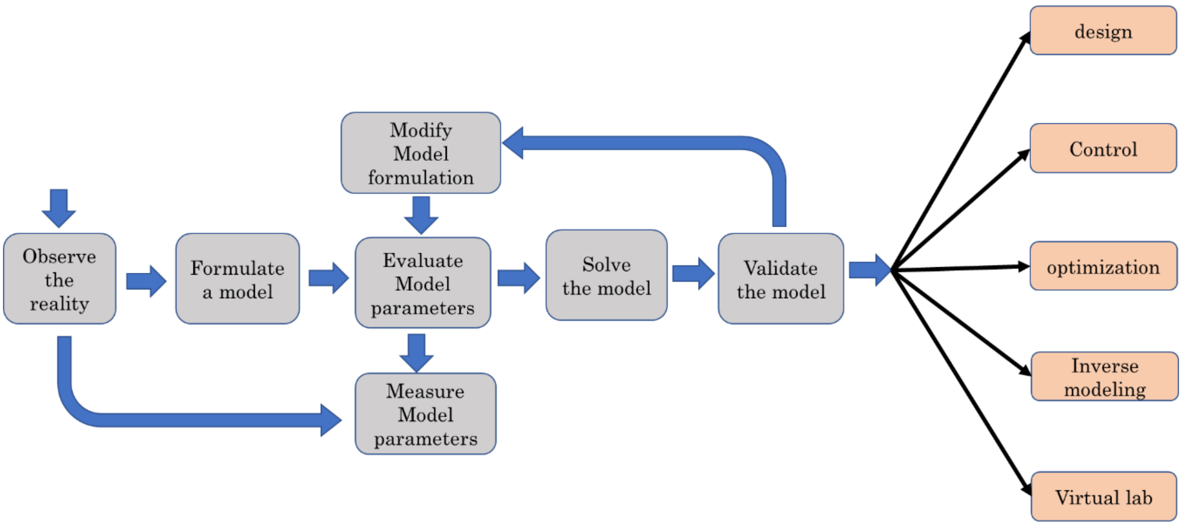

Figure 3. Framework of steps in modeling (design to validation) (Erdogdu *et al.*, 2017).

In the current generation, the physics-based models are becoming more realistic by including more detailed physics, which is possible due to advances in computer technology, making modeling more user-friendly, thus increasing its scope and application in product, process, and research and equipment design. Due to the unavailability of correlations between food quality, food safety and process parameters in physics-based models, physics-based models have not been made ready for food quality and safety yet. In observational models, physical processes are not required; hence these are highly preferred for modeling food safety and quality. In food industries, most of the problems of fluid mechanics are solved by modeling which use the application of laws of motion of Newton in fluids, such as dryer, cooler, designs for ovens, refrigerated vehicles, pasteurization systems, extrusion, heating and cooling systems. In the food sector, our focus is mainly upon quality or safety and, in many cases, both of them. Figure 4 represents that the modeling can be of two types: Safety model or Quality model. Product, process, and equipment designs are covered in the physical processes of a process model. The quality/safety model includes a quantitative description of quality and safety characteristics such as colour or microbes' concentration as a function of temperature.

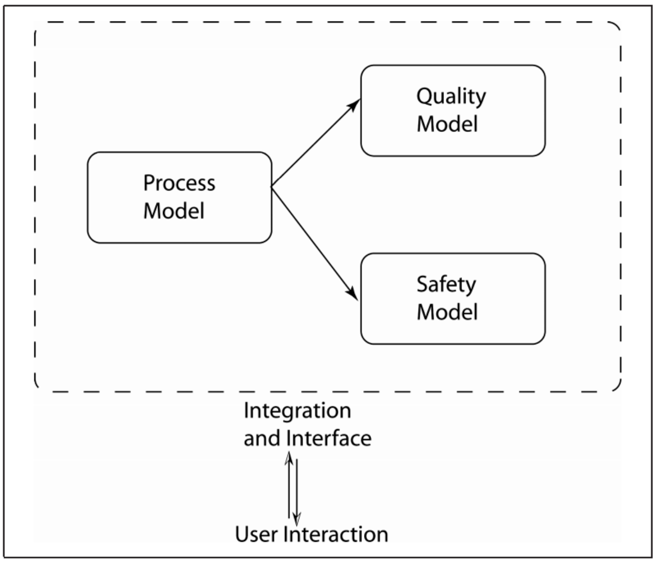

Figure 4. Modeling in food manufacturing as process model and quality/safety model.

Many qualities of food products during dryings like bulk density, porosity, stress cracking, shrinkage, crust formation, and case hardening are major concerns. Final texture during drying is relied upon the following factors: (1) different process parameters during drying and their histories like deformation, temperature, and moisture content, (2) the material's state: whether it is rubbery or glassy, and (3) the conditions of drying along with supporting external environment. Gulati *et al.* (2015) studied the phase transition between rubbery and glassy states of materials along which the deformation developed. Using these, a fundamental-based model for the coupled physics of transport was developed. The case hardening concept, along with various other quality attributes was known using this model. Due to the versatility of this modeling framework, the same can be applied in various other drying conditions to develop enhanced quality food products and highly efficient designed equipment based on food quality. Thus, food processing technologies' characterization, improvisation, and optimization can be easily achieved with the help of modeling. There are various fields where we can apply mathematical modeling like a model for HPP, canning, baking, roasting, grilling, frying, fouling, extrusion, brewing, microbial and enzymatic inactivation, heat & mass transfer (pasteurization, UHT etc.) models, which will be useful in the prediction of temperature profiles in a large solid-type of food, freezing, modeling of growth of ice crystal especially for ice-cream and lot more others. This paper contains modeling of conventional processing (freezing, retorting, pasteurization) which is mostly used in all food processing industries, and their effect on food (microbial inactivation) and equipment (Fouling). As food undergoes a complex transformation during processing, it makes the physics-based model less common in food processing (equipment design) compared to other sectors having major use like aerospace and automobile industry.

Scope of modeling

The standard methods used for food design, known as mathematical modeling, have not been effectively utilized in food process engineering; thus, enormous scope underlies there (Erdogdu *et al.,* 2017). A wide range of scales is covered in food processing based on physics-based models, which are,

(a) Molecular dynamics - micro-scale,
(b) Lattice-Boltzmann models - meso-scale, and s
(c) Macro-scale continuum models which are completely dependent on fluid flow, structural mechanics, heat and mass transfer and kinetics.

Empirical Methods were used where mutual relationships between variables are not unclear specially in a complex situation. MacFie (1994) theorized that new product development, a complex manufacturing process, can be developed using mathematical models, and it involved less work and more computing.

FOULING

Fouling is an unintended accumulation of food on a channel wall while processing which causes deposition. Fouling is a very common problem in food processing, majorly occurring in heat exchangers during thermal treatment due to the thermal instability of food ingredients. There are many structural and chemical changes during the heat treatment of food products. These changes result in making the constituents like proteins and minerals forming a film-like structure and being sticked upon the surface of food equipment. The layer which has been deposited is foulant, and the whole process is known as fouling. For example, milk fouling occurs in PHE, mainly due β-lactoglobulin (Georgiadis *et al.,* 2000). When milk temperature goes above 65^{o}C, β-lactoglobulin becomes thermal unstable and starts forming insoluble particles by irreversible polymerization.

It directly impacts operation performance by reducing heat exchanger efficiency, causing an increase in production loss, energy, and water losses because of repetitive cleaning operation and reduced system performance because of the pressure drop through the heat exchanger. Lots of authors have studied these deposits prevalent across different products. Optimization to the best-operating conditions can be achieved by using a specific model to minimize undesirable effects while heating in fouling. The prevalent models check upon the increasing heat transfer resistance and relate it with a rate of fouling. In the induction period, the thermal resistance doesn't increase with time; hence it is not considered except for boiling conditions. The basic mechanism of the film forming process is very much required for understanding the concept. There are two major thoughts regarding this fouling process: there is no role of mass transfer, a bulk controlled homogeneous reaction or the surface reaction process, and the mass transfer between the fluid containing protein and the thermal boundary layer. The deposition of protein is due to aggregated protein adhering to surface equipment which causes fouling, and this rate is directly proportional to the concentration of aggregation of proteins (Balsubramanian *et al.*, 2008). Few authors assumed that fouling is only due to protein aggregation, and few assuming that it is due to denaturation of protein. Few others believe that fouling is due to both protein denaturation and aggregation. So, based on different assumptions, there are various models. Manika *et al.* (2004) have mentioned that understanding heat exchanger fouling during pasteurization was still incomplete. So, he proposed a new model whose objective was to achieve profit per unit during one heating-cleaning cycle.

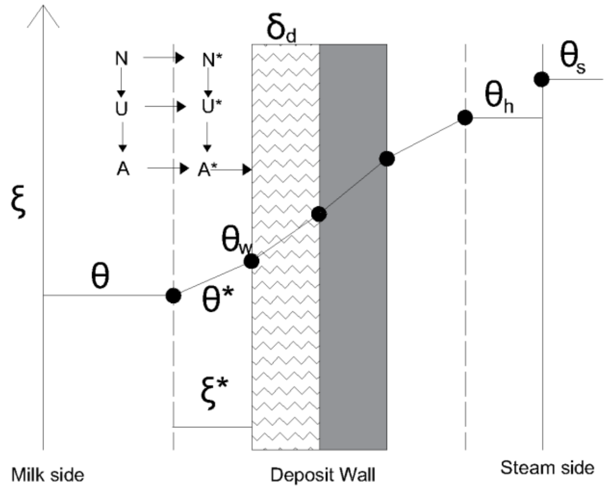

Figure 5. Fouling model.

The model considers two regions of fluid where fouling takes place: bulk and thermal layer. Deposit thickness at the end of the heating stage is

$$\delta_h(\xi) = \delta_0(\xi) + r(\xi) \cdot t_h \qquad (1)$$

Model of cleaning phase assumes two steps, namely swelling of deposit with the rate ω_y, and removal of swell deposit with the rate w_x.

$$\delta_c(\xi) = \frac{\omega_y}{\omega_x} \cdot \exp(-\omega_x t_c) \cdot [\, \exp\left(\frac{\omega_x}{\omega_y} \cdot \delta_h(\xi)\right) - 1\,] \qquad (2)$$

Where, t= time (c= cleaning time, h = heating time), ξ = Bulk layer condition ($0 \leq \xi \leq 1$), $\omega = deposit\ rate$ (y =for swelling, x= removal), δ = Deposit thickness (0 =initial condition, c = after cleaning, h= at the end of heating).

This model gives the dependence of deposit thickness after cleaning δ_c, versus initial (at the end of heating) thickness δ_h and cleaning time t_c. This model concludes that optimum pasteurization units consist of a long heating stage followed by short, infrequent cleaning.

Hydrodynamic and Thermodynamic Model

Jun *et al.* (2005b) have categorized fouling model into hydrodynamic and thermodynamic models, and integrated fouling dynamics models. In hydrodynamic and thermodynamic models, their patterns of flow streams made the mechanism of fouling clear in heat exchangers. Dynamic performance of plate heat exchangers was simulated by cinematic models; the time and space both are independent here. The main advantage here is the number of simulations is minimum which leads to an evaluation of performance in heat exchangers. A very simple approach to design problems is covered through one sweep of steady-state as well as dynamic profile view with the help of this model. Fouling behaviour in plate heat exchangers was studied with the help of 3-dimensional model in which the calculation gives virtual flow velocity fields, which further helped in forming new plate designs, and thus, the fouling on the surface was reduced, which was not possible in 2-dimensional configuration. In the 3-dimensional model, a region of turbulent backflow was also found, which emerged as a cause for elevated temperatures in specific regions and these regions were finally responsible for fouling. So, the CFD model has immense potential to help design a better plate that minimizes such occurrence of regions in a plate for reducing fouling. Continuity and momentum equations are used for describing flow equations in a 3-dimensional model. The continuity equation is defined below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

Momentum equations in all directions (x, y and z) are described below

$$\text{x-momentum: } \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \tag{4}$$

$$\text{y-momentum: } \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \vartheta\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \tag{5}$$

$$\text{z-momentum: } \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \vartheta\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \tag{6}$$

Transient energy equation for incompressible flow is given below

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{7}$$

Where, u, v, w is velocity of component in x, y, and z direction respectively, t is time, ϑ is kinematic viscosity, ρ is density, P is pressure, k is thermal conductivity, C_p is the specific heat capacity of the substance. The above model can be used by coupling with CFD software, fouling behaviour in PHE system can be understood. Jun *et al.* (2005a) have studied 3-dimensional milk fouling using the computational fluid dynamics package FLUENT. Thus, the new surface configuration can be designed with the help of a 3-dimensional model, which would help in reducing the fouling.

Dynamic Fouling Model Incorporating Physico-Chemical Changes

The basis for developing the dynamic model is that the fouling is a transient process. The model was developed across various phases of fouling as it collects over the equipment with the time. The study of the denaturation of β- LG is conducted with the help of dynamic models, and the observations are recorded for the fouling. Delplace *et al.* (1996) conducted an experiment to calculate the dry mass of deposits with the help of a complex configuration of 13 plates. Native β- LG's amount at the outlet of the heat exchanger can be predicted with the help of the model. The numerical calculation of temperature profiles for each channel and steady-state conditions was used as a basis for simulating the amount of denatured protein and was used for testing the model. β- LG at the outlet of the exchanger was found with an experimental error of less than 10% (Balsubramanian *et al.*, 2008).

$$C(t) = \frac{C_0}{1+kC_0t} \tag{8}$$

For T less than 363.15 K, log k = 37.95 – 14.51 (10^3/T),

For T greater than 363.15 K, log k = 5.98 – 2.86 (10^3/T)

Where, C and C_0 = concentration of β- LG at time t and initial condition respectively. k = second order constant rate, t = time, T = temperature.

Milk fouling in heat exchanger has been clearly understood, because of tremendous volume of milk and dairy product being processed worldwide, so intensive studies were done on it and most of the literature is limited to dairy products. Fouling problem has been a critical issue for other food products such as grape juice, tomato and orange juice. For cleaning and heat exchanging, understanding of interaction between foulants and contact surface is very critical. In food processing industries fouling may also be occurring across multiphase flowing foods through pipe. Kim *et al.* (2020) have developed a model by adopting the most common fouling model for a heat exchanger to predict deposition thickness of food channel.

DRYING

Drying involves coupled heat and mass transfer among the all-other food processing methods, making it a multi-physics model. However, food structure has many scales, but in food processing, specially drying, microstructural features play an important role. The quality of food materials and the structure have a good correlation. To design any food drying process, it is imperative to know the structure and behavior of the products & relationship between the structure and properties of food materials. However, the fundamental structure-property relationships are not very clearly understood till now; it may be due to the heterogeneous nature of food. Hence, food process engineers design the drying process by relying upon empirical data. The microstructure and product properties are completely different for different products. A recent trend approximates material's structure by using food drying models. A representative elementary volume exists that considers fluid phases as a fictitious continuum by adopting a larger description scale. Thus, models have a limited ability to capture the heterogeneity of the material (Welsh *et al.*, 2021).

The micro-level transport processes involved in multiscale modeling overcomes the above limitation. A simple macroscopic model will describe very less of drying phenomena. To describe all drying phenomena, multiscale modeling is required (Rahman *et al.*, 2018). Multiscale modeling is powerful technique that incorporate cellular homogeneity with micro-scale heat and mass transfer during drying. Multiscale model is combination of macro-scale model and micro-scale model (Figure 6). Thus, in the field of drying technology, multiscale modeling is a new yet very advanced approach. Welsh *et al.* (2018) has gave complete overview of multiscale modeling, there frameworks, and how factor affecting this model.

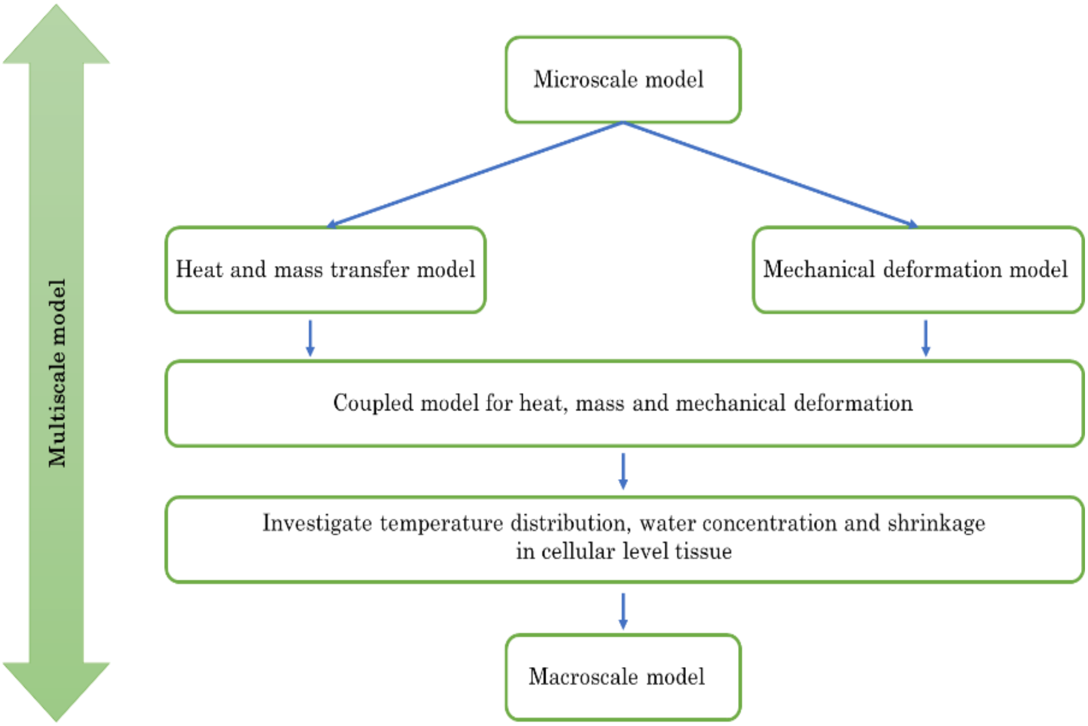

Figure 6. Multiscale approach in drying technology (Rahman *et al.*, 2018).

Effective Moisture Diffusion

Diffusion is a very complex phenomenon to understand during drying which includes majorly Knudsen flow, molecular diffusion, surface diffusion, and hydrodynamic flow. Welsh *et al.* (2021) have considered two spatial scales, which are microscale and macroscale. The macroscale model included applications of temperature and moisture-dependent diffusivity, while both microscale and macroscale included homogenized diffusivity. Effective homogenized diffusion was calculated by application of homogenization to the microscale domain. In macroscale mass transport phenomena, Fick's law of diffusion was used to derive the transport equation, given below:

$$\frac{\partial C}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left[-D_{i,\ eff}\, r\,\frac{\partial C}{\partial r}\right] + \frac{\partial}{\partial z}\left[-D_{i,\ eff}\frac{\partial C}{\partial z}\right] = 0 \qquad (9)$$

A kind of Arrhenius relation is used to define the temperature-dependent effective diffusivity approach used for food drying in the macroscopic model, which is:

$$D_{T,\ eff} = D_0 e^{\frac{-E_a}{RT}} \qquad (10)$$

Similarly, for products undergoing deformation, a shrinkage dependent effective diffusivity as a function of moisture content in relation to moisture dependent effective diffusivity in macroscopic model is defined as:

$$\frac{D_{ref}}{D_{M,\ eff}} = \left(\frac{b_0}{b}\right)^2 \tag{11}$$

The need of representation of heterogenous structure of food material was satisfied by introduction of microscale model. The previous macroscale mass transport equation has a coupled microscale mass transport equation defined as,

$$\frac{\partial}{\partial t} c(x, y, t) + \nabla \cdot \left(-D(x, y)\, \nabla \mathrm{c}(\mathrm{x}, \mathrm{y}, \mathrm{t})\right) = 0 \tag{12}$$

2-D homogenized effective tool called as tensor is used for calculation of microscopic effective diffusivity defined as,

$$D_{H,\ eff} = \begin{bmatrix} \frac{1}{\omega}\int_{\omega} D(x,y)\nabla(u_1(x,y) + e_1)d\omega & \frac{1}{\omega}\int_{\omega} D(x,y)\nabla(u_2(x,y) + e_2)d\omega \\ \frac{1}{\omega}\int_{\omega} D(x,y)\nabla(u_1(x,y) + e_1)d\omega & \frac{1}{\omega}\int_{\omega} D(x,y)\nabla(u_2(x,y) + e_2)d\omega \end{bmatrix} \tag{13}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Where, C = instantaneous moisture concentration (mol m^{-3}), t = time (s), $D_{i,\ eff}$ = effective moisture diffusivity (m^2 s^{-1}) with I = *T, M, H.*

Where T = temperature dependent, M = moisture dependent, H = homogenized diffusivity respectively, D = cellular diffusivity (m^2 s^{-1}), D_0 = integration constant (m^2 s^{-1}), E_a = activation energy (J mol^{-1}), D_{ref} = reference diffusivity (m^2 s^{-1}), b_0 = initial product's thickness (m), b = product's thickness (m) at time t (s), ω = area of the macroscale domain, u_1 and u_2 = corrective factors and the solution of the periodic cell problem.

The knowledge of microstructure evolution is applied to provide new insights in consideration of the distribution of water molecules with the help of a homogenous diffusivity model. A more mechanistic approach is involved in determining diffusivity using the above model, which will further assist in the advanced food drying process. Welsh *et al.* (2021) have proven that there was greatly reduced error in convective drying of apples at 45°C and 60°C using this model, while at 70°C, the errors were increased due to more microstructure deformation resulting from an increase in temperature.

ROASTING PROCESS

Roasting is a process of cooking that uses dry heat where the hot air covers the food. To get optimum culinary from roasting process, setting of process parameters is done by experiencing judgment or skill of an individual. In such a case, it is really tough to upscale the oven roasting process, especially while transferring the technology to a new equipment or setting up a new process design. Numerous researchers have developed models of mass transfer during roasting based on different hypotheses, and most of them are based on diffusion. A pure diffusion-based model cannot fully describe the moisture transport phenomenon while the food is being roasted because of the shrinkage effect and capacity of water binding, both of which are not considered. During roasting of meat, protein denaturation occurs, which reduces the water holding capacity and the protein network has also induced shrinkage. This phenomenon greatly accelerates the gradient of pressure inside muscles, and modeling of such phenomena requires poro-elastic theory. Van der Sman, (2007) used this theory to explain water transport phenomena during meat cooking. As per his statement, there was a quite large increment in moisture content at the centre of whole meat and another author has disagreed on this. Feyissa *et al.* (2013) studied roasting of meat, developed a coupled heat and mass transfer model in a convection oven, and solved it using the COMSOL Multiphysics model. He concluded that there was no rise in water content at the centre, which contradicted the previous statement. Temperature and moisture distribution was understood as a function of moisture content and time in this model. The change in microstructure water-holding capacity and elastic modulus in meat could be easily incorporated with the help of this model. The physics of meat while roasting can be easily understood with the help of this process, and the temperature along with the moisture content can be predicted quickly. Aiming at correct and effective moisture loss maintaining the quality of food safety during roasting, this model will completely fit the purpose.

Microbial Biomass Modeling

Microbial modeling is used to predict microbial growth or decline under specific environmental conditions with the help of mathematical expression. The journey of microbial modeling started in 1920s with thermal death time, D and Z value calculation for canned food to ensure that food was free from *Clostridium botulinum* (food poisoning bacteria) (Whiting and Whiting, 2009). Microbial models are a function of mainly many extrinsic (Temperature, gaseous atmosphere) and intrinsic variables (pH, a_w) and are mainly the mathematical expressions that describe the number of microorganisms at a macroscopic scale in a given environment. There is a major role of probability of a microorganism while defining any microbial model since it lies in the context of population of microorganisms. Mathematical model of microbial growth estimation is most commonly used in the field of food safety research, but along with this, few alternative models like Monte Carlo simulation modeling, artificial neural network, individual based model etc., also used widely in the field of risk assessment studies (Esser *et al.*, 2015). Using this mathematical model, one can determine the mathematical growth curve; for this, we have to find a function relationship between the number of live cells and temperature. While collecting data of above mention parameters for understanding growth kinetics, it is necessary to maintain a controlled environment like temperature, pH, water activity (a_w), salt and sugar of medium. As per doing this, only principal factors play significant role in growth kinetics. Other factors like changes in solubility, gas diffusion, nutrients, and heat transfer were neglected. As of now, there are several models have been developed for microbial prediction. However, the model selection will depend on the quality and quantity of the generated data and the complete knowledge of the system. Currently, the data and knowledge are not adequate to form mechanistic or fundamental microbial models; hence there are only empirical, and some phenological models are currently available. There are mainly three classifications: primary, secondary, and tertiary.

The primary model is a mathematical expression which describes microbial growth as a function of time or survival curve (after heat treatment, i.e., sterilization). The primary model is either empirical, phenological, growth rate model, inactivation or survival model, or combination. This model does not explain the reason behind a particular pattern. Among empirical growth models, the Gompertz model is most widely used (McKellar *et al.*, 2003). This model can be written in the following equation.

$$log_e N\ (t) = A + C\ exp\ \{\ exp[-B\ (t - m)]\} \quad (14)$$

Where *N(t)* is no of a cell at time t, *A* is the logarithmic value of initial population, *c* is asymptomatic logarithmic growth ratio, *B* is the relative growth rate, *t* is time, and *M* is time for maximum growth rate.

Major parameters that are considered in this are initial population size, or asymptotic population size, and relative growth rate. Because of flexibility in the Gompertz model, it has numerous applications in various sectors. As per Esser *et al.* (2015), Gompertz model, another model like the three-phase linearized growth or Buchanam model, and the logistic model are also widely used. These models do not describe all phase of microbial growth curve. Those models were used to predict density of population, but growth rate model is used to predict population growth rate as the name itself implies. As shown in equation no 14, the Gompertz model can be converted into a rate model (Peleg *et al.*, 2011). Verhulst (logistic) and Baranyi-Roberts model is the most commonly known rate growth model. Chick, (1908) has proposed a primary model to understand microbial inactivation kinetics and this model can be used if it is assumed that inactivation follows first-order kinetics. In the secondary model we apply the concept of D-value, though this concept may be insufficient. In secondary inactivation model, it describes k or D with respect to product T, pH and a_w.

$$D = D_{ref} \times 10^{\frac{T_{ref}-T}{Z_T}} \times 10^{\frac{pH_{ref}-pH}{Z_{pH}}} \times 10^{\frac{1-a_w}{z_{a_w}}} \quad (15)$$

Where, D = decimal reduction at T temperature, D_{ref} = required time at the reference temperature (T_{ref}) to decrease the bacterial concentration to 1/10, Z_T = required elevation in temperature to reduce the D value to 1/10, Z_{pH} = pH variation that enhances a reduction of 1/10th of the D value, z_{a_w} = difference of water activity (a_w) from 1 resulting in10-fold reduction of the D value.

Secondary mathematical model describe microorganism growth rate is affected by external factors such as pH and temperature, and water activity (a_w) (Peleg *et al.*, 2011). So, this model establishes a relationship between primary models and other intrinsic or extrinsic parameters. For quantitative study of microbial growth, Arrhenius model is still used as most prominently. McMeekin *et al.* (1993) have mentioned that applicability of Arrhenius model for microbial growth is because of inability to account for optimal growth temperature. Limitation of this model is that, it validated for restricted temperature range (i.e., growth rate of microorganism increases monotonically with temperature). Other mathematical models like Ratkowsky model (Ratkowsky *et al.*, 1983) or peleg-Corradini-Model (Peleg *et al.*, 2011) has explained how growth rate is dependent on temperature, pH and other factors. Primary and secondary models may be characterized as segregated or non-segregated, linear or non-linear, structured or non-structured (Whiting and Whiting, 2009). Tertiary models make the prediction tool complete by combining the other two models (primary and/or secondary model) with a computer interface. Computer program calculate microbial response with changing conditions, and do their comparison, or contrast the behaviour of several microorganisms. These models will help in various sectors like industries, extension, academia in food safety to understand, design or modify the outcome of several "what if" scenario.

In food sector, microbial growth plays important role in respect to food safety and quality, especially when we talk about perishable product like meat, milk etc. Contamination and growth are two major concerns for food industries. Traditionally, inspection and sampling used to maintain quality and safety of food products. Now using model one can predict microbial growth and their population, and how environmental factor (pH, storage temperature, packaging material) affect them. Cárdenas *et al.*, (2008) has shown the effect of a range of pH (6.1,5.8, and 5.6) at a variety of storage temperatures (0, 4, and 10°C) and was packed two films having different gas permeabilities (polyethylene and EVA SARAN EVA) on beef muscle extracted three bacteria (*Klebsiella* sp., E. *coli* and *Pseudomonas* sp.), which was inoculated on sample of ground meat and Gompertz model was used for microbial growth and linear equation and effect of temperature on specific microbial growth and lag phase duration value were modelled through Arrhenius type equation, to determining the corresponding activation energy. Swinnen *et al.* (2004) has covered well predictive modeling of microbial lag phase.

RETORT PROCESS

Packaged self-stable food products generally utilise thermal processing. An advanced form of food preservation is canning which includes retorting in semi-rigid flexible pouches. This process of thermal sterilization consists of heating food packed container in pressurized retort at specific temperature for particular length of time, depending upon nature of food. This time is calculated based on inactivation of bacteria in each container. If process fails in delivering sufficient lethality, causing risk of public health, probability of spoilage, further it will damage industries reputation. Loss of heat sensitive ingredient is always associated with any thermal process. So, both quality and safety require attention, while calculating process time and temperature, to avoid over or under-processing. Understanding temperature profile inside food container is very important, especially cold point, and it is majorly affected by heat transfer. The process time in canning is based on a specific degree of sterilization, while sensory properties and the texture present define the degradation of nutrients. Thermal processing has two opposing effects (desirable and undesirable), which are both time and temperature dependent. Thus, temperature change of any product is determined for a target sterilization value with the help of mathematical modeling of heat transfer focused on decreasing the rate of reduction of nutrients simultaneously assuring food safety. Some of author tried to understand retort process by experimentally, but due to high cost and time consumption in experiment did not find in-depth result (Mosna *et al.*, 2015). But in recent years, with the help numerical simulation these problems are understood deeply and allow us to set best thermal setting of particular process. Mosna *et al.* (2015) mentioned that among all other different numerical approach, CFD (computational fluid dynamics) is the most suitable for understanding the temperature profile inside the container during the retort process. Using this approach, most of the studies were done by considering the real configuration of the packaging; they all avoided the flow of steam-water movement inside a retort chamber. Mosna *et al.* (2015) have used three-dimensional geometric configuration to study internal temperature of cold spot in order to control time-temperature trend. Based on the results, they mention that chamber's internal configuration affects the heat exchange process. Cold point temperature is also affected by water content and total solids (plant origin); Gokhale *et al.* (2014) have suggested that must be a unified approach to mathematical modeling. They also have developed a semi-empirical unified model that is successfully extrapolated to predict product temperature over a range of process conditions for various products with a relative error is <10%.

BAKING PHENOMENA

There are a series of reactions taking place physically, chemically and biologically while baking which include volume expansion, formation of pores, starch gelatinization, protein denaturation, formation of crust and browning. Thus, there is simultaneously heat and water transport inside the chambers' environment as well as the product. Heat and water transport to product occur mainly through conduction from mold, radiation, natural and forced convection, condensing steam and evaporation of water. Baking is a high energy consuming process, so, overall energy consumption can be reduced with improving of product quality with the help of optimization of operating conditions (Sablani *et al.*, 1998). These transformations took place due to heat and mass transfer as well as mechanical deformation during baking of bread. The overall mechanism of baking not yet being clear makes it really difficult for predicting models under baking conditions. Any attempt to modify or alter the baking process requires an understanding of the physico-chemical changes involved in the process, so usually empirical and mathematical modeling approach is used. Model was solved by finite difference numerical method. Zhang *et al.*, (2016) has used spatial reaction engineering approach (S-REA) for modeling of browning kinetics while miniature bread baking using. Many equations of heat and mass transfer are used in S-REA which uses multiphase model approach where evaporation and condensation rates locally are described by the reaction engineering approach (REA). The models, as well as browning kinetics, are changed when the combination of S-REA approach with equations of moisture content and temperature with respect to colour change takes place. *Golchin et al.*, (2021) have developed mathematical models of weight loss and crust temperature of toast bread containing guar gum during baking process, and the predicted bread crust temperature and weight loss of bread (control and containing guar gum) were in a reasonable agreement with the experimental temperature with a coefficient of determination of 0.98 and 0.99, respectively. Zhang *et al*., (2006) have developed fully coupled model, it is developed for deformation and multiphase heat and mass transfer in bread baking. This model considers heat and moisture transport that is fully coupled with large volume change. Viscoelastic material property is used and is a function of temperature. The model displays the same transport and deformation characteristics as that observed from a baking experiment in terms of temperature, moisture loss, and surface browning. So, this model shows the interaction between transport and deformation and enables us to understand the baking process better.

INTEGRATION OF MODEL

Many processing steps are included in food processing which are all dependent. The models are present at all the stages of food processing, but they are built for specific purposes and the hypotheses are set accordingly having limitations to their applicability. The integration of all of these models is also complex (Figure 7). Considering these, three areas are only viable (1) Process engineering (mass transfer and heat transfer's characterization and modeling), (2) Microbiology, (production of metabolite along with characterization and modeling of bacterial load), and (3) Applied thermodynamics (modeling of physico-chemical characteristics in food). Physics-based models can be integrated with microbial growth or inactivation models. This type of model may be partially kept in the category of tertiary models under food microbiology literature (Perez-Rodriguez *et al*., 2013). Amezquita *et al.* (2005) have modeled integrated heat transfer and dynamic growth of clostridium *Perfringens* in boneless ham and the results were validated for three cooling scenarios. There was one input to this growth model, which was predicted temperature history of the microbes; as a consequence, it was known that the observed and predicted readings were same, thus validating the model sufficiently where the largest value of deviation came out to be 0.37 $\log_{10}$ CFU/g in 6 out of 7 test scenarios. Pradhan *et al*., (2007) have integrated kinetic and heat-mass transfer models to measure different properties like temperature, moisture content, survival of microbes, etc. under specific conditions in air stream impingement oven. There was difference between observed and predicted values. The absolute reason for this was that the water bath cooking for all products except chicken breasts used kinetic parameters along with the temperature histories which had been used for survival of bacteria. Marks, (2008) has given some limitations for integration models to be used with food processing models as: 1) The range of parameters are limited against collected data; 2) standard methodologies are absent; and 3) inadequate information regarding variability and uncertainty.

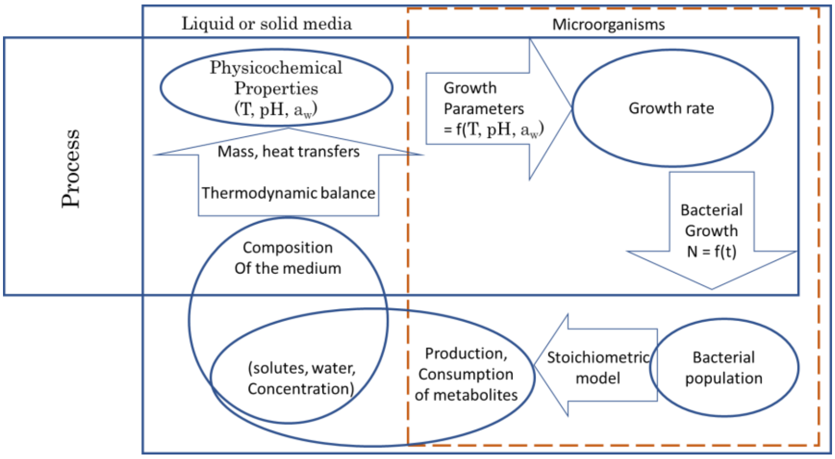

Figure 7. Variables and models involved in an integrated modeling approach, (Isabelle *et al.*, 2006).

CONCLUSION

This paper has appraised a fundamental understanding of mathematical models and how modeling has vast food and process design scope. The designing, optimizing, and innovation of the food processes underlie the very high potentials of modeling. Several parameters on quality and safety have enhanced effects because the number of overall experiments is reduced, which is conducted, being possible due to modeling. Hence, giving the consumer the exact food they want to eat with the help of reducing the microbial growth completely not affecting the quality and safety standards is the main aim of food processing, and how modeling could help to achieve such final quality has been explained in this paper. Implementing a mode in food processing will be a key feature for uncovering the actual micro-level physics involved in any process. So, modeling is undoubtedly one of the main areas for research in the food sector in the coming years.

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DECLARATION OF COMPETING INTEREST

The authors declare that they have no conflict of interest.

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Manibhushan Kumar: Conceptualization, writing -original draft, review, editing and visualization.
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