Some results on soft topological notions

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Abstract — Recently, the generalizations of soft open sets have become a popular subject. These generalizations define based on the concepts of the soft interior and soft closure. Therefore, the properties related to these concepts play an essential role in propositions concerning the generalizations. To this end, we consider the soft interior and soft closure through the concept of the soft element, and thus we clarify the relationships between a soft topological space and its soft subspace topologies. Afterwards, we mention soft α-open sets, soft α-closed sets, and soft α-𝕋₀ space via soft elements. Finally, we discuss soft α-separation axioms for further research.

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1. Introduction

The topological notions of the soft sets [1] first were introduced in two different studies in 2011. In the first of these studies, Çağman et al. [2] have defined soft topology on a soft set. They have proposed basic topological concepts, such as soft open set, soft interior, soft closure, soft limit point, and soft boundary with elements of a soft set and investigated some of their basic properties. On the other, Shabir and Naz [3] have described the concept of soft topology on a universal set. Moreover, they have suggested the basic definitions and properties of the concept regarding a universal set’s elements. Afterwards, Enginoğlu et al. [4] have updated the definition of the soft closed set provided in [2] and several theorems related to it to eliminate inconsistencies between definitions and theorems. So far, many researchers have conducted studies [5-14] on various topological concepts ranging from soft separation axioms to soft compactness.

Recently, the researchers have focused on soft α-open sets [15], soft pre-open sets [16], soft semi-open sets [17], and soft β-open sets [16]. Akdağ and Özkan [15] have defined soft α-continuous and soft α-open functions and investigated their relationships among the other soft continuous structures. After that, Akdağ and Özkan [18] have introduced soft α-separation axioms by using the elements of a universal set. Moreover, they have studied some of their fundamental properties and compared the soft α-separation axioms with the soft separation axioms. Khattak et al. [19] have propounded soft α-separation axioms in terms of soft points. Saleh and Sur [20] have proposed novel separation axioms called soft α-R₀, soft α-symmetric, and soft α-R₁ using soft points.

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The present study's primary goal is to study some relationships concerning the soft interior and soft closure of a soft set in a soft topological space and its soft subspace to avail from them in the next research. The second one is to define soft $\alpha$-$T_0$ space by using the soft element's concept [5] and the soft topological notions provided in [2,4] and analyse some of its fundamental properties.

In Section 2 of the present paper, we present the soft topological notions, such as soft element, soft open set, soft interior, and soft closure, and some of their basic properties to be utilised in the following sections. In Section 3, we study the relationships related to soft interior and soft closure concepts between soft topological space and its soft subspace. In Section 4, as different from the literature, we propose some properties related to soft $\alpha$-open sets and soft $\alpha$-closed sets via the concept of the soft element. Furthermore, we describe soft $\alpha$-$T_0$ space via the soft element concept and investigate several of its basic properties. In Section 5, we discuss soft $\alpha$-separation axioms for further research.

2. Preliminaries

In this section, firstly, we present the concepts of soft sets [1,21] and soft element [5] and some of their basic properties [4,8,21] to be employed in the following sections. Throughout this study, let $U$ be a universal set, $E$ be a parameter set, and $P(U)$ be the power set of $U$.

**Definition 2.1.** [1] Let $f$ be a function from $E$ to $P(U)$. Then, the set $F := \{(x, f(x)) : x \in E\}$ is called a soft set parameterized via $E$ over $U$ (or briefly over $U$).

**Definition 2.2.** [21] Let $A \subseteq E$ and $f_A$ be a function from $E$ to $P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Then, the set $F_A := \{(x, f_A(x)) : x \in E\}$ is called a soft set parameterized via $E$ over $U$ (or briefly over $U$).

In the present paper, the set of all the soft sets over $U$ is denoted by $S(U)$. Besides, as long as it causes no confusion, we do not display the elements $(x, \emptyset)$ in a soft set.

**Definition 2.3.** [21] Let $F_A \in S(U)$. For all $x \in A$, if $f_A(x) = \emptyset$, then $F_A$ is called empty soft set and is denoted by $F_\emptyset$, and if $f_A(x) = U$, then $F_A$ is called $A$-universal soft set and is denoted by $F_A$. Moreover, $E$-universal soft set is called universal soft set and is denoted by $F_E$.

**Definition 2.4.** [21] Let $F_A, F_B \in S(U)$. For all $x \in E$, if $f_A(x) \subseteq f_B(x)$ then, $F_A$ is called a soft subset of $F_B$ and is denoted by $F_A \subseteq F_B$, and if $f_A(x) = f_B(x)$, then $F_A$ and $F_B$ are called equal soft sets and is denoted by $F_A = F_B$.

**Proposition 2.5.** [21] Let $F_A, F_B, F_C \in S(U)$. Then, $F_A \subseteq F_E, F_\emptyset \subseteq F_A, F_A \subseteq F_A, (F_A \subseteq F_B \land F_B \subseteq F_C) \Rightarrow F_A \subseteq F_C, (F_A = F_B \land F_B = F_C) \Rightarrow F_A = F_C$, and $(F_A \subseteq F_B \land F_B \subseteq F_A) \Rightarrow F_A = F_B$.

**Definition 2.6.** [21] Let $F_A, F_B, F_C \in S(U)$. For all $x \in E$, if $f_C(x) = f_A(x) \cup f_B(x)$, then $F_C$ is called soft union of $F_A$ and $F_B$ and is denoted by $F_A \cup F_B$, and if $f_C(x) = f_A(x) \cap f_B(x)$, then $F_C$ is called soft intersection of $F_A$ and $F_B$ and is denoted by $F_A \cap F_B$.

**Definition 2.7.** [21] Let $F_A, F_\emptyset, F_C \in S(U)$. For all $x \in E$, if $f_C(x) = f_A(x) \setminus f_B(x)$, then $F_C$ is called soft difference between $F_A$ and $F_B$ and is denoted by $F_A \setminus F_B$.

**Definition 2.8.** [21] Let $F_A, F_B \in S(U)$. For all $x \in E$, if $f_B(x) = f_A^c(x)$, then $F_B$ is called soft complement of $F_A$ and is denoted by $F_A^c$. Here, $f_A^c(x) := f_B(x) \setminus f_A(x)$.

**Proposition 2.9.** [21] Let $F_A, F_B, F_C \in S(U)$. Then,

1. $F_A \cup F_A = F_A$ and $F_A \cap F_A = F_A$
2. $F_A \cup F_\emptyset = F_A$ and $F_A \cap F_\emptyset = F_A$
3. $F_A \cup F_E = F_E$ and $F_A \cap F_E = F_E$
4. $F_A \cup F_B = F_B \cup F_A$ and $F_A \cap F_B = F_B \cap F_A$
v. \((F_A \cup F_B) \setminus F_C = F_A \setminus (F_B \cup F_C)\) and \((F_A \cap F_B) \cap F_C = F_A \cap (F_B \cap F_C)\)

vi. \(F_A \cup (F_B \cap F_C) = (F_A \cup F_B) \cap (F_A \cup F_C)\) and \(F_A \cap (F_B \cup F_C) = (F_A \cap F_B) \cup (F_A \cap F_C)\)

vii. \(F_A \cup F_A^c = F_B\) and \((F_A \cap F_B)^c = \emptyset\)

viii. \((F_A \cup F_B)^c = F_A^c \cap F_B^c\) and \((F_A \cap F_B)^c = F_A^c \cup F_B^c\)

ix. \((F_A)^c = F_A\)

x. \(F_A \setminus F_B = F_A \cap F_B^c\)

**Proposition 2.10.** [4,8] Let \(F_A, F_B, F_C, F_D \in S(U)\). Then,

i. \((F_A \subseteq F_B \land F_C \subseteq F_D) \Rightarrow F_A \cap F_C \subseteq F_B \cap F_D\)

ii. \((F_A \subseteq F_B \land F_C \subseteq F_D) \Rightarrow F_A \cup F_C \subseteq F_B \cup F_D\)

iii. \((F_A \subseteq F_B \land F_B \cap F_C = F_D) \Rightarrow F_B \supseteq (F_A \setminus F_C)\)

**Example 2.11.** [2] Let \(F_A = \{\{x_1, \{u_1, u_2\}\}, \{x_2, \{u_2, u_3\}\}\}\) such that \(U = \{u_1, u_2, u_3\}, E = \{x_1, x_2, x_3\}\), and \(A = \{x_1, x_2\}\). Then, all the soft subsets of \(F_A\) are as follows:

- \(F_{A_1} = \{\{x_1, \{u_1\}\}\}\)
- \(F_{A_2} = \{\{x_1, \{u_2\}\}\}\)
- \(F_{A_3} = \{\{x_1, \{u_1, u_2\}\}\}\)
- \(F_{A_4} = \{\{x_2, \{u_2\}\}\}\)
- \(F_{A_5} = \{\{x_2, \{u_1, u_2\}\}\}\)
- \(F_{A_6} = \{\{x_2, \{u_2, u_3\}\}\}\)
- \(F_{A_7} = \{\{x_1, \{u_1\}, \{x_2, \{u_2\}\}\}\}\)
- \(F_{A_8} = \{\{x_1, \{u_1\}, \{x_2, \{u_2\}\}\}\}\)
- \(F_{A_9} = \{\{x_1, \{u_2\}, \{x_2, \{u_2, u_3\}\}\}\}\)
- \(F_{A_10} = \{\{x_1, \{u_2\}, \{x_2, \{u_2\}\}\}\}\)
- \(F_{A_{11}} = \{\{x_1, \{u_2\}, \{x_2, \{u_2, u_3\}\}\}\}\)
- \(F_{A_{12}} = \{\{x_2, \{u_2, u_3\}\}\}\)
- \(F_{A_{13}} = \{\{x_1, \{u_1, u_2\}\}, \{x_2, \{u_2\}\}\}\)
- \(F_{A_{14}} = \{\{x_1, \{u_1, u_2\}\}, \{x_2, \{u_3\}\}\}\)
- \(F_{A_{15}} = F_A\)
- \(F_{A_{16}} = F_B\)

**Definition 2.12.** [22] Let \(F_A \in S(U)\) and \(F_B \not\subseteq F_A\). If, for \(x_0 \in B, f_B(x_0) \neq \emptyset\), and for all \(y \in B \setminus \{x_0\}\), \(f_B(y) = \emptyset\), then \(F_B\) is called a soft point of \(F_A\) and is denoted by \(F_B \not\subseteq F_A\).

**Definition 2.13.** [5] Let \(F_A \in S(U)\) and \(F_B \not\subseteq F_A\). If, for \(x_0 \in B, f_B(x_0)\) is a single point set and for all \(y \in B \setminus \{x_0\}\), \(f_B(y) = \emptyset\), then \(F_B\) is called a soft element of \(F_A\) and is denoted by \(F_B \not\subseteq F_A\).

This study exploits Definition 2.13. Throughout this paper, the set of all the soft elements of \(F_A\) is denoted by \(S(F_A)\) or briefly \(S\). It is clear that \(F_B \not\subseteq F_A\) and \(F_B \not\subseteq S(F_A)\) are the same.

**Example 2.14.** For Example 2.11, \(\{\{x_1, \{u_1\}\}\}, \{\{x_1, \{u_2\}\}\}, \{\{x_2, \{u_2\}\}\}, \{\{x_2, \{u_3\}\}\}\) \(\subseteq F_A\) and \(\{\{x_1, \{u_1\}\}\}, \{\{x_2, \{u_2\}\}\}\) \(\subseteq F_A\).

**Proposition 2.15.** [8] Let \(F_A, F_B \in S(U)\). Then, \(F_A \not\subseteq F_B\) iff \(\varepsilon \not\subseteq F_B\), for all \(\varepsilon \not\subseteq F_A\). Besides, \(S(F_A \cup F_B) = \{\varepsilon : \varepsilon \not\subseteq F_A \lor \varepsilon \not\subseteq F_B\}\). Namely, \(\varepsilon \not\subseteq F_A \cup F_B \iff \varepsilon \not\subseteq F_A \lor \varepsilon \not\subseteq F_B\). Similarly, \(\varepsilon \not\subseteq F_A \cap F_B \iff \varepsilon \not\subseteq F_A \land \varepsilon \not\subseteq F_B\). and \(\varepsilon \not\subseteq F_A^c \iff \varepsilon \not\subseteq F_A \land \varepsilon \not\subseteq F_B\).

Secondly, we provide some of the soft topological notions [2,4,5,7,15,23], such as soft open sets, soft interior, soft closure, and soft \(\alpha\)-open sets and some of their basic properties to be used in the next sections.

**Definition 2.16.** [2] Let \(F_A \in S(U)\) and \(\tilde{\tau}\) be a collection of soft subsets of \(F_A\). If

i. \(F_\emptyset, F_A \in \tilde{\tau}\)

ii. Countable soft unions of the soft subsets of \(\tilde{\tau}\) belong to \(\tilde{\tau}\).

iii. Finite soft intersections of the soft subsets of \(\tilde{\tau}\) belong to \(\tilde{\tau}\).

then \(\tilde{\tau}\) is called a soft topology on \(F_A\). Moreover, the ordered pair \((F_A, \tilde{\tau})\) is referred to as a soft topological space.
Here, this study utilizes the arbitrary soft unions instead of the countable soft unions provided in Definition 2.16.

**Definition 2.17.** [2,4] Let \((F_A, \bar{\tau})\) be a soft topological space. Then, every element of \(\bar{\tau}\) is called a \(\bar{\tau}\)-soft open set (or briefly soft open). Furthermore, if \(F_B \in \bar{\tau}\), then \(F_A \setminus F_B\) is referred to as a \(\bar{\tau}\)-soft closed set (or briefly soft closed).

From now on, set of all the soft closed sets of \((F_A, \bar{\tau})\) is denoted by \(\bar{\tau}^k\).

**Proposition 2.18.** [2,4] In \((F_A, \bar{\tau})\), \(F_B\) and \(F_A\) are both soft open and soft closed.

**Example 2.19.** For Example 2.11, \(\bar{\tau} = \{F_{\Phi}, F_{A_1}, F_{A_7}, F_{A_{14}}\}\) is a soft topology on \(F_A\) and so \((F_A, \bar{\tau})\) is a soft topological space.

**Definition 2.20.** [2] Let \((F_A, \bar{\tau})\) be a soft topological space and \(F_B \subseteq F_A\). Then, the collection \(\{F_{A_i} \cap F_B : F_{A_i} \in \bar{\tau}, \ i \in I\}\) is called a soft subspace topology on \(F_B\) and is denoted by \(\bar{\tau}_{F_B}\). Moreover, \((F_B, \bar{\tau}_{F_B})\) is referred to as a soft topological subspace of \((F_A, \bar{\tau})\).

**Proposition 2.21.** [2] Let \((F_A, \bar{\tau})\) be a soft topological space and \(F_B \subseteq F_A\). Then, a soft subspace topology on \(F_B\) is soft topology on \(F_B\).

**Example 2.22.** For Example 2.19, \(\bar{\tau}_{F_{A_7}} = \{F_{\Phi}, F_{A_7}, F_{A_1}\}\) and \(\bar{\tau}_{F_{A_{12}}} = \{F_{\Phi}, F_{A_{12}}, F_{A_4}, F_{A_{11}}\}\) are soft subspace topologies on \(F_{A_7}\) and \(F_{A_{12}}\), respectively. Therefore, \((F_{A_7}, \bar{\tau}_{F_{A_7}})\) and \((F_{A_{12}}, \bar{\tau}_{F_{A_{12}}})\) are soft topological subspaces of \((F_A, \bar{\tau})\).

**Theorem 2.23.** [4] Let \((F_B, \bar{\tau}_{F_B})\) be a soft topological subspace of \((F_A, \bar{\tau})\). If \(F_C \in \bar{\tau}_{F_B}\), then there exists at least one \(F_D \in \bar{\tau}\) such that \(F_C \subseteq F_D\).

**Theorem 2.24.** [4] Let \((F_A, \bar{\tau})\) be a soft topological space. Then, \(\bar{\tau}^k\) provides the following conditions:

i. \(F_B\) and \(F_A\) are soft closed.

ii. Arbitrary soft intersections of the soft closed sets are soft closed.

iii. Finite soft unions of the soft closed sets are soft closed.

**Definition 2.25.** [5] Let \((F_A, \bar{\tau})\) be a soft topological space, \(F_B \subseteq F_A\), and \(\varepsilon \in F_B\). If there exists at least one \(F_C \in \bar{\tau}\) such that \(\varepsilon \subseteq F_C\) and \(F_C \subseteq F_B\), then \(\varepsilon\) is called a \(\bar{\tau}\)-soft interior point (or briefly soft interior point) of \(F_B\). Moreover, the soft union of all the soft interior points of \(F_B\) is called \(\bar{\tau}\)-soft interior (or briefly soft interior) of \(F_B\) and is denoted by \(F_B^\ast\).

**Definition 2.26.** [2] Let \((F_A, \bar{\tau})\) be a soft topological space and \(F_B \subseteq F_A\). Then, the soft intersection of all the soft closed sets containing \(F_B\) is called \(\bar{\tau}\)-soft closure (or briefly soft closure) of \(F_B\) and is denoted by \(\overline{F_B}\).

**Proposition 2.27.** [2] Let \((F_A, \bar{\tau})\) be a soft topological space and \(F_B \subseteq F_A\). Then, the soft interior of \(F_B\) is soft union all the soft open subsets of \(F_B\). In other words, \(F_B^\ast\) is the biggest soft open set contained by \(F_B\). Moreover, \(\overline{F_B}\) is the smallest soft closed set containing \(F_B\).

**Proposition 2.28.** [2] Let \((F_A, \bar{\tau})\) be a soft topological space and \(F_B, F_C \subseteq F_A\). Then,

i. \(F_B \in \bar{\tau} \iff F_B = F_B^\ast\)

ii. \(F_B \in \bar{\tau}^k \iff F_B = \overline{F_B}\)

iii. \((F_B^\ast)^\ast = F_B^\ast\) and \((\overline{F_B}) = \overline{F_B}\)

iv. \((F_B \subseteq F_C \Rightarrow F_B^\ast \subseteq F_C^\ast)\) and \((F_B \subseteq F_C \Rightarrow F_B \subseteq \overline{F_C})\)

v. \(F_B^\ast \cap F_C^\ast = (F_B \cap F_C)^\ast\) and \((\overline{F_B \cap F_C}) = \overline{F_B} \cap \overline{F_C}\)
vi. \( F^*_B \setminus F^*_C \subseteq (F_B \setminus F_C) \) and \( 
abla F^*_B \setminus F^*_C = (F_B \setminus F_C) \)

vii. \( F^*_B \subseteq F_B \subseteq F^*_B \)

**Theorem 2.29.** [2, 4] Let \((F_A, \tau)\) be a soft topological space and \(F_B, F_C \subseteq F_A\). Then, \( \varepsilon \in \overline{F_B} \) iff, for all \( F_C \in \tau \) such that \( \varepsilon \in \overline{F_C}, F_B \cap F_C \neq F_B \).

**Proposition 2.30.** [7] Let \((F_B, \tau_{F_B})\) be a soft topological subspace of \((F_A, \tau)\) and \(F_C \subseteq F_B \subseteq F_A\).

i. \( F_C \in \tau_{F_B} \) iff there exists at least one \( F_B \in \tau^{k} \) such that \( F_C = F_B \cap F_B \).

ii. If \( F_C \in \tau \), then \( F_C \in \tau_{F_B} \).

iii. If \( F_C \in \tau^{k} \), then \( F_C \in \tau_{F_B} \).

**Proposition 2.31.** [23] Let \((F_A, \tau)\) be a soft topological space and \(F_B, F_C \subseteq F_A\). If \( F_B \) is soft open, then \( F_B \cap F_C \subseteq F_B \cap F_C \). Moreover, the arbitrary soft intersections of soft open sets and all the soft \( \alpha \)-open sets are soft \( \alpha \)-open sets.

Hereinafter, the set of all the soft \( \alpha \)-open sets and all the soft \( \alpha \)-closed sets in \((F_A, \tau)\) are denoted by \( S\alpha O(\tau) \) and \( S\alpha C(\tau) \), respectively.

**Proposition 2.33.** [15] In a soft topological space, the arbitrary soft unions of soft \( \alpha \)-open sets are soft \( \alpha \)-open set. Moreover, the arbitrary soft intersections of soft \( \alpha \)-closed sets are soft \( \alpha \)-closed set.

**Definition 2.34.** [15] Let \((F_A, \tau)\) be a soft topological space and \(F_B \subseteq F_A\). Then, \( \tau \)-soft \( \alpha \)-interior (or briefly soft \( \alpha \)-interior) of \( F_B \) is defined by \( \bigcup \{ F_C \in S\alpha O(\tau) : F_C \subseteq F_B \} \) and is denoted by \( (F_B)_{\alpha} \), and \( \tau \)-soft \( \alpha \)-closure (or briefly soft \( \alpha \)-closure) of \( F_B \) is defined by \( \bigcap \{ F_C \in S\alpha C(\tau) : F_B \subseteq F_C \} \) and is denoted by \( (F_B)_{\alpha} \).

**Proposition 2.35.** [15] Let \((F_A, \tau)\) be a soft topological space and \(F_B, F_C \subseteq F_A\). Then,

i. \( F_B \in S\alpha C(\tau) \Leftrightarrow (F_B)_{\alpha} = F_B \)

ii. \( F_B \subseteq F_C \Rightarrow (F_B)_{\alpha} \subseteq (F_C)_{\alpha} \)

**Theorem 2.36.** [15] Let \((F_A, \tau)\) be a soft topological space and \(F_B \subseteq F_A\). Then, \( F_B \in S\alpha O(\tau) \) iff \( (F_B)^{-} \subseteq F_B \).

**Theorem 2.37.** [23] Let \((F_A, \tau)\) be a soft topological space and \(F_B, F_C \subseteq F_A\). Then, \( F_B \in S\alpha O(\tau) \) iff there exists at least one \( F_C \in \tau \) such that \( F_C \subseteq F_B \subseteq (F_C)^{\overline{C}} \).


This section studies several properties containing relationships between the soft interior and soft closure of a soft set according to a soft topological space and its soft subspace.

**Theorem 3.1.** Let \((F_A, \tau)\) be a soft topological space and \(F_B \subseteq F_A\).

i. If \( F_B \) is soft open, then \( F_B \subseteq F_B^\overline{C} \).

ii. If \( F_B \) is soft closed, then \( (F_B)^{\overline{C}} \subseteq F_B \).

**Proof.** Let \((F_A, \tau)\) be a soft topological space and \(F_B \subseteq F_A\).

i. If \( F_B \) is soft open, then \( F_B = F_B^\overline{C} \). Moreover, from Proposition 2.28 (vii), \( F_B \subseteq F_B^\overline{C} \). Thus, \( F_B \subseteq F_B^{\overline{C}} \).

ii. If \( F_B \) is soft closed, then \( F_B = F_B^\overline{C} \). Moreover, from Proposition 2.28 (vii), \( F_B^\overline{C} \subseteq F_B \). Thus, \( (F_B)^{\overline{C}} \subseteq F_B \).
The converse of the propositions provided in Theorem 3.1 is not always correct. This situation is proved with the following example.

**Example 3.2.** For Example 2.19, \( \overline{F_{A_9}} = \overline{F_A} = F_A \). Thus, \( F_{A_9} \subseteq \overline{F_{A_9}} \), but \( F_{A_9} \) is not soft open. Similarly, \( (\overline{F_{A_5}})_\circ = F_{A_{11}} = F_\Phi \). Thus, \( (\overline{F_{A_5}})_\circ \subseteq \overline{F_{A_5}} \), but \( F_{A_5} \) is not soft closed.

**Theorem 3.3.** Let \( (F_A, \tau) \) be a soft topological space and \( F_B, F_C \subseteq F_A \). If \( F_B \) is soft open, then \( \overline{F_B} \cap \overline{F_C} = \overline{F_B} \cap F_C \).

**Proof.** Let \( (F_A, \tau) \) be a soft topological space, \( F_B, F_C \subseteq F_A \), and \( F_B \) be a soft open. Then, from Proposition 2.31, \( \overline{F_B} \cap \overline{F_C} \subseteq \overline{F_B} \cap F_C \) and from Proposition 2.28 (iv), \( \overline{F_B} \cap F_C \subseteq \overline{F_B} \cap F_C \). Therefore, \( \overline{F_B} \cap \overline{F_C} \subseteq \overline{F_B} \cap F_C \). Moreover, since \( F_B \) is soft open, then \( \overline{F_B} \subseteq \overline{F_B} \). From Proposition 2.28 (vii), \( F_C \subseteq F_C \). Therefore, \( \overline{F_B} \cap F_C \subseteq \overline{F_B} \cap F_C \) and so \( \overline{F_B} \cap \overline{F_C} \subseteq \overline{F_B} \cap F_C \). Consequently, \( \overline{F_B} \cap \overline{F_C} = \overline{F_B} \cap \overline{F_C} \).

**Theorem 3.4.** Let \( (F_B, \tau_{F_B}) \) be a soft topological subspace of \( (F_A, \tau) \) and \( F_C \subseteq F_B \subseteq F_A \).

i. If \( F_B \in \tau \) and \( F_C \in \tau_{F_B} \), then \( F_C \in \tau \).

ii. If \( F_B \in \tau_k \) and \( F_C \in \tau_{F_B}^k \), then \( F_C \in \tau_k \).

**Proof.** Let \( (F_B, \tau_{F_B}) \) be a soft topological subspace of \( (F_A, \tau) \) and \( F_C \subseteq F_B \subseteq F_A \).

i. Let \( F_B \in \tau_k \) and \( F_C \in \tau_{F_B} \). Then, there exists at least one \( F_D \in \tau_1 \) such that \( F_C = F_D \cap F_B \). Moreover, since \( F_B, F_D \in \tau_k \), then \( F_D \in \tau_k \). Therefore, \( F_C \in \tau_k \).

ii. Let \( F_B \in \tau_k \) and \( F_C \in \tau_{F_B}^k \). Then, from Proposition 2.30 (i), there exists at least one \( F_D \in \tau_k \) such that \( F_C = F_D \cap F_B \). Moreover, since \( F_B, F_D \in \tau_k \), then \( F_D \in \tau_k \). Therefore, \( F_C \in \tau_k \).

Henceforth, \( F_B^{\tau_1} \) and \( \overline{F_B}^{\tau_1} \) indicate \( \tau_1 \)-soft interior and \( \tau_1 \)-soft closure of \( F_B \), respectively.

**Theorem 3.5.** Let \( (F_A, \tau_1) \) and \( (F_A, \tau_2) \) be two soft topological spaces and \( F_B \subseteq F_A \). Then,

i. \( \tau_1 \subseteq \tau_2 \Rightarrow F_B^{\tau_1} \subseteq F_B^{\tau_2} \)

ii. \( \tau_1 \subseteq \tau_2 \Rightarrow \overline{F_B}^{\tau_1} \subseteq \overline{F_B}^{\tau_2} \)

**Proof.** Let \( (F_A, \tau_1) \) and \( (F_A, \tau_2) \) be two soft topological spaces and \( F_B \subseteq F_A \).

i. \( (\Rightarrow) \): Let \( \tau_1 \subseteq \tau_2 \) and \( \epsilon \in F_B^{\tau_1} \). Then, there exists at least one \( F_C \in \tau_1 \) such that \( \epsilon \in F_C \) and \( F_C \subseteq F_B \).

ii. \( (\Leftarrow) \): Let \( F_B^{\tau_1} \subseteq F_B^{\tau_2} \), for all \( F_B \subseteq F_A \). Then, for all \( F_C \in \tau_1 \), the hypothesis is valid. Since \( F_C = F_B^{\tau_1} \), then \( F_C \subseteq F_B^{\tau_2} \). Besides, \( F_B^{\tau_2} \subseteq F_C \). That is, \( F_C^{\tau_2} = F_C \). Therefore, \( F_C \in \tau_1 \Rightarrow F_C \in \tau_2 \). Hence, \( \tau_1 \subseteq \tau_2 \).

**Theorem 3.6.** Let \( (F_B, \tau_{F_B}) \) be a soft topological subspace of \( (F_A, \tau) \) and \( F_C \subseteq F_B \).

i. \( F_C^{\circ} \subseteq F_C^{\tau_{F_B}} \)
Proof. Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_C \subseteq F_B\).

i. Let \(e \in F_C^\circ\). Then, there exists at least one \(F_D \in \tilde{\tau}\) such that \(e \in F_D\) and \(F_D \subseteq F_C\). Therefore, from Proposition 2.30 (ii), there exists at least one \(F_D \in \tilde{\tau}_{FB}\) such that \(e \in F_D\) and \(F_D \subseteq F_C\). Therefore, \(e \in F_C^{\tau_{FB}}\) and \(F_C^{\tau_{FB}} \subseteq F_C\).

ii. Let \(e \in F_C^{\tau_{FB}}\). Then, from Theorem 2.29, for all \(F_D \in \tilde{\tau}_{FB}\) such that \(e \in F_D\), \(F_C \cap F_D \neq F_\emptyset\). Besides, for all \(F_D \in \tilde{\tau}_{FB}\), there exists at least one \(F_K \in \tilde{\tau}\) such that \(F_D = F_K \cap F_B\). Since \(F_K \cap F_B \subseteq F_K\) and \(F_K \in \tilde{\tau}\), then for all \(F_K \in \tilde{\tau}\) such that \(e \in F_K\), \(F_C \cap F_K \neq F_\emptyset\). Therefore, \(e \in F_C\). Hence, \(F_C^{\tau_{FB}} \subseteq F_C\).

**Theorem 3.7.** Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_C \subseteq F_B\). Then,

i. If \(F_B \in \tilde{\tau}\), then \(F_C^{\tau_{FB}} \subseteq F_C^\circ\).

ii. If \(F_B \in \tilde{\tau}\), then \(F_C^{\tau_{FB}} \subseteq F_C\).

**Proof.**

i. Let \(F_B \in \tilde{\tau}\) and \(e \in F_C^{\tau_{FB}}\). Then, there exists at least one \(F_D \in \tilde{\tau}_{FB}\) such that \(e \in F_D\) and \(F_D \subseteq F_C\). Therefore, from Theorem 3.4 (i), there exists at least one \(F_D \in \tilde{\tau}\) such that \(e \in F_D\) and \(F_D \subseteq F_C\). Thus, \(e \in F_C^\circ\). Hence, \(F_C^{\tau_{FB}} \subseteq F_C^\circ\).

ii. Let \(F_B \in \tilde{\tau}\) and \(e \in F_C^{\tau_{FB}}\). Then, from Theorem 2.29, for all \(F_D \in \tilde{\tau}\) such that \(e \in F_D\), \(F_C \cap F_D \neq F_\emptyset\). Moreover, for all \(F_D \cap F_B \in \tilde{\tau}\) such that \(e \in F_D\cap F_B\), \(F_C \cap (F_D \cap F_B) \neq F_\emptyset\). Therefore, for all \(F_D \cap F_B \in \tilde{\tau}_{FB}\) such that \(e \in F_D\cap F_B\), \(F_C \cap (F_D \cap F_B) \neq F_\emptyset\). Thus, \(e \in F_C^{\tau_{FB}}\). Hence, \(F_C^{\tau_{FB}} \subseteq F_C^{\tau_{FB}}\).

**Corollary 3.8.** Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\), \(F_B \in \tilde{\tau}\), and \(F_C \subseteq F_B\). Then,

i. \(F_C^\circ \cap F_B \subseteq F_C^{\tau_{FB}}\)

ii. \(\overline{F_C^\circ} \subseteq F_C^{\tau_{FB}}\)

**Corollary 3.9.** Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_C \subseteq F_B \subseteq F_A\). Then,

i. \(F_C^\circ \cap F_B \subseteq F_C^{\tau_{FB}}\)

ii. \(\overline{F_C^{\tau_{FB}}} = \overline{F_C} \cap F_B\)

**Proof.**

Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_C \subseteq F_B \subseteq F_A\). Then,

i. From Theorem 3.6 (i), \(F_C^\circ \cap F_B \subseteq F_C^{\tau_{FB}}\).

ii. From Theorem 3.6 (ii), \(\overline{F_C^{\tau_{FB}}} \subseteq \overline{F_C} \cap F_B\). On the other hand, let \(e \in \overline{F_C} \cap F_B\). Then, \(e \in \overline{F_C}\) and \(e \in F_B\). That is, for all \(F_K \in \tilde{\tau}\) such that \(e \in F_K\), \(F_C \cap F_K \neq F_\emptyset\) and \(e \in F_B\). Therefore, for all \(F_K \cap F_B \in \tilde{\tau}_{FB}\) such that \(e \in F_K \cap F_B\), \(F_C \cap (F_K \cap F_B) \neq F_\emptyset\) and so \(e \in F_C^{\tau_{FB}}\). Hence, \(\overline{F_C^{\tau_{FB}}} = \overline{F_C} \cap F_B\).

**Theorem 3.10.** Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_B, F_C \subseteq F_A\). Then, \(\overline{F_B \cap F_C^{\tau_{FB}}} \subseteq F_B \cap \overline{F_C}\).

**Proof.**

Let \((F_B, \tilde{\tau}_{FB})\) be a soft topological subspace of \((F_A, \tilde{\tau})\) and \(F_B, F_C \subseteq F_A\). We have \(F_B \cap F_C \subseteq F_B \subseteq F_A\). Because of Corollary 3.9 (ii) and \(F_B \subseteq \overline{F_B}\),

\[
\overline{F_B \cap F_C^{\tau_{FB}}} = \overline{F_B \cap F_C} \cap F_B \subseteq \overline{F_B} \cap \overline{F_C} \cap F_B = (\overline{F_B} \cap F_B) \cap \overline{F_C} = F_B \cap \overline{F_C}
\]

Hence, \(\overline{F_B \cap F_C^{\tau_{FB}}} \subseteq F_B \cap \overline{F_C}\).
4. Soft $\alpha$-open Sets and Soft $\alpha$-$T_0$ Spaces

In this section, we introduce some properties of soft $\alpha$-open sets and soft $\alpha$-closed sets. Moreover, we define soft $\alpha$-$T_0$ space and study some of its basic properties.

**Theorem 4.1.** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \subseteq F_A$. If $F_B \in \tilde{\tau}$ and $F_C \in S\alpha O(\tilde{\tau})$, then $F_B \cap F_C \in S\alpha O(\tilde{\tau})$.

**Proof.** Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B, F_C \subseteq F_A$, $F_B \in \tilde{\tau}$, and $F_C \in S\alpha O(\tilde{\tau})$. Since

$$F_B \in \tilde{\tau} \land F_C \in S\alpha O(\tilde{\tau}) \Rightarrow F_B = F_B^\circ \land F_C \subseteq (F_C^\circ)^\circ,$$

$$\Rightarrow F_B \cap F_C \subseteq F_B^\circ \cap (F_C^\circ)^\circ,$$

$$\Rightarrow F_B \cap F_C \subseteq (F_B \cap F_C)^\circ,$$

then $F_B \cap F_C \in S\alpha O(\tilde{\tau})$.

**Theorem 4.2.** Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \subseteq F_B \subseteq F_A$. If $F_C \in S\alpha O(\tilde{\tau})$, then $F_C \in S\alpha O(\tilde{\tau}_{F_B})$.

**Proof.** Let $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace of $(F_A, \tilde{\tau})$ and $F_C \subseteq F_B \subseteq F_A$. Then,

$$F_C \in S\alpha O(\tilde{\tau}) \Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ}, \text{ from Theorem 2.37}$$

$$\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ} \cap F_B$$

(Since $F_D \subseteq F_C \subseteq F_B$, then $F_D \cap F_B = F_D$ and $F_C \cap F_B = F_C$.)

$$\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ} \cap F_B,$$ from Theorem 3.6 (i)

$$\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ} \cap F_B$$

(Since $F_D \in \tilde{\tau}_{F_B}$, then $F_B = F_B^{\tilde{\tau}_{F_B}}$)

$$\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ} \cap F_B,$$ from Proposition 2.28 (v)

$$\Rightarrow \exists F_D \in \tilde{\tau}_{F_B} \exists F_D \subseteq F_C \subseteq \overline{(F_D)^\circ} \cap F_B$$

(Since $F_D \subseteq F_C \subseteq F_B \subseteq F_A$, then $\overline{F_D} = F_B \cap F_B$

$$\Rightarrow F_C \in S\alpha O(\tilde{\tau}_{F_B})$$

The converse of the Theorem 4.2 is not always correct. In other words, a soft $\alpha$-open set in a soft subspace of a soft topological space is may not soft $\alpha$-open set in a soft topological space. This situation is shown in the following example.

**Example 4.3.** For $(F_A, \tilde{\tau})$ and $(F_{A_{12}}, \tilde{\tau}_{F_{A_{12}}})$ provided in Example 2.19 and Example 2.22, $S\alpha O(\tilde{\tau}) = \{F_B, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_A\}$ and $S\alpha O(\tilde{\tau}_{F_{A_{12}}}) = \{F_B, F_{A_4}, F_{A_{11}}, F_{A_{12}}\}$. Hence, $F_{A_4} \in S\alpha O(\tilde{\tau}_{F_{A_{12}}})$ but $F_{A_4} \notin S\alpha O(\tilde{\tau})$.

**Theorem 4.4.** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \subseteq F_A$. Then, $F_B \in S\alpha C(\tilde{\tau})$ if and only if there
exists at least one \( F_C \in \hat{\tau}^k \) such that \( \overline{F_C} \subseteq F_B \subseteq F_C \).

**Proof.** Let \((F_A, \hat{\tau})\) be a soft topological space and \( F_B \subseteq F_A \).

\((\Rightarrow):\) Let \( F_B \in \text{Sao}(\hat{\tau}) \). From Theorem 2.36, \( \overline{(F_B)^\circ} \subseteq F_B \). Since \( F_B \subseteq F_A \) and \( F_B \in \hat{\tau}^k \), then \( \overline{(F_B)^\circ} \subseteq F_B \subseteq F_B \).

\((\Leftarrow):\) Let there exists at least one \( F_C \in \hat{\tau}^k \) such that \( \overline{F_C} \subseteq F_B \subseteq F_C \). Since \( F_C \in \hat{\tau}^k \), then \( F_C = F_C \). Thus, \( \overline{(F_C)^\circ} \subseteq F_B \subseteq F_C \). Since \( \overline{(F_C)^\circ} \subseteq F_B \), then \( F_B \in \text{Sao}(\hat{\tau}) \).

**Definition 4.5.** Let \((F_A, \hat{\tau})\) be a soft topological space, \( F_B \subseteq F_A \), and \( \epsilon \in F_B \). If there exists at least one \( F_C \in \text{Sao}(\hat{\tau}) \) such that \( \epsilon \in F_C \) and \( F_C \subseteq F_B \), then \( \epsilon \) is called a \( \hat{\tau} \)-soft \( \alpha \)-interior point (or briefly soft \( \alpha \)-interior point) of \( F_B \).

**Theorem 4.6.** Let \((F_A, \hat{\tau})\) be a soft topological space and \( F_B, F_C \subseteq F_A \). Then, \( F_A \setminus \overline{(F_B)^\circ} = (F_A \setminus F_B)^\circ \).

**Proof.** Let \((F_A, \hat{\tau})\) be a soft topological space and \( F_B, F_C \subseteq F_A \). Then,

\[
F_A \setminus \overline{(F_B)^\circ} = F_A \setminus \left( \bigcap_{F_A \setminus F_B \subseteq F_A \setminus F_B \in \text{Sao}(\hat{\tau})} F_A \right) = \bigcup_{F_A \setminus F_B \subseteq F_A \setminus F_B \in \text{Sao}(\hat{\tau})} (F_A \setminus F_B)^\circ
\]

**Theorem 4.7.** Let \((F_A, \hat{\tau})\) be a soft topological space and \( F_B, F_C \subseteq F_A \). Then, \( \epsilon \in (F_B)^\circ \) iff, for all \( F_C \in \text{Sao}(\hat{\tau}) \) such that \( \epsilon \notin F_C \) and \( F_B \cap F_C = F_B \).

**Proof.** Let \((F_A, \hat{\tau})\) be a soft topological space and \( F_B, F_C \subseteq F_A \).

\((\Rightarrow):\) Let \( \epsilon \notin (F_B)^\circ \). Then, \( \epsilon \notin F_A \setminus (F_B)^\circ \). From Theorem 4.6, \( \epsilon \notin (F_A \setminus F_B)^\circ \). Thus, there exists at least one \( F_C \in \text{Sao}(\hat{\tau}) \) such that \( \epsilon \notin F_C \) and \( F_B \subseteq F_C \). From Proposition 2.10 (iii), there exists at least one \( F_C \in \text{Sao}(\hat{\tau}) \) such that \( \epsilon \notin F_C \) and \( F_B \cap F_C = F_B \).

\((\Leftarrow):\) Let there exists at least one \( F_C \in \text{Sao}(\hat{\tau}) \) such that \( \epsilon \notin F_C \) and \( F_B \cap F_C = F_B \). From Proposition 2.10 (iii), there exists \( F_A \setminus F_C \in \text{Sao}(\hat{\tau}) \) such that \( F_B \subseteq F_A \setminus F_C \). In this case, from Proposition 2.35, \( (F_B)^\circ \subseteq (F_A \setminus F_C)^\circ = F_A \setminus F_C \). Hence, \( \epsilon \notin (F_B)^\circ \).

**Definition 4.8.** Let \((F_A, \hat{\tau})\) be a soft topological space. For all \( \epsilon_1, \epsilon_2 \in F_A \) such that \( \epsilon_1 \neq \epsilon_2 \), if there exists at least one \( F_B, F_C \in \text{Sao}(\hat{\tau}) \) such that \( (\epsilon_1 \notin F_B \land \epsilon_2 \notin F_B) \) or \( (\epsilon_1 \notin F_C \land \epsilon_2 \notin F_C) \), then \((F_A, \hat{\tau})\) is called a soft \( \alpha \)-T\(_0\) space.

**Example 4.9.** Let us consider Example 4.3. Since \( F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\} \) and \( F_{A_1}, F_{A_3}, F_{A_8} \in \text{Sao}(\hat{\tau}) \), then

\[
\{\{x_1, \{u_1\}\}\} \in F_{A_1} \quad \text{and} \quad \{\{x_1, \{u_2\}\}\} \notin F_{A_1} \quad \{\{x_1, \{u_3\}\}\} \notin F_{A_1}
\]

\[
\{\{x_2, \{u_3\}\}\} \notin F_{A_3} \quad \text{and} \quad \{\{x_2, \{u_4\}\}\} \notin F_{A_3} \quad \{\{x_2, \{u_5\}\}\} \notin F_{A_3}
\]

\[
\{\{x_3, \{u_3\}\}\} \notin F_{A_8} \quad \text{and} \quad \{\{x_3, \{u_4\}\}\} \notin F_{A_8} \quad \{\{x_3, \{u_5\}\}\} \notin F_{A_8}
\]

Therefore, \((F_A, \hat{\tau})\) is a soft \( \alpha \)-T\(_0\) space. On the other hand, for \( \{\{x_1, \{u_2\}\}\}, \{\{x_2, \{u_3\}\}\} \in F_{A_{12}}, \) there do not exist \( F_B, F_C \in \text{Sao}(\hat{\tau}) \) such that \( \{\{x_1, \{u_2\}\}\} \in F_B \land \{\{x_2, \{u_3\}\}\} \in F_B \) or \( \{\{x_1, \{u_2\}\}\} \in F_C \land \{\{x_2, \{u_3\}\}\} \in F_C \). Hence, \((F_{A_{12}}, \hat{\tau}_{F_{A_{12}}})\) is not a soft \( \alpha \)-T\(_0\) space.
Corollary 4.10. Every soft subspace of a $\alpha$-$T_0$ space is not always a soft $\alpha$-$T_0$ space. Therefore, being soft $\alpha$-$T_0$ space is not a hereditary property.

Theorem 4.11. Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, $(F_A, \tilde{\tau})$ is a soft $\alpha$-$T_0$ space if and only if for all $\varepsilon_1, \varepsilon_2 \in F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $(\varepsilon_1)_a \neq (\varepsilon_2)_a$.

Proof.

$(\Rightarrow)$: Let $(F_A, \tilde{\tau})$ be a soft $\alpha$-$T_0$ space. Then, for all $\varepsilon_1, \varepsilon_2 \in F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, there exists at least one $F_B, F_C \in SaO(\tilde{\tau})$ such that $(\varepsilon_1 \in F_B \land \varepsilon_2 \in F_B)$ or $(\varepsilon_1 \in F_C \land \varepsilon_2 \in F_C)$. Let $\varepsilon_1 \in F_B$ and $\varepsilon_2 \in F_B$. Since $\varepsilon_2 \notin F_B$, then $\varepsilon_2 \cap F_B = F_B \phi$. Thus, there exists at least one $F_B \in SaO(\tilde{\tau})$ such that $\varepsilon_1 \in F_B$ and $\varepsilon_2 \cap F_B = F_B \phi$. Because of Theorem 4.7, $\varepsilon_1 \notin (\varepsilon_2)_a$. Moreover, since $\varepsilon_1 \notin (\varepsilon_2)_a$, then for all $\varepsilon_1, \varepsilon_2 \in F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $(\varepsilon_1)_a \neq (\varepsilon_2)_a$.

$(\Leftarrow)$: Let for all $\varepsilon_1, \varepsilon_2 \in F_A$ such that $\varepsilon_1 \neq \varepsilon_2$, $(\varepsilon_1)_a \neq (\varepsilon_2)_a$. Since $\varepsilon_1 \notin (\varepsilon_2)_a$ and $(\varepsilon_1)_a \neq (\varepsilon_2)_a$, then $\varepsilon_1 \notin (\varepsilon_2)_a$. Because of Theorem 4.7, there exists at least one $F_B \in SaO(\tilde{\tau})$ such that $\varepsilon_1 \notin F_B$ and $\varepsilon_2 \nexists F_B = F_B \phi$. Thus, there exists at least one $F_B \in SaO(\tilde{\tau})$ such that $\varepsilon_1 \notin F_B$ and $\varepsilon_2 \notin F_B$. Therefore, $(F_A, \tilde{\tau})$ is a soft $\alpha$-$T_0$ space.

Example 4.12. For $(F_A, \tilde{\tau}_F, F_{A_7})$ provided in Example 2.22, $SaO(\tilde{\tau}_F, F_{A_7}) = \{F_\phi, F_{A_1}, F_{A_7}\}$ and $SaC(\tilde{\tau}_F, F_{A_7}) = \{F_\phi, F_{A_4}, F_{A_7}\}$. For $\{(x_1, \{u_1\}), (x_2, \{u_2\})\} \in F_{A_7}$, since $\{(x_1, \{u_1\})_a = F_{A_7}$ and $\{(x_2, \{u_2\})_a = F_{A_4}$, then $\{(x_1, \{u_1\})_a \neq \{(x_2, \{u_2\})_a$. Therefore, $(F_{A_7}, \tilde{\tau}_F, F_{A_7})$ is a soft $\alpha$-$T_0$ subspace of $(F_A, \tilde{\tau})$.

5. Conclusion

This paper studied relationships between the soft interior and soft closure of a soft set in soft topological spaces and their soft subspace through the concept of the soft element. Thus, the validity of many propositions on various generalizations of soft open sets can be proved easier. We then provided a few theorems concerning soft $\alpha$-open sets and soft $\alpha$-closed sets. Moreover, we defined soft $\alpha$-$T_0$ space and investigated its basic properties. We showed that every soft subspace of a soft $\alpha$-$T_0$ space is not always a soft $\alpha$-$T_0$ space.

In the future, researchers can study that every soft subspace of a soft $\alpha$-$T_0$ space is a soft $\alpha$-$T_0$ space under which condition or conditions. Moreover, they can define through the concept of the soft element the other soft $\alpha$-separation axioms, i.e., soft $\alpha$-$T_1$ space, soft $\alpha$-$T_2$ space (soft $\alpha$-Hausdorff space), soft $\alpha$-regular space, and soft $\alpha$-normal space.

Author Contributions

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References


