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Modeling of Telecommunication Revenue as a Percentage of Gross Domestic Product's for Countries with Fractional Calculus

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ABSTRACT

This study explores the modeling of the share of telecommunication revenues in gross domestic product from the year 2000 to 2018 for 5 countries including France, Germany, Italy, Turkey, the UK, and the OECD average. First, a new mathematical model based on Fractional Calculus and Least Square Method is proposed. Later, the telecommunication revenues in GDP dataset is modeled. Further, we compare the new Fractional approach to the classical Polynomial approach in three different settings. The results show that employing Fractional Calculus yields better modeling performance when compared to the classical Polynomial Approach in terms of Mean Absolute Percentage Error (MAPE). The Fractional approach outperforms the Polynomial approach by 0.1329 % MAPE on average. The largest MAPE is found for Turkey while the smallest MAPE is obtained for Italy in all settings.

1. INTRODUCTION

TELECOMMUNICATIONS can be defined as the exchange of any signals such as written messages, images, icons, sounds, or information by utilizing various media such as wire, radio, optical, or other electromagnetic systems [1, 2]. Early days of communication consist of visual and audial signals such as horns, drums, signal flakes, smoke signals. In 20^{th} and 21^{th} centuries, long-distance communication technologies evolved with the help of inventions such as the telegraph, the telephone, the radio, network, antenna systems, optical fiber, and communication satellites. In parallel, the demand for data exchange increased too. Therefore, understanding the changes of this demand and communication need in specific intervals are important and crucial for scientists, companies, and states for modeling and analyzing the pattern of progress.

In the last several decades, telecommunication need is increased drastically with the help of advancing technology. Communication systems used previously cannot support the state of the art technologies. Increasing data sizes and the need for reaching more than one user at a time leads to make progress. This progress has its advantages and disadvantages. One advantage is, communication quality and comfort have increased and the desired data can be accessed in a short time. The disadvantage is, legacy systems needs to be replaced by new technological tools which increases the cost for investors. These innovations or technology replacements affect firms and therefore the income of the countries. The expenses and revenues of the systems that have changed over the years constitute an important share of the countries' economy.

Previously, the "Marginal Revolution" and "Keynesian revolution" offered fundamental economic methods. Regarding these, the concepts of "marginal value", "economic multiplier", "economic accelerator", "elasticity" were studied [3-5]. These revolutions prompted the researchers, companies, and institutes to employ mathematical tools such as derivatives and integrals in modeling each specific case for their problem. Then, the economic models were understood and tested very easily with these equations including the differential, integral, or difference equations [3].

Integer-order derivatives and integrals are well-known and studied for centuries by researchers [3–8]. Fractional Calculus is the theory of non-integer or complex-valued derivatives and integrals [9–20]. The applications of such mathematical tools in practical and engineering problems are relatively new [3-11]. The fractional approach has the flexibility, hereditary, and dynamicity and therefore it can be applied to areas such as mathematical economics, management, and finance [3, 7, 12]. The main goals, notions, effects, and objectives of mathematical economics can be generalized, widen, and improved by including such new approaches [3].

In this study, we propose a new mathematical model based on Fractional Calculus and model the telecommunication revenue as a percentage of GDP for 5 countries including France, Germany, Italy, Turkey, the UK, and the OECD average. We name this new approach as Fractional Model-2. Later, we assess the performance of the newly proposed approach with the help of conventional Polynomial model and compare these two models.

The structure of this study is as follows. Section 2 provides the foundations of the employed fractional model. Then, in Section 3, Dataset and Performance Metrics are presented. Later, Section 4 reports the experimental results and lastly, the conclusion is given in Section 5.

2. MATHEMATICAL MODEL

The main motivation is to model the given discrete dataset and obtained a continuous curve representing the dataset with the minimum error. To achieve this goal, the Taylor expansion is employed at the first stage of the mathematical manipulations [18-21].

An arbitrarily chosen, continuous and analytical function $g(x)$ can be expanded as

An arbitrarily chosen, continuous and analytical function $g(x)$ can be expanded as

$$g(x) = \sum_{n=0}^{\infty} \tilde{a}_n \, x^{n+\alpha} \qquad (1)$$

Then, the first derivative of the function with respect to x becomes $g'(x) = \sum_{n=0}^{\infty} \tilde{a}_n (n+\alpha) \, x^{n+\alpha-1}$. From (1), we would like to mimic the same approach for the function $f(x)$ which stands for the income of the telecommunication sector in years. Note that, in this case, x corresponds to years. To have a better modeling approach utilizing the non-locality and heredity properties of fractional calculus, the fractional derivative of $f(x)$ is expressed as Equation (2).

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} = \sum_{n=0}^{\infty} a_n (n+\alpha) x^{n+\alpha-1} \qquad (2)$$

Here, α is the fractional-order and ranges from [0,1] [21]. The main motive is to find $f(0)$, a_n, and α representing $f(x)$ with minimum error. Before, going into details, it is better to define the fractional derivative. Caputo's definition of the fractional derivative is provided below [14, 18, 22].

$$\mathfrak{D}_x^{\alpha} f(x) = \frac{d^{\alpha} f(x)}{dx^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \int_{-\infty}^{x} \frac{f'(t)}{(x-t)^{\alpha}} dt \qquad (3)$$

where fractional derivative $\mathfrak{D}_x^{\alpha}$ states that the derivative is taken with respect to x in the order of α ($\alpha \in [0,1]$), and f' stands for the first derivative.

Note that, $\Gamma(1-\alpha)$ is called Gamma function and given as Equation (4)

$$\Gamma(1-\alpha) = \int_{0}^{\infty} t^{-\alpha} e^{-t} dt \qquad (4)$$

By generalizing the derivative operator, more flexible and fast converging modeling becomes possible.

To solve the fractional-order differential equation given in Equation (2), the Laplace Transform is taken and the differential equation is converted into an algebraic equation. In Equation (5) and Equation (6), two properties of Laplace Transform ($\mathcal{L}$) are listed [6, 19, 22].

$$x^{\alpha} \xrightarrow{\mathcal{L}} \frac{\Gamma(\alpha+1)}{s^{\alpha+1}} \qquad (5)$$

$$\mathcal{L}\frac{d^{\alpha} f}{dx^{\alpha}} = \xrightarrow{\mathcal{L}} s^{\alpha} F(s) - s^{\alpha-1} f(0) \qquad (6)$$

Note that, $F(s)$ is the Laplace Transform of $f(x)$. The properties are employed in Equation (2) and the following procedure is tracked.

$$\mathcal{L}\frac{d^{\alpha} y}{dx^{\alpha}} = \mathcal{L} \sum_{n=0}^{\infty} a_n (n+\alpha) x^{n+\alpha-1}$$

$$s^{\alpha} F(s) - s^{\alpha-1} f(0) = \sum_{n=0}^{\infty} a_n (n+\alpha) \frac{\Gamma(n+\alpha)}{s^{n+\alpha}} \Gamma(n+\alpha+1)$$

$$F(s) = s^{-1} f(0) + \sum_{n=0}^{\infty} a_n \frac{\Gamma(n+\alpha+1)}{s^{n+2\alpha}} \qquad (7)$$

After obtaining the algebraic equation for $F(s)$ as given in Equation (7), the inverse Laplace Transform ($\mathcal{L}^{-1}$) is employed to obtain $f(x)$ which is provided in Equation (8).

$$f(x) = f(0) + \sum_{n=0}^{\infty} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x^{n+2\alpha-1} \qquad (8)$$

For the numerical calculation, the infinite sum is truncated to N and approximate value of $f(x)$ is given in (9).

$$f(x) \cong f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x^{n+2\alpha-1} \qquad (9)$$

At this point, theoretically, $f(x)$ function is achieved. To obtain the unknowns $f(0)$, a_n, and α, the discrete dataset is employed and then, by error minimization, continuous $f(x)$ function representing that specific dataset would be acquired. The dataset consists of Telecommunication GDP income per year. Here, telecommunication GDP income was defined as P_i and x_i represents the telecommunication income in years as expressed below.

$$P_i = [\; p_0 \, p_1 \dots p_{K-1}]$$
$$x_i = [\; x_0 \, x_1 \dots x_{K-1}]$$

Note that K values exist. At this point, function $f(x_i)$ will be the expected value for x_i^{th} year. According to the least square method ϵ_i, which is defined as the error between p_i and $f(x_i)$ values, is shown as follows.

$$(\epsilon_i)^2 = \left(p_i - f(x_i)\right)^2 \tag{10}$$

The total square of the error is defined as Equation (11) and according to the least-squares method, the sum of error squares ϵ_T^2 is tried to be minimized [18-20].

$$\epsilon_T^2 = \epsilon_0^2 + \epsilon_1^2 + \epsilon_2^2 + \cdots \epsilon_{K-1}^2 = \sum_{i=0}^{K-1} \epsilon_i^2 \tag{11}$$

For the sake of simplicity, the square of error for each point in the dataset can be obtained as follows:

$$(\epsilon_0)^2 = \left[p_0 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} {x_0}^{n+2\alpha-1}\right\}\right]^2$$

$$(\epsilon_1)^2 = \left[p_1 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} {x_1}^{n+2\alpha-1}\right\}\right]^2$$

$$\dots$$

$$\epsilon_i^2 = \sum_{i=0}^{K-1} \left[p_i - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} {x_i}^{n+2\alpha-1}\right\}\right]^2$$

$$(\epsilon_{K-1})^2 = \left[p_{K-1} - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} {x_{K-1}}^{n+2\alpha-1}\right\}\right]^2 \tag{12}$$

For minimizing the total error given in Equation (11), the Least Squares Method is employed as given in (13) [18, 21]:

$$\frac{\partial \epsilon_T^2}{\partial f(0)} = 0, \quad \frac{\partial \epsilon_T^2}{\partial a_0} = 0, \quad \frac{\partial \epsilon_T^2}{\partial a_1} = 0, \quad \frac{\partial \epsilon_T^2}{\partial a_2} = 0, \quad \frac{\partial \epsilon_T^2}{\partial a_N} = 0 \tag{13}$$

Implementing Equation (13) leads to having $N+2$ equations and also Equation (9) has the same number of unknowns. Therefore, this problem can be solved. The Least Squares method leads to having a System of Linear algebraic equations (SLAE). Several specific derivative operations in Equation (13) are given below for the readers.

First example:

$$\frac{\partial \epsilon_T^2}{\partial f(0)} = -2\left[p_0 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_0^{n+2\alpha-1}\right\}\right]$$

$$-2\left[p_1 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_1^{n+2\alpha-1}\right\}\right]$$

$$-2\left[p_2 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_2^{n+2\alpha-1}\right\}\right]$$

$$\dots$$

$$-2\left[p_{K-1} - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_{K-1}^{n+2\alpha-1}\right\}\right] = 0$$

Then, the procedure above can be written in the compact form as Equation (14):

$$\frac{\partial \epsilon_T^2}{\partial f(0)} = \sum_{i=0}^{K} p_i - \Bigg[(K+1)f(0) + a_0 \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha)} \sum_{i=0}^{K} x_i^{2\alpha-1} + a_1 \frac{\Gamma(\alpha+2)}{\Gamma(2\alpha+1)} \sum_{i=0}^{K} x_i^{2\alpha} + \cdots + a_N \frac{\Gamma(N+\alpha+1)}{\Gamma(N+2\alpha)} \sum_{i=0}^{K-1} x_N^{N+2\alpha-1}\Bigg] = 0 \tag{14}$$

Second Example:

$$\frac{\partial \epsilon_T^2}{\partial a_N} = -2\left[p_0 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_0^{n+2\alpha-1}\right\}\right] x_0^{N+2\alpha-1}$$

$$-2\left[p_1 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_1^{n+2\alpha-1}\right\}\right] x_1^{N+2\alpha-1}$$

$$-2\left[p_2 - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_2^{n+2\alpha-1}\right\}\right] x_2^{N+2\alpha-1}$$

$$\dots$$

$$-2\left[p_{K-1} - \left\{f(0) + \sum_{n=0}^{N} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x_{K-1}^{n+2\alpha-1}\right\}\right] x_{K-1}^{N+2\alpha-1} = 0 \tag{15}$$

Then, the procedure above can be summarized in the compact form as Equation (15):

$$\frac{\partial \epsilon_T^2}{\partial a_N} = \left[\sum_{i=0}^{K} p_i\, x_i^{N+2\alpha-1}\right] - \Bigg[f(0) \sum_{i=0}^{K} x_i^{N+2\alpha-1} + a_0 \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha)} \sum_{i=0}^{K} x_i^{4\alpha+N-2} + a_1 \frac{\Gamma(\alpha+2)}{\Gamma(2\alpha+1)} \sum_{i=0}^{K} x_i^{4\alpha+N-1} + \cdots + a_N \frac{\Gamma(N+\alpha+1)}{\Gamma(N+2\alpha)} \sum_{i=0}^{K} x_i^{4\alpha+2N-2}\Bigg] = 0$$

The procedure is repeated for all cases in Equation (13). Then, the following SLAE is achieved.

$$[A]_{N+2\mathrm{x}N+2}[\Omega]_{N+2\mathrm{x}1} = [B]_{N+2\mathrm{x}1} \tag{16}$$

Here,

$$A = \begin{bmatrix} K+1 & \sum_{i=1}^{k} c_0(x_i) & \sum_{i=1}^{k} c_1(x_i) & \dots & \sum_{i=1}^{k} c_N(x_i) \\ \sum c_0(x_i) & \sum c_0(x_i)c_0(x_i) & \sum c_0(x_i)c_1(x_i) & \dots & \sum c_0(x_i)c_N(x_i) \\ \sum c_1(x_i) & \sum c_1(x_i)c_0(x_i) & \sum c_1(x_i)c_1(x_i) & \dots & \sum c_1(x_i)c_N(x_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum c_m(x_i) & \sum c_m(x_i)c_0(x_i) & \sum c_m(x_i)c_1(x_i) & \dots & \sum_{i=1}^{k} c_m(x_i)c_N(x_i) \end{bmatrix}$$

$$[\Omega] = [f(0) \quad a_0 \quad a_1 \quad \dots \quad a_N]^T$$

$$[\mathrm{B}] = \left[\sum_{i=0}^{K} P_i \quad \sum_{i=0}^{K} P_i c_0(x_i) \quad \sum_{i=0}^{K} P_i c_1(x_i) \quad \dots \quad \sum_{i=0}^{K} P_i c_N(x_i)\right]^T$$

where,

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$$c_m(x,\alpha) = \frac{\Gamma(m+\alpha+1)}{\Gamma(m+2\alpha)} x^{m+2\alpha-1}$$

Here, $m = 1,2, \dots N$

The vector Ω consist of unknowns $(f(0), a_n)$. By inversion of $[A]$, Ω vector can be obtained. Then, Equation (9) allows one to obtain $f(x)$ which represents the discrete dataset with minimum error. The optimum value of α is found by implementing a grid search. Note that, when the fractional order α is equal to one, the fractional approach is equal to the polynomial method.

3. DATASET AND PERFORMANCE METRICS

In this study, we model the telecommunication revenues as a percentage of GDP for countries and compare them from the year 2000 to 2018. The dataset of the telecommunication revenues for each country is extracted from OECD [24]. The dataset is reported in Figure 1 and Table A.1 of the Appendix for five countries (France, Germany, Italy, Turkey, and UK) and the average of the OECD members.

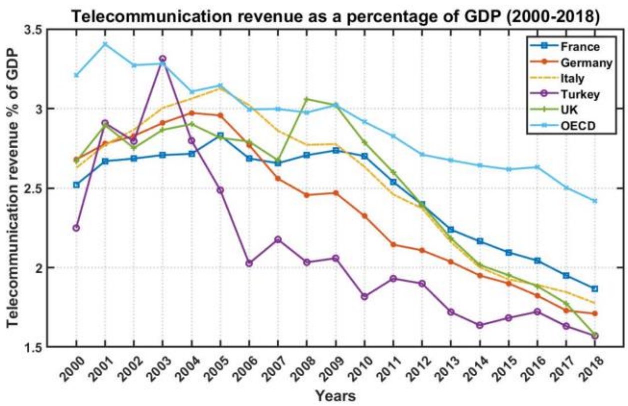

Fig.1. Telecommunication revenues as a percentage of GDP of the countries.

It is important to know that these selected countries are comparable and similar regarding the population, the number of subscribers, technological infrastructure. However, as expected, there are also differences among the selected countries such as the percentage of young people or adults over the total population which can affect the revenue of the telecommunication sector and total economical size of the country. Nevertheless, a key point in the present study is how the telecommunication share affects the countries economy.
All the results reported in tables are in terms of Mean Absolute Percentage Error (MAPE). The MAPE is calculated as in (19).

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{P(i)-f(x_i)}{P(i)}\right| \times 100 \qquad (19)$$

The average error in all percentiles is calculated by the formula in (20).

$$AMAPE = \frac{\sum MAPE}{M} \qquad (20)$$

4. EXPERIMENTAL RESULTS

This section will provide modeling results of Fractional and Polynomial methods with three different modeling settings. Each set has a different exponent value. Table 1 illustrates the modeling results of three N's for both fractional and polynomial models. When $N = 5$, the performance of the fractional approach outperforms the polynomial approach by 0.3152 % MAPE where the first model yields 2.2909 % and the latter yields 2.6061 % AMAPE.

When the exponent is 8, the fractional approach produces %1.7202 AMAPE where the polynomial approach yields 1.7861% AMAPE. Lastly, when the exponent is equal to 10, the fractional approach results in 1.7031% MAPE while the polynomial method results in 1.7208% AMAPE.

TABLE I
MODELING RESULTS OF THE TELECOMMUNICATION REVENUE AS A PERCENTAGE OF GDP (2000-2018).

N value	Models	Results	France	Germany	Italy	Turkey	UK	OECD
N=5	Fract. Model	MAPE	1.69	1.59	1.33	4.12	3.59	1.40
		α	0.97	1	0.50	0.50	0.95	0.50
	Pol. Model	MAPE	1.70	1.59	1.44	5.80	3.61	1.50
N=8	Fract. Model	MAPE	1.16	1.23	1.05	3.71	2.13	1.01
		α	0.50	0.50	0.92	0.50	0.50	1
	Pol. Model	MAPE	1.25	1.28	1.07	3.88	2.22	1.02
N=10	Fract. Model	MAPE	0.76	0.89	0.59	3.24	1.59	0.67
		α	0.50	0.82	1	0.96	0.50	0.50
	Pol. Model	MAPE	0.82	0.93	0.60	3.25	1.94	0.80

For all three settings, the Fractional approach outperforms the polynomial approach. Also, for all exponent values, the largest MAPE is observed in Turkey and the smallest is observed for Italy. As expected, increasing the exponent value decreases the error rate.

The largest MAPE difference between the two models is observed when $N = 5$. Note that, when the fractional order α is equal to one, the fractional approach is equal to the polynomial approach. For Germany where $N = 5$, for OECD average where $N = 8$, and lastly for Italy where $N = 10$, the optimized fractional order is found as 1. In these three cases, the MAPE results of the two models are equal as reported in Table 1.

Figures 2, 3, and 4 illustrate the actual and the modeled data curves for both Fractional Model and Polynomial Model. In most cases, Fractional and Polynomial modeled curves are similar to each other. The biggest difference is observed for Turkey in Figure 2. As seen from the plot, the fractional model fits the data better. This is consistent with the MAPE results reported in Table 1. It can be seen from Figure 1, the Polynomial and Fractional Models produce similar results. Numerically, Italy has the highest revenue USD in millions among the others.
From the figures, one can see that Italy has the highest telecommunication revenue percentage while Turkey has the lowest revenue percentage among others in 2000. Germany started with 2.6 percent telecommunication revenue and decreased to 1.7%. Initially, Italy had 2.6% revenue in 2000 and decreased to around 1.77%. France started with 2.5% revenue and ended up at 1.8%. Turkey started with 2.24% revenue and decreased to 1.57%. As seen from the figure, the telecommunication revenue % of GDP decreased for all

modeled countries and OECD average. The largest difference in percentage from 2000 to 2018 is observed for the UK. Also, Italy's trend is smoother compared to the others.

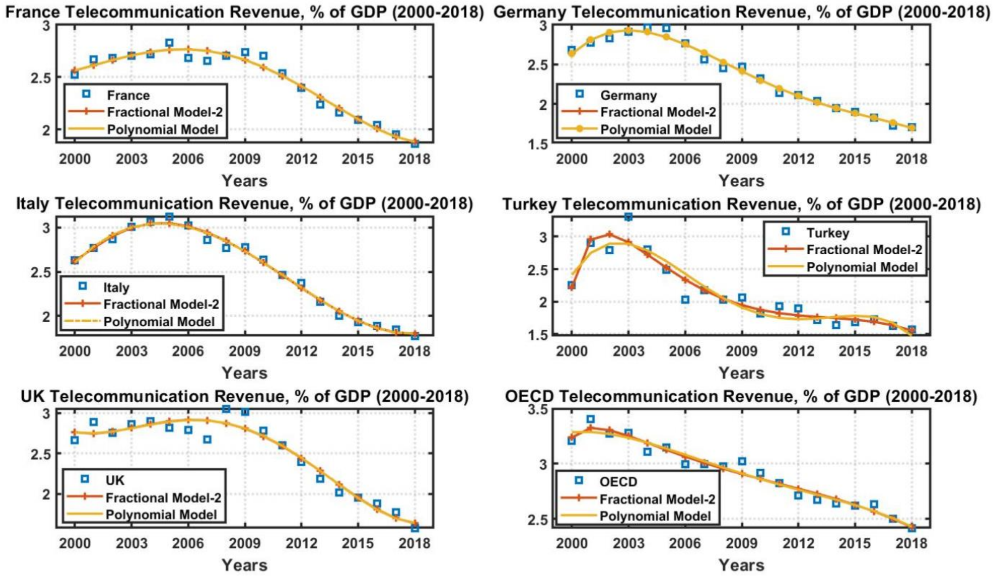

Fig. 2. Modeling of the countries using the Fractional model for $N = 5$.

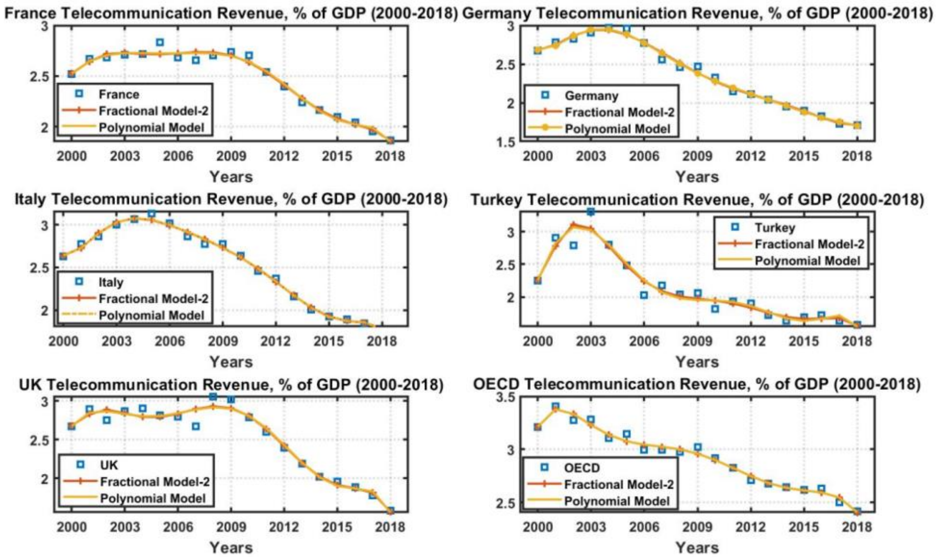

Fig. 3. Modeling of the countries using the Fractional model for $N = 8$.

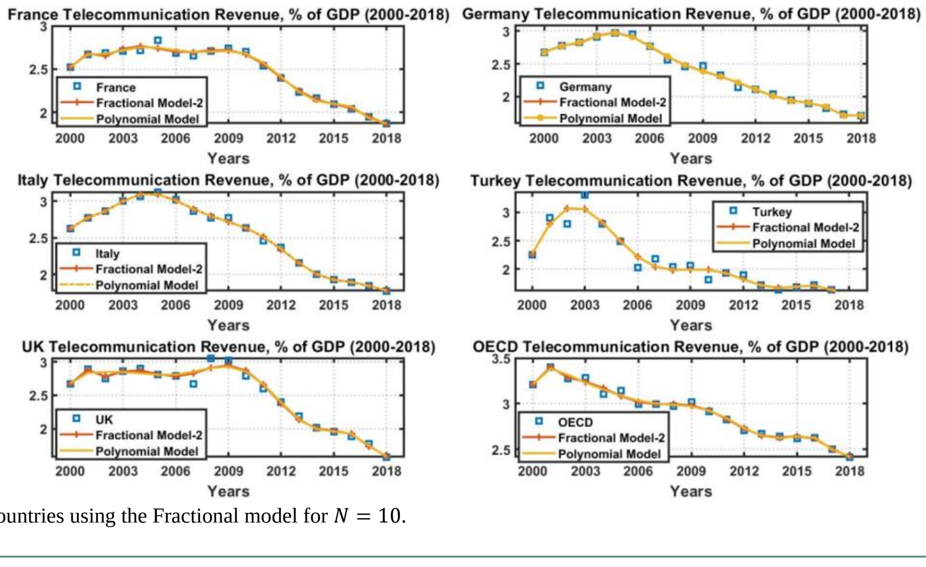

Fig. 4. Modeling of the countries using the Fractional model for $N = 10$.

5. CONCLUSIONS

In this study, we employed Fractional Calculus to model the telecommunications revenue as a percentage of the Gross Domestic Product of 5 countries (France, Germany, Italy, Turkey, and the UK) and OECD average. First, we proposed a new Fractional mathematical model that employs Least Square Methods named Fractional Model-2. Later, we compared the performances of the Fractional Model-2 and the Polynomial model. Results are reported for three experimental settings with three different exponent N values (5, 8, 10). As expected, increasing the exponent value decreased the error rate for both models. For all three settings, Fractional Approach resulted in better performance compared to the Polynomial Approach. The largest modeling error is obtained for Turkey while the smallest modeling error is observed in Italy. On average, the Fractional approach is superior to the Polynomial approach with 0.1329% MAPE.

APPENDIX

TABLE A.I
TELECOMMUNICATION REVENUE AS A PERCENTAGE OF GDP (2000-2018).

Years	France	Germany	Italy	Turkey	UK	OECD members
2000	2.520	2.678	2.626	2.249	2.668	3.209
2001	2.669	2.779	2.772	2.908	2.894	3.404
2002	2.685	2.828	2.866	2.794	2.752	3.272
2003	2.707	2.909	3.004	3.313	2.866	3.282
2004	2.715	2.972	3.062	2.798	2.902	3.106
2005	2.830	2.956	3.126	2.487	2.815	3.145
2006	2.686	2.770	3.020	2.026	2.792	2.995
2007	2.656	2.559	2.8597	2.1767	2.675	2.997
2008	2.707	2.456	2.771	2.033	3.059	2.974
2009	2.736	2.469	2.776	2.059	3.021	3.021
2010	2.701	2.324	2.634	1.817	2.787	2.916
2011	2.538	2.144	2.459	1.931	2.599	2.826
2012	2.396	2.108	2.372	1.899	2.394	2.710
2013	2.239	2.036	2.160	1.720	2.184	2.674
2014	2.165	1.950	2.001	1.637	2.017	2.642
2015	2.095	1.900	1.925	1.684	1.952	2.617
2016	2.043	1.823	1.891	1.722	1.883	2.631
2017	1.950	1.730	1.845	1.631	1.774	2.503
2018	1.866	1.710	1.776	1.570	1.577	2.418

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