Hacettepe Journal of
Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 51 (1) (2022), 1 – 7
DOI : 10.15672/hujms.911145

Research Article

β-Menger and β-Hurewicz spaces

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Abstract

Recently, some papers on weaker forms of classical covering properties of Hurewicz and Menger have been published. In this paper, using the covers formed by β-open sets, we introduce and study the properties of β-Menger and β-Hurewicz topological spaces. We give counterexamples that show the interrelations between those properties. The subject of our investigation is also the preservation of β-Menger and β-Hurewicz properties under subspaces, products, and mappings.

Mathematics Subject Classification (2020). 54D20, 54B05, 54C08

Keywords. β-open set, β-Menger space, β-Hurewicz space

1. Introduction

In this paper, we consider β-Menger and β-Hurewicz spaces which are similar to the well-known properties of Menger and Hurewicz. We will show that the β-Menger and β-Hurewicz properties are different from the (semi-)Menger and the (semi-)Hurewicz properties. The properties of Menger and Hurewicz take a long history going back to the papers by [18] and [8]. Both of them are generalizations of σ-compactness. A topological space X is said to have the *Menger* (resp. *Hurewicz*) property if for every sequence $\mathcal{U}_1, \mathcal{U}_2, \ldots$ of open covers of the space X, there are finite subfamilies $\mathcal{V}_1 \subseteq \mathcal{U}_1, \mathcal{V}_2 \subseteq \mathcal{U}_2, \ldots$ (resp. $\mathcal{W}_1 \subseteq \mathcal{U}_1, \mathcal{W}_2 \subseteq \mathcal{U}_2, \ldots$) such that the collection $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an open cover of the space X (resp. each element of X belongs to all but finitely many of the sets $\cup\mathcal{W}_n$). It is easy to see that each σ-compact space has the Hurewicz property and the latter implies the Menger property. Each space with the Menger property is Lindelöf.

Several weak variants of selection principles have been studied by applying the closure and the interior operators in the definition of a selection property and the other types have been investigated when sequences of open covers are replaced by some generalized open sets. On the study of the weaker versions of Hurewicz spaces, the readers can see the references [9, 10, 19–21]. Also, the Menger property has the weakest one among the so-called selection principles, see, e.g., [4, 11, 22] for detailed introductions to the topic. This is a growing area of general topology, see, e.g., [24]. Moreover, Menger spaces have recently found applications in such areas as forcing [6].

On the other hand, generalized open sets and their counterpart generalized closed sets are one of the significant areas of research in general topology. Many authors studied these

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Received: 08.04.2021; Accepted: 28.07.2021

notions with their various properties. Semi-open sets and semi-continuity which are early notions related to this area were introduced by Levine [16], and the other one was pre-open sets introduced by Mashhour et al. [17]. The concepts of pre-open sets and semi-open sets are independent. The role of generalized open sets (semi-open, pre-open, α-open, θ-open and so on) in selection principles theory in topological spaces was discussed in several papers (see, for example, [4], [12], [14], [19], [25] and the survey paper [13] and references therein).

Apart from introducing β-open set in topology, Abd El Monsef et al. [1], and the equivalent notion of semi-pre-open set was given by Andrijevic in [3] and examined by Ganster and Andrijevic [7]. After that many researchers studied these concepts and related notions. For more detailed studies of β-open sets, we refer the reader to Caldas and Jafari [5].

The paper is organized in such a way that after this introduction in Section 2 we give information about terminology and notation. In Section 3 we show that we can replace open sets with β-open sets in the definition of β-Menger and β-Hurewicz spaces. We also give several examples that β-Menger and β-Hurewicz properties are not equivalent to the (semi-)Menger and the (semi-)Hurewicz properties. In Section 4 we investigate the behavior of β-Menger and β-Hurewicz properties with respect to subspaces and products. We also look at the behavior of the β-Menger (β-Hurewicz) under β-continuous mappings. Section 5 concludes the paper.

2. Preliminaries

Throughout the paper (X, τ), or shortly X, will denote an infinite topological space on which no separation axioms are assumed unless otherwise stated. The symbols $\mathbb{R}$ and $\mathbb{N}$ are respectively the set of real numbers and natural numbers.

We begin recalling that for a subset A of (X, τ), $cl(A)$ and $int(A)$ will denote the closure and the interior of A, respectively, while τ_A denotes the subspace topology on A inherited from (X, τ). A set A in a topological space X is said to be β-open if and only if $A \subseteq cl\,(int\,(cl\,(A)))$ and β-closed if its complement is β-open. A set A in a space X is called semi-open if and only if $A \subseteq cl\,(int\,(A))$ and semi-closed if its complement is semi-open. Every open set is β-open, whereas a β-open set may not be open. The union of any number of β-open sets is β-open. The collection of all β-open sets of X contain in A is called β-interior of A and is denoted by $\beta int(A)$. Dually, the β-closure $\beta cl(A)$ of $A \subset X$ denotes the intersection of all β-closed sets containing A. A set A is β-open if and only if $\beta int(A) = A$, and A is β-closed if and only if $\beta cl(A) = A$. Note that for any subset A of X

$$int(A) \subset \beta int(A) \subset A \subset \beta cl(A) \subset cl(A).$$

For more details on β-open sets and β-continuity, we refer to [1].

Definition 2.1. A topological space (X, τ) is said to be:

(1) β-compact [2] if every cover $\{A_j \mid j \in J\}$ of X by β-open sets A_j has a finite subcover.
(2) β-Lindelöf if every cover $\{A_j \mid j \in J\}$ of X by β-open sets A_j has a countable subcover.

We give now definitions of generalized continuous (open) mappings related to β-open sets.

A mapping $f : X \to Y$ is called:

(1) β-continuous if the preimage of any open subset of Y is β-open in X;
(2) β-irresolute if the preimage of any β-open subset of Y is β-open in X;
(3) β-open if the image of any β-open subset of X is β-open in Y;
(4) strongly β-open if the image of any β-open subset of X is open in Y;

(5) β-perfect if for each β-closed set $A \subset X$, the set $f(A)$ is β-closed in Y and for each $y \in Y$, its preimage $f^{-1}(y)$ is β-compact relative to X;
(6) β-homeomorphism if f is a bijection and images and preimages of β-open sets are β-open.

A topological space X is said to have the semi-*Menger* [19] (resp. semi-*Hurewicz* [14]) property if for every sequence $\mathcal{U}_1, \mathcal{U}_2, \ldots$ of semi-open covers of the space X, there are finite subfamilies $\mathcal{V}_1 \subseteq \mathcal{U}_1, \mathcal{V}_2 \subseteq \mathcal{U}_2, \ldots$ (resp. $\mathcal{W}_1 \subseteq \mathcal{U}_1, \mathcal{W}_2 \subseteq \mathcal{U}_2, \ldots$) such that the collection $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is a semi-open cover of the space X (resp. each element of X belongs to all but finitely many of the sets $\cup\mathcal{W}_n$).

3. β-Menger and β-Hurewicz spaces

In this section, we define β-Menger and β-Hurewicz selection properties in topological spaces and some examples are given.

Definition 3.1. Let X be a topological space and A be a subset of X.

(1) A is said to have the β-Menger property if for any sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of β-open covers of A there exists a sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $A \subseteq \bigcup_{n\in\mathbb{N}} \cup\mathcal{V}_n$. We say that X is β-Menger if the set X is β-Menger.
(2) A is said to have the β-Hurewicz property if for any sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of β-open covers of A there exists a sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in A$ for all but finitely many n, $x \in \cup\mathcal{V}_n$. We say that X is β-Hurewicz if the set X is β-Hurewicz.

Evidently, we have the following implications between these notions:

$$\begin{array}{ccccccc}
\beta-compact & \Rightarrow & \beta-Hurewicz & \Rightarrow & \beta-Menger & \Rightarrow & \beta-Lindelöf \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
compact & \Rightarrow & Hurewicz & \Rightarrow & Menger & \Rightarrow & Lindelöf \\
\Uparrow & & \Uparrow & & \Uparrow & & \Uparrow \\
semi-compact & \Rightarrow & semi-Hurewicz & \Rightarrow & semi-Menger & \Rightarrow & semi-Lindelöf
\end{array}$$

Neither of these implications is reversible. The β-Hurewicz property does not imply the β-compact property in general as the following example shows.

Example 3.2. Let X be the set of positive integers and define the discrete topology. Here, every subset in this space β-open. Thus X is β-Hurewicz space and also is β-Menger space. On the other hand, $\{\{x\} : x \in X\}$ is a β-open cover of X which has no finite subcover. Hence, X is not β-compact.

Example 3.3. The Sorgenfrey line has the topology generated by all half-open intervals $[p, q)$. The Sorgenfrey line is a hereditarily Lindelöf space which is not β-Menger (because it is not Menger, as it is well-known). Therefore this space is not a β-Hurewicz space.

There exists a Hurewicz space which is not β-Hurewicz.

Example 3.4. Endow the real line $\mathbb{R}$ with the Euclidean topology τ. $(\mathbb{R}, \tau)$ is a Hurewicz space. However, this space is not a β-Hurewicz space, because from a sequence of covers whose elements are sets of the form $[a, b]$, $a, b \in \mathbb{R}$, one can not choose finite subfamilies whose union covers $\mathbb{R}$.

From the definition, it is obvious that every β-Menger space is a Menger space. The following example shows that the inverse does not hold.

Example 3.5. Let X be an uncountable set with the topology $\tau = \{X, \emptyset, \{a\}\}$, where a is a fixed point in X. This space is compact, so Menger. All sets containing a are β-open. The β-open cover $U = \{\{a, x\}, x \in X \setminus \{a\}\}$ does not contain a countable subcover, so that this space is not β-Lindelöf and thus can not be β-Menger.

There exists a semi-Hurewicz (semi-Menger) space which is not β-Hurewicz (β-Menger).

Example 3.6. Endow the real line $\mathbb{R}$ with the indiscrete topology τ. Obviously, $(\mathbb{R}, \tau)$ is compact thus, it is a Hurewicz space. Also, this space is semi-compact hence, it is a semi-Hurewicz space, because the open sets of $(\mathbb{R}, \tau)$ coincides with semi-open sets. But consider β-open cover $\{\{x\} : x \in \mathbb{R}\}$ of $\mathbb{R}$ (because every subset of $(\mathbb{R}, \tau)$ is β-open). This cover does not have a countable subcover. Thus $(\mathbb{R}, \tau)$ is not β-Lindelöf. So, $(\mathbb{R}, \tau)$ is neither β-Hurewicz space nor β-Menger space.

4. Properties of β-Menger and β-Hurewicz spaces

In this section, we examine some properties of β-Menger and β-Hurewicz spaces. Principally, we consider the preservation of these properties under subspaces, products, and mappings. We begin the section with a simple fact.

Theorem 4.1. *The β-Menger property (the β-Hurewicz property) is closed under countable unions.*

Proof. We will prove only the β-Menger case. Let $X = \bigcup \{X_k : k \in \mathbb{N}\}$, where each X_k has β-Menger property and $(\mathcal{U}_n)_{n\in\mathbb{N}}$ be a sequence of β-open covers of X. For each $k \in \mathbb{N}$, let us consider finite subfamilies $(\mathcal{V}_{n,k})_{n\in\mathbb{N}} \subset (\mathcal{U}_n)_{n\in\mathbb{N}}$ such that for every nonempty β-open subset U of X_k, $U \cap (\bigcup \mathcal{V}_{n,k}) \neq \emptyset$ for all but finitely many $(n \geqslant k)$. Let $\mathcal{V}_n = \{\mathcal{V}_{n,k} : n \geqslant k\}$. Then each $\mathcal{V}_n$ is a finite subfamily of $\mathcal{U}_n$ and we have $X = \bigcup_{n\in\mathbb{N}} \cup \mathcal{V}_n$. □

Menger's property is hereditary for closed subsets but β-Mengerness is not a hereditary property. The following example actually shows that a β-open subspace of a β-Menger space need not be β-Menger.

Example 4.2. Let X be an uncountable space and p a particular point of X. Define on X the topology $\tau = \{U \subseteq X : p \notin U\} \cup \{U \subseteq X : \text{the set } X \setminus U \text{ is finite}\}$, known as uncountable Fort space on X [23]. All one-point subsets of X, except the set $\{p\}$, are β-open because all singletons $\{x\}$, $x \in X \setminus \{p\}$ are β-open. This space is β-Menger as it is easily checked. However, the β-open subspace $Y = X \setminus \{p\}$ is not. Consider the β-open cover $\mathcal{U} = \{\{x\} : x \in Y\}$ of Y. This cover contains no countable subcover so that Y cannot be β-Menger.

Proposition 4.3. *Every β-clopen subspace of a β-Menger (β-Hurewicz) space is also β-Menger (β-Hurewicz).*

Proof. Let A be a β-clopen subspace of a β-Menger space X and let $(\mathcal{U}_n)_{n\in\mathbb{N}}$ be a sequence of β-open covers of (A, τ_A). It is easy to see that every β-open subset of β-clopen subspace A of X is the intersection of a β-open subset of X with A. Therefore, for each $n \in \mathbb{N}$ and each $U \in \mathcal{U}_n$ there exists a β-open set G_U in X such that $U = A \cap G_U$. Let $\mathcal{G}_n = \{G_U : U \in \mathcal{U}_n\} \cup \{X \setminus A\}$, $n \in \mathbb{N}$. Then $(\mathcal{G}_n)_{n\in\mathbb{N}}$ is a sequence of β-open covers of X. β-Mengerness of X implies the existence of a sequence $(\mathcal{W}_n)_{n\in\mathbb{N}}$ with $\mathcal{W}_n$ is a finite subset of $\mathcal{G}_n$ for each $n \in \mathbb{N}$, and $X = \bigcup_{n\in\mathbb{N}} \cup \mathcal{W}_n$. If we put for each n, $\mathcal{V}_n = \{U : G_U \in \mathcal{W}_n\}$ we obtain the sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ witnessing for $(\mathcal{U}_n)_{n\in\mathbb{N}}$ that (A, τ_A) is β-Menger.

Quite similarly we prove the β-Hurewicz part of the theorem. □

Menger's property is hereditary continuous images. Now, we consider preservation (in the image or preimage direction) of the β-Menger property. Thus we consider some kinds of mappings.

Theorem 4.4. *A β-continuous image of a β-Menger space is a Menger space.*

Proof. Let X be a β-Menger space and $Y = f(X)$ its image under a β-continuous mapping $f : X \to Y$. Let $(\mathcal{V}_n)_{n\in\mathbb{N}}$ be a sequence of open covers of Y. Since f is β-continuous, setting $\mathcal{U}_n = f^{-1}(\mathcal{V}_n)$, $n \in \mathbb{N}$, we get the sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of β-open covers of X. Use the fact X is β-Menger and for each n, find a finite subset $\mathcal{H}_n$ of $\mathcal{U}_n$ such that $X = \bigcup_{n\in\mathbb{N}} \cup\mathcal{H}_n$. Let $\mathcal{W}_n = f(\mathcal{H}_n)$, $n \in \mathbb{N}$. Then the sequence $(\mathcal{W}_n)_{n\in\mathbb{N}}$ verifies for $(\mathcal{V}_n)_{n\in\mathbb{N}}$ that Y is a Menger space. □

Corollary 4.5. *Let $f : X \to Y$ be a continuous surjection mapping. If X is a β-Menger space, then Y is a Menger space.*

Similarly to the proof of Theorem 4.4 one proves the following theorem.

Theorem 4.6. *A β-irresolute image of a β-Menger space is also β-Menger space.*

Recall that a property which is preserved by β-homeomorphisms is called a β-topological property.

Remark 4.7. From the previous theorem, we see that the β-Mengerness is a β-topological property. On the other hand, the Menger property is not a β-topological property.

Theorem 4.8. *Let (X, τ) be a space. If X admits a strongly β-open bijection onto a Menger space Y, then the space X is β-Menger.*

Proof. Let $f : X \to Y$ be a strongly β-open bijection from a space X onto a Menger space Y. Let $(\mathcal{U}_n)_{n\in\mathbb{N}}$ be a sequence of β-open covers of (X, τ). Then $f(\mathcal{U}_n)$ is a sequence of open covers of Y. Choose for each n a finite subset $\mathcal{V}_n$ such that $Y = \bigcup_{n\in\mathbb{N}} \cup f(\mathcal{V}_n)$. Clearly, we have $X = \bigcup_{n\in\mathbb{N}} \cup\mathcal{V}_n$, so that X is β-Menger. □

Theorem 4.9. *Let $f : X \to Y$ be a β-perfect mapping. If Y is a β-Menger space, then X is a β-Menger space.*

Proof. Assume that $(\mathcal{U}_n)_{n\in\mathbb{N}}$ is a sequence of β-open covers of X. For each $y \in Y$ and each n, there exists a finite subcollection $\mathcal{V}_n^y$ of $\mathcal{U}_n$ covering $f^{-1}(y)$. Set $V_n^y = \bigcup \mathcal{V}_n^y$ and $W_n^y = Y \backslash f(X \backslash V_n^y)$. Then $y \in W_n^y$, and $\mathcal{W}_n = \{W_n^y : y \in Y\}$ is a β-open cover of Y for each $n \in \mathbb{N}$. As Y is a β-Menger space, there is a sequence $(\mathcal{H}_n)_{n\in\mathbb{N}}$ such that $\mathcal{H}_n$ is a finite subcollection of $\mathcal{W}_n$, $n \in \mathbb{N}$, and $Y = \bigcup_{n\in\mathbb{N}} \bigcup \mathcal{H}_n$. To each $H \in \mathcal{H}_n$ associate finitely many sets from $\mathcal{V}_n^y$ which occur in the representation of V_n^y for which $H = Y \backslash f(X \backslash V_n^y)$. Thus, for each n we have chosen a finite subcollection $\mathcal{V}_n$ of $\mathcal{U}_n$. Clearly, we have $X = \bigcup_{n\in\mathbb{N}} \bigcup \mathcal{V}_n$, so that X is a β-Menger space. □

Now, we consider preservation of the β-Hurewicz property.

Theorem 4.10. *Let $f : X \to Y$ be a β-continuous surjection mapping. If X is a β-Hurewicz space, then Y is a Hurewicz space.*

Proof. Let $\left(\mathcal{U}_n^Y\right)_{n\in\mathbb{N}}$ be a sequence of open covers of Y and $x \in X$. If $\mathcal{U}_n^X = \left\{f^{-1}(U) : U \in \mathcal{U}_n^Y\right\}$, then $\left(\mathcal{U}_n^X\right)_{n\in\mathbb{N}}$ is a sequence of β-open covers of X. Since X is a β-Hurewicz space, there is a finite subset $\left(\mathcal{V}_n^X\right)_{n\in\mathbb{N}}$ of $\left(\mathcal{U}_n^X\right)_{n\in\mathbb{N}}$ with for each $x \in X$ for all but finitely many n such that $x \in \cup\mathcal{V}_n^X$.

Let $\mathcal{V}_n^Y = \left\{f(V) : V \in \mathcal{V}_n^X\right\}$ and $x \in X$. For each $n \in \mathbb{N}$ there is a set $f(V) \in \mathcal{V}_n^Y$ containing $f(x)$. Since f is β-continuous there is a β-open set $V \in \mathcal{V}_n^X$ containing x such that $V = f^{-1}(U)$ for some $U \in \mathcal{U}_n^X$. Then we have $f(V) = f\left(f^{-1}(U)\right) = U \in \mathcal{U}_n^X$ and so each $y \in Y$ belongs to all but finitely many sets $\mathcal{V}_n^Y$. It is clear that $Y = f(X) = f(\cup\mathcal{V}_n^X) = \cup\mathcal{V}_n^Y$ and so Y is a Hurewicz space. □

Corollary 4.11. *Let $f : X \to Y$ be a continuous surjection. If X is a β-Hurewicz space, then Y is a Hurewicz space.*

It is well known that the Lindelöf property is not preserved even by squares, as witnessed by the Sorgenfrey line. Also, the fact that a product of Hurewicz (resp. Menger) spaces can fail to be Hurewicz/Menger (resp. Menger). When we consider a Menger space and a compact space, the product of them is Menger. Now we examine the above information for β-Menger in a topological space.

Theorem 4.12. *If X is β-Menger and Y is β-compact, then $X \times Y$ is β-Menger.*

Proof. Let $(\mathcal{U}_n)_{n\in\mathbb{N}}$ be a sequence of covers of $X \times Y$ by β-open sets in $X \times Y$. Hence for each $n \in \mathbb{N}$ there exist β-open covers $\mathcal{V}_n$ and $\mathcal{W}_n$ of X and Y, respectively, such that $\mathcal{U}_n = \mathcal{V}_n \times \mathcal{W}_n$. By β-Menger of X there are finite subsets $\mathcal{V}'_n \subseteq \mathcal{V}_n$, $n \in \mathbb{N}$, so that $S = \bigcup_{n\in\mathbb{N}} \bigcup \mathcal{V}'_n$. Since Y is β-compact, choose a finite subset A_n of $(\mathcal{W}_n)_{n\in\mathbb{N}}$ which is a β-open cover of Y. Now let us consider the family

$$\mathcal{X}_n = \mathcal{V}'_n \times A_n.$$

Hence for each $n \in \mathbb{N}$, $\mathcal{X}_n$ is a finite subset of $\mathcal{U}_n$. That implies

$$X \times Y = \bigcup_{n\in\mathbb{N}} \bigcup \mathcal{V}'_n \times \bigcup A_n = \bigcup_{n\in\mathbb{N}} \bigcup (\mathcal{V}'_n \times A_n) = \bigcup_{n\in\mathbb{N}} \bigcup \mathcal{X}_n$$

which concludes the proof. □

5. Closing remarks

This study is devoted to introducing and investigating the concepts of β-Menger and β-Hurewicz spaces. We did not consider β-Rothberger spaces, but all properties from Section 4 concerning the β-Menger property can be investigated for the β-Rothberger property applying quite similar techniques for their proofs. On the other hand, the covering properties of Menger and Hurewicz can be characterized game theoretically and Ramsey theoretically [21] and [15]. So far, as the author knows, both of these problems are still open for the β-Menger and β-Hurewicz properties.

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