Gazi Mühendislik Bilimleri Dergisi
2021, 7(2): 169-174
Araştırma Makalesi/Research Article
https://dergipark.org.tr/gmbd

Application of Differential Transformation Method and Dunkerley Formula for Stability Analysis of Bars in Water

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ARTICLE INFO

Received: 10.04.2021
Accepted: 10.08.2021

Keywords:
Stability,
Buckling,
DTM,
Dunkerley,
Bar in water

ABSTRACT

In this study, two methods have been proposed to find the critical buckling load of bars that are in water and buckled under the effect of P singular force and distributed self-weight. Three different Euler's cases were taken into account in the study. In the first method, the solution of the stability differential equation of the bar in water is solved by the Differential Transformation Method (DTM), while in the second method, the Dunkerley formula is applied to determine the critical buckling load factor of the buckled bar in water. Finally, the results obtained by solving an example with the two proposed methods were compared with the finite element method. SAP 2000 program was used for finite element analysis. From the results obtained, it was observed that the two methods gave results in good agreement to the finite element method.

https://dx.doi.org/10.30855/gmbd.2021.02.10

Su İçeresindeki Çubuğun Stabilite Analizi için Diferansiyel Dönüşüm Yöntemi ve Dunkerley Formülünün Uygulanması

MAKALE BİLGİSİ

Alınma: 10.04.2021
Kabul: 10.08.2021

Anahtar Kelimeler:
Stabilite,
Burkulma,
DTM,
Dunkerley,
Suda kolon

ÖZ

Bu çalışmada su içerisinde olan ve P tekil kuvvet ve yayılı kendi ağırlığı etkisinde burkulan çubukların kritik burkulma yükünün bulunması için iki yöntem önerilmiştir. Çalışmada üç farklı Euler durumu dikkate alınmıştır. Sunulan yöntemlerden ilkinde su içerindeki çubuğun stabilite diferansiyel denklemin çözümü Diferansiyel dönüşüm yöntemi (DTM) ile çözülürken, ikinci yöntemde Dunkerley Formülü su içerisinde burkulan çubuğun kritik burkulma yük faktörünün belirlenmesi için uygulanmıştır. Çalışmanın sonunda bir örnek önerilen iki yöntem ile çözülerek elde edilen sonuçlar sonlu elamanlar yöntemi ile karşılaştırılmıştır. Sonlu elemanlar yöntemi ile analiz için SAP 2000 programı kullanılmıştır. Elde edilen sonuçlardan iki yönteminde sonlu elemanlar yöntemine yeter uygunlukta sonuç verdiği gözlenmiştir.

https://dx.doi.org/10.30855/gmbd.2021.02.10

1. INTRODUCTION *(GİRİŞ)*

The use of columns has become widespread in civil, mechanical, and aerospace engineering and has a significant place among engineering structures. Therefore, extensive studies have been performed in recent decades to determine elastic columns' critical buckling load. Euler was the first researcher that studies the buckling of a prismatic column under a compressive force or self-weight [1].

Wei et al. [2] investigated elastic bars stability with variable cross-section under self-weight and concentrated end load. They transformed governing

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To cite this article: F. Khosravt, K.B. Bozdoğan “Application of Differential Transformation Method and Dunkerley Formula for Stability Analysis of Bars in Water”, *Gazi Journal of Engineering Sciences,* vol.7, no.2, pp. 169-174, 2021. doi:https://dx.doi.org/10.30855/gmbd.2021.02.10

equation for EB(Euler–Bernoulli) columns into an integral equation and evaluated critical buckling load by investigating the lowest eigenvalue of the resulting integral equation. They validated the effectiveness of this method by numerical examples.

Darbandi et al. [3] studied the static stability of the variable cross-section columns under distributed axial force. They used the EB beam theory to model the column and used the singular perturbation method of Wentzel-Kramers-Brillouin to solve the problem. Wang [4] investigated the stability of columns under self-weight and a tip load. The governing equation was integrated by turning the two-point boundary value problem into an initial value problem. Four boundary conditions were studied. Optimum locations of the support for maximum load-bearing capacity were obtained. Huang and Li [5] investigated analytic approach to solve EB columns' buckling instability with arbitrarily axial nonhomogeneity and/or varying cross-section. Differential equations of buckling for different support conditions (pinned-pinned, clamped, and cantilevered columns) were derived with varying flexural rigidity and reduced to a Fredholm integral equation. The impact of the method was approved by comparing our results with existing closed-form solutions and numerical results.

Krutii and vandynskyi [6] proposed a new method to study the uniform column's buckling problem under self-weight. The method base was on the exact solution of the appropriate differential equation for a column's buckling. The solution was stated with dimensionless fundamental functions and initial parameters. The stress-strain state of the column was defined. The analytical form of the load is defined because of the dimensionless nature of the fundamental functions. The values of the buckling coefficient were determined. Wahrhaftig et al. [7] Investigated mathematical solutions of a column, including the self-weight, to obtain the critical buckling load. They Compared the analytical solutions with computation modeling.

The bar's critical buckling strength, whose weight is considered, will differ depending on whether the bar is in an air or water environment. Pekbey [8] developed self-weight buckling of the column at its top fixed and lower end fixed-roller supported in water and presented an analytical solution to determine the heavy column's critical buckling load.

The Differential Transformation Method (DTM) is an alternative method for solving differential equations. DTM was first introduced by Zhou [9]. Several studies on the method have been presented in the literature [10–16]. Holubowski and Tarczewska [17] presented the application of the DTM to the stability analysis of nonuniformly loaded beams subjected to bending. They solved two coupled ordinary differential equations with variable coefficients and parameters to obtain the critical load. Rajasekaran [18] investigated the stability of fully or partially supported heavy prismatic piles and fully supported non-prismatic piles by using DTM. Chai and Wang [19] used DTM to study the critical buckling load of axially compressed heavy columns of various support conditions. They compared obtained results with an analytical solution. Therefore, the method was also shown to be very accurate.

In this study, different from the literature, the DTM and Dunkerley formula approach is proposed to determine the critical buckling load of a rectiliner bar in water under the singular load and its weight. It is accepted that the material is linear elastic in the study.

2. METHOD *(METOD)*

In this study, three different Euler cases are considered in a bar under the influence of their weight in water and with a singular force P above it, as shown in figure 1.

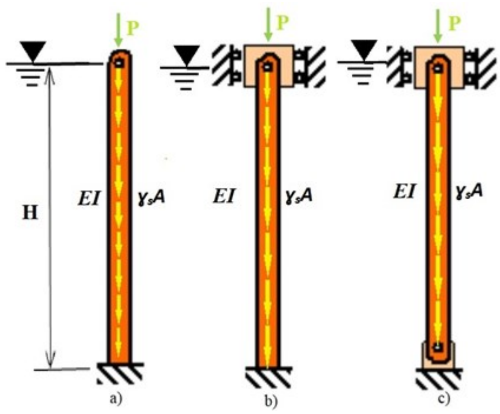

Figure 1. Three different Euler cases a) Free-Clamped b) Sliding -Clamped c) Sliding-Pinned *(Üç farklı Euler durumu a) Serbest-ankastre b)Kayıcı ankastre - Ankastree c) Kayıcı ankastre-mafsallı)*

According to the given set of axes, the differential equation of stability is written as follows, using the literature [8].

$$EI\frac{d^4y}{dz^4}+P\frac{d^2y}{dz^2}+\frac{d}{dz}[(\gamma_s-\gamma_f)Az\frac{dy}{dz}]=0 \quad (1)$$

where EI is the bending stiffness, P is axial singular load, and A is cross-section area, ɣs and ɣf are the bar and air's unit weight, respectively. y indicates horizontal displacement, and z indicates the axis along the bar axis. The differential Eq.(2) is obtained by integrating the differential Eq.(1).

$$EI\frac{d^3y}{dz^3}+[P+[(\gamma_s-\gamma_f)Az]\frac{dy}{dz}=c \quad (2)$$

Since the shear force is zero at the beginning of the bar for the Euler states considered according to the selected axes. The constant integration c in the differential Eq. (2) is zero. In this case, the differential Eq. (2) can be written as

$$EI\frac{d^3y}{dz^3}+[P+[(\gamma_s-\gamma_f)Az]\frac{dy}{dz}=0 \quad (3)$$

To make the differential Eq. (3) dimensionless, the transformation given in Eq. (4) is applied, and Eq. (5) is obtained.

$$\varepsilon=\frac{z}{H} \quad (4)$$

$$\frac{d^3y}{d\varepsilon^3}+[\frac{PH^2}{EI}+[(\gamma_s-\gamma_f)\frac{A\varepsilon H}{EI}H^2]\frac{dy}{d\varepsilon}=0 \quad (5)$$

The necessary arrangements are made in the differential Eq. (5), and the differential Eq. (6) is obtained.

$$\frac{d^3y}{d\varepsilon^3}+[n+n*a*(1-b)\varepsilon]\frac{dy}{d\varepsilon}=0 \quad (6)$$

Here n, a, and b are defined as follows.

$$n=\frac{PH^2}{EI} \quad (7)$$

$$a=\frac{\gamma_s AH}{P} \quad (8)$$

$$b=\frac{\gamma_f}{\gamma_s} \quad (9)$$

Eq. (11) is obtained by substituting Eq. (10) in Eq. (5).

$$\theta=\frac{dy}{d\varepsilon} \quad (10)$$

$$\frac{d^2\theta}{d\varepsilon^2}+[n+n*a*(1-b)\varepsilon]\theta=0 \quad (11)$$

The boundary conditions required for the differential Eq. solution (11) are given below for three different Euler cases.

Free Clamped:

$$\varepsilon=0\Rightarrow\frac{EI}{H^2}\frac{d\theta}{d\varepsilon}=0 \quad (12)$$

$$\varepsilon=1\Rightarrow\theta=0 \quad (13)$$

Sliding- Clamped:

$$\varepsilon=0\Rightarrow\theta=0 \quad (14)$$

$$\varepsilon=1\Rightarrow\theta=0 \quad (15)$$

Sliding-Pinned:

$$\varepsilon=0\Rightarrow\theta=0 \quad (16)$$

$$\varepsilon=1\Rightarrow\frac{EI}{H^2}\frac{d\theta}{d\varepsilon}=0 \quad (17)$$

3. APPLICATION OF THE DTM METHOD

(DTM UYGULAMASI)

If the DTM method detailed in the literature [16,20] is applied to Eq. (11) the following equation is obtained.

$$\theta[k+2]=-\frac{[n*\theta[k]+n*a*(1-b)*\theta[k-1]}{(k+2)*(k+1)} \quad (18)$$

If DTM is applied to the boundary conditions, the following relations are obtained.

Free Clamped:

$$\theta[1]=0 \quad (19)$$

$$\sum_{k=0}^{N}\theta[k]=0 \quad (20)$$

Clamped Sliding:

$$\theta[0]=0 \quad (21)$$

$$\sum_{k=0}^{N}\theta[k]=0 \quad (22)$$

Sliding-Pinned:

$$\theta[0]=0 \quad (23)$$

$$\sum_{k=0}^{N} k * \theta[k] = 0 \qquad (24)$$

If the boundary conditions are applied for each different Euler case, the resulting transcendent equations are obtained as follows.

$$A_1(n)\theta[0] = 0 \text{ (Free Clamped)} \qquad (25)$$
$$A_2(n)\theta[1] = 0 \text{ (Sliding- Clamped)} \qquad (26)$$
$$A_3(n)\theta[1] = 0 \text{ (Sliding -Pinned)} \qquad (27)$$

In Eq. (25), (26), and (27), n values that make θ [0] and θ [1] non-zero are the buckling load factor of the system.

4. DUNKERLEY FORMULA *(DUNKERLEY FORMÜLÜ)*

Dunkerley formula, whose theory and basic principles are given in the literature [21-22], has been adapted to the Stability Analysis of Bars in Water problem.With the Dunkerley formula [21, 22] for the critical buckling loads of the given systems, the system's total buckling load can be found with the help of the buckling loads found separately for the case of buckling in water with the effect of singular force and self-weight (Fig. 2).

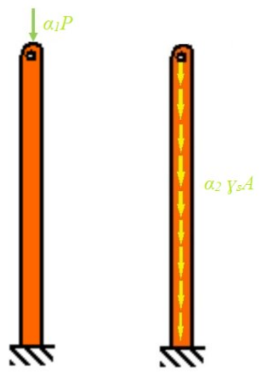

Figure 2. Application of the Dunkerley formula *(Dunkerley formülünün uygulanması)*

The critical buckling load of the given system is found by the following equation.

$$N_{cr} = \alpha_{cr} \frac{EI}{H^2} \qquad (28)$$

For the single load case, the critical buckling load is written as

$$N_{cr1} = \alpha_{cr1} \frac{EI}{H^2} \qquad (29)$$

The critical buckling load of a bar under the influence of its own weight is found by Eq. (30).

$$N_{cr2} = \gamma AH = \alpha_{cr2\gamma} \frac{EI}{H^2} \qquad (30)$$

With the appliction of Dunkerley formula α buckling load factor is found by the help of Eq. (31).

$$\frac{1}{\alpha_{cr}} = \frac{1}{\alpha_{cr1}} + \frac{1}{\alpha_{cr2}} \qquad (31)$$

α's buckling load factors and are given in Table 1 for three different Euler cases considered.

Table 1. α values for Euler cases. *(Euler durumları için α değerleri)*

α_{cr} values		
Euler	α_{cr1}	α_{cr2}
F-C	2.4674	7.8373
C-S	9.6696	18.9563
S-P	2.4674	3.4766

5. RESULTS *(SONUÇLAR)*

To investigate the suitability of the methods presented in the study, the buckling load factors of bars in air and water for three different Euler cases were calculated with the DTM and Dunkerley Formula presented in this study, and the obtained results were compared with the results of the SAP 2000 program in Table 2. Convergence of DTM method according to the number of elements is given in Table 3. In order to investigate the suitability of the methods presented in the study, the SAP 2000 program, which analyzes with the finite element method, was chosen. As the number of elements in the solution increases with SAP 2000, the obtained solution approaches the exact solution. For this reason, sufficient number of elements has been taken into account in SAP 2000 and the SAP 2000 program has been taken as a reference for the convenience of the presented methods. In the modeling with SAP2000 in water, the water effect is considered as the distributed load along the rod. In SAP 2000, the lengths of the bars are defined as 1 m and flexural stiffness EI = 1 kNm^2. Bars are modeled with 5 elements and frame element type is used in modeling. In the study, the first mode was considered as the buckling mode.

Table 2. Buckling load factors of bars in the air and water. *(Havada ve su içindeki çubuğun burkulma yük faktörü)*

In the air					
Euler	DTM (a)	Dunkerley (b)	FEM SAP2000 (c)	%(a-c)/c	%(b-c)/c
F-C	1.8960	1.877	1.8820	0.7439	-0.2657
C-S	6.5484	6.403	6.5497	-0.0198	-2.2398
S-P	1.4466	1.4432	1.4525	-0.4062	-0.6403
In the water					
Euler	DTM (a)	Dunkerley (b)	FEM SAP2000 (c)	%(a-c)/c	%(b-c)/c
F-C	1.9540	1.9355	1.9579	-0.1992	-1.1440
C-S	6.8433	6.6903	6.84471	-0.0206	-2.256
S-P	1.5270	1.5235	1.5328	-0.3784	-0.6067

Table 3. Convergence of DTM method according to the number of elements *(Eleman sayısına bağlı olarak DTM yönteminin yakınsaklığı)*

Euler	5 elements	10 elements	15 elements	16 elements
F-C	1.9782	1.9539	1.9540	1.9540
C-S	5.0724	6.7586	6.8430	6.8433
S-P	1.3619	1.5266	1.5270	1.5270

6. CONCLUSION *(SONUÇ)*

In this paper, two methods are proposed to find the critical buckling load for three different Euler cases of bars in water and buckled under the singular load P at the top and the distributed self-weight. In the first method, the solution of the differential equation representing the stability was performed by the DTM method, while in the second method, the Dunkerley formulation was used to find the critical buckling load. It was observed that the two methods presented from the solved example gave results compatible with the finite element method. It is seen that the DTM method gives results closer to the finite element method than the Dunkerley method. The presented methods are particularly useful in understanding the stability behavior with few parameters.

CONFLICT OF INTEREST STATEMENT *(ÇIKAR ÇATIŞMASI BİLDİRİMİ)*

No potential conflict of interest was reported by the authors.

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