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Research Article

Some commutativity criteria involving endomorphism conditions on prime ideals

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Abstract

In this paper we initiate a new approach consisting to characterize the commutativity of a quotient ring R/P by endomorphisms of R satisfying some algebraic identities involving the prime ideal P. Some well-known results concerning the commutativity of prime (semi-prime) rings have been improved.

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1. Introduction

Throughout this paper R will represent an associative ring with center $Z(R)$. Recall that a proper ideal P of R is said to be prime if for any $x, y \in R$, $xRy \subseteq P$ implies that $x \in P$ or $y \in P$. The ring R is a prime ring if and only if (0) is a prime ideal of R. For any $x, y \in R$ the symbol $[x, y]$ will denote the commutator $xy - yx$; while the symbol $x \circ y$ will stand for the anti-commutator $xy + yx$. An additive mapping $d : R \longrightarrow R$ is a *derivation* if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. Let $a \in R$ be a fixed element. A map $d : R \longrightarrow R$ defined by $d(x) = [a, x] = ax - xa$, $x \in R$, is a derivation on R, which is called *inner derivation* defined by a. Recently, many results in literature indicate how the global structure of a ring R is often tightly connected to the behaviour of additive mappings defined on R (for example, see [13] and [11]). Herstein [3] showed that a 2-torsion free prime ring R must be a commutative integral domain if it admits a nonzero derivation d satisfying $[d(x), d(y)] = 0$ for all $x, y \in R$, and if the characteristic of R equals two, the ring R must be commutative or an order in a simple algebra which is 4-dimensional over its center. Several authors have proved commutativity theorems for prime rings admitting derivations which are centralizing on R. We begin recalling that a mapping $f : R \longrightarrow R$ is called centralizing on R if $[f(x), x] \in Z(R)$ for all $x \in R$. We say that a mapping $f : R \longrightarrow R$ preserves commutativity if $[f(x), f(y)] = 0$ whenever $[x, y] = 0$ for all $x, y \in R$. The study of commutativity preserving mappings has been an active research area in matrix theory, operator theory and ring theory. A mapping $f : R \longrightarrow R$ is said to be strong

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commutativity preserving (SCP) on a subset S of R if $[f(x), f(y)] = [x, y]$ for all $x, y \in S$. A well known result of Posner [14] states that if d is a derivation of the prime ring R such that $[d(x), x] \in Z(R)$, for any $x \in R$, then either $d = 0$ or R is commutative. Mayne [10] obtained the analogous result of Posner for non identity centralizing automorphisms. In [5] Lanski generalizes the result of Posner to a Lie ideal.

More recently several authors considered similar situation in the case the derivation d is replaced by a generalized derivation. More specifically an additive map $F : R \longrightarrow R$ is said to be a generalized derivation if there exists a derivation d of R such that, for all $x, y \in R$, $F(xy) = F(x)y + xd(y)$. Basic examples of generalized derivations are the usual derivations on R and left R-module mappings from R into itself. An important example is a map of the form $F(x) = ax + xb$, for some $a, b \in R$; such generalized derivations are called *inner*. Generalized derivations have been primarily studied on operator algebras. Therefore any investigation from the algebraic point of view might be interesting (see for example [11] and [6]).

During the last two decades, many authors have studied commutativity of prime and semi-prime rings admitting suitably constrained additive mappings acting on appropriate subsets of the rings. Moreover, many of obtained results extend other ones proven previously just for the action of the considered mapping on the entire ring. In this direction, the recent literature contains numerous results on commutativity in prime and semi-prime rings admitting suitably constrained derivations and generalized derivations (for example, see [7–9, 12]).

The purpose of this paper is to study the commutativity of a quotient ring R/P where R is an arbitrary ring and P is a prime ideal of R. The originality in this work is that we will consider endomorphisms of R satisfying some specific algebraic identities involving P and without primeness (semi-primeness) assumption on the ring R.

2. Main results

Throughout this section, the symbol id_R will denote the identity map on ring R. We recall the following Facts which are essential for developing the proof of our results.

Fact 2.1. *Let R be a ring, I a nonzero ideal of R and P a prime ideal of R such that $P \subsetneq I$. If $aIb \subseteq P$, with $a, b \in R$, then $a \in P$ or $b \in P$.*

Fact 2.2. *Let R be a semi-prime ring, I a nonzero ideal of R and $a \in I$ such that $aIa = 0$, then $a = 0$.*

In [[2], Theorem 2] Bell and Daif established that a prime ring R must be a commutative integral domain if it admits a non-identity endomorphism T which is SCP on a nonzero right ideal U of R.

Our aim in the following theorem is to generalize this result in two directions. First of all, we will assume that for all x, y in a nonzero ideal I, "$[T(x), T(y)] - [x, y]$" belongs to a prime ideal P rather than $[T(x), T(y)] - [x, y] = 0$ and the ring is not necessary prime. Secondly, we will treat a more general differential identity involving two endomorphisms f and g. More precisely we will prove the following result.

Theorem 2.3. *Let R be a ring, I a nonzero ideal of R and P a prime ideal of R such that $P \subsetneq I$. If f and g are endomorphisms of R such that*
$[f(x), g(y)] - [x, y] \in P$ for all $x, y \in I$, then one of the following holds:
1) $(f - id_R)(R) \subseteq P$ and $(g - id_R)(R) \subseteq P$
2) R/P is a commutative integral domain.

Proof. Suppose that R/P is non commutative. We are given that

$$[f(x), g(y)] - [x, y] \in P \quad \text{for all } \; x, y \in I. \tag{2.1}$$

Replacing y by yx in (2.1), we get

$$[f(x), g(y)]g(x) + g(y)[f(x), g(x)] - [x, y]x \in P \quad \text{for all} \;\; x, y \in I. \tag{2.2}$$

Since $[f(x), g(x)] \in P$ by (2.1), the relation (2.2) becomes

$$[f(x), g(y)]g(x) - [x, y]x \in P \quad \text{for all} \;\; x, y \in I, \tag{2.3}$$

so that

$$([f(x), g(y)] - [x, y])g(x) + [x, y](g(x) - x) \in P \quad \text{for all} \;\; x, y \in I. \tag{2.4}$$

Using the relation (2.1), one can see that

$$[x, y](g(x) - x) \in P \quad \text{for all} \;\; x, y \in I, \tag{2.5}$$

and thus

$$[x, y]I(g(x) - x) \subseteq P \quad \text{for all} \;\; x, y \in I. \tag{2.6}$$

Applying Fact 2.1, we have either $[x, I] \subseteq P$ or $(g(x) - x) \in P$ for all $x \in I$. Let us set $I_1 = \{x \in I \,/\, [x, I] \subseteq P\}$ and $I_2 = \{x \in I \,/\, g(x) - x \in P\}$. Then it can be seen that I_1 and I_2 are two additive subgroups of I whose union is I. According to Brauer's trick we have either $I = I_1$ or $I = I_2$. Since R/P is non commutative, then $(g - id_R)(R) \subseteq P$ and our hypothesis leads to

$$[f(x), y] - [x, y] \in P \quad \text{for all} \;\; x, y \in I. \tag{2.7}$$

Substituting xy for x, one can verify that

$$[f(x), y]f(y) + f(x)[f(y), y] - [x, y]y \in P \quad \text{for all} \;\; x, y \in I. \tag{2.8}$$

In such a way that

$$[f(x), y]f(y) - [x, y]y \in P \quad \text{for all} \;\; x, y \in I.$$

Thereby obtaining

$$([f(x), y] - [x, y])f(y) + [x, y](f(y) - y) \in P \quad \text{for all} \;\; x, y \in I. \tag{2.9}$$

By view of the expression (2.7), we obtain

$$[x, y](f(y) - y) \in P \quad \text{for all} \;\; x, y \in I.$$

Reasoning as above, one can easily see that $(f - id_R)(R) \subseteq P$. This completes the proof of our result. □

If we let R to be a prime ring, then the ideal $P = (0)$ is prime. In this case we obtain the following result which is an improved version of [[2], Theorems 2].

Corollary 2.4. *Let R be a prime ring and I a nonzero ideal of R. If f and g are endomorphisms of R not both trivial, then the following assertions are equivalent:*
1) $[f(x), g(y)] - [x, y] = 0$ *for all* $x, y \in I$;
2) *R is a commutative integral domain.*

In [[4], Theorem 2], it is proved that if R is a 2-torsion free semi-prime ring admitting endomorphisms f and g such that $[f(u), g(v)] - [u, v] = 0$ for all u, v in a nonzero ideal U of R, then g is commuting on U. The following corollary proves that the similar conclusion remains valid without the characteristic assumption on the semi-prime ring R.

Corollary 2.5. *Let R be a semi-prime ring and I a nonzero ideal of R. If f and g are endomorphisms of R such that*

$$[f(x), g(y)] - [x, y] = 0 \quad \textit{for all} \;\; x, y \in I$$

then g is commuting on I.

Proof. Assume that $[f(x),g(y)]-[x,y]=0$ for all $x,y\in I$. By view of the semi-primeness of the ring R, there exists a family Γ of prime ideals such that $\bigcap_{P\in\Gamma}P=(0)$, thereby obtaining $[f(x),g(y)]-[x,y]\in P$ for all $P\in\Gamma$ and for all $x,y\in I$. Invoking the proof of Theorem 2.3 by relation (2.6) we get

$$[x,y]r(g(x)-x)\in P \quad \text{for all } r,x,y\in I. \tag{2.10}$$

So that

$$[x,y]r(g(x)x-x^2)\in P \quad \text{for all } r,x\in I. \tag{2.11}$$

Replacing r by rx in (2.10), we can see that

$$[x,y]r(xg(x)-x^2)\in P \quad \text{for all } r,x,y\in I. \tag{2.12}$$

Combining relation (2.11) with (2.12), we can see that

$$[x,y]r[x,g(x)]\in P \quad \text{for all } r,x,y\in I. \tag{2.13}$$

Writing $g(x)y$ instead of y in the last expression, we arrive at

$$[x,g(x)]yr[x,g(x)]\in P \quad \text{for all } r,x,y\in I. \tag{2.14}$$

As a special case of the above relation, when we put $y=y[x,g(x)]$, we can easily obtain for all $P\in\Gamma$

$$[x,g(x)]I[x,g(x)]I[x,g(x)]\subseteq P \quad \text{for all } x\in I. \tag{2.15}$$

Therefore $[x,g(x)]I[x,g(x)]I[x,g(x)]=0$ for all $x\in I$. According to Fact 2.2, we conclude that g is commuting on I. □

Putting $f=g$ and applying the above result, then we can give the following corollary.

Corollary 2.6 ([4], Corollary 3)**.** *Let R be a semi-prime ring and I a nonzero ideal of R. If f is an endomorphism of R such that $f(xy)=xy$ for all $x,y\in I$, then f is commuting on I.*

In light of Corollary 2.6 and with no further assumption to the semi-primeness of the considered ring, it is natural to ask: what can we conclude if we consider two endomorphisms f and g satisfying the more general condition $\overline{f(x)g(y)-xy}\in Z(R/P)$? The purpose of the following theorem is to treat this problem.

Theorem 2.7. *Let R be a ring, I a nonzero ideal of R and P be a prime ideal of R such that $P\subsetneq I$. If f and g are endomorphisms of R such that $\overline{f(x)g(y)-xy}\in Z(R/P)$ for all $x,y\in I$, then $[g(x),x]\in P$ for all $x\in I$.*

Proof. By assumption, we have

$$\overline{f(x)g(y)-xy}\in Z(R/P) \quad \text{for all } x,y\in I. \tag{2.16}$$

Replacing y by yr in the above relation, we get

$$\overline{(f(x)g(y)-xy)g(r)+xy(g(r)-r)}\in Z(R/P) \quad \text{for all } r,x,y\in I. \tag{2.17}$$

Commuting the relation (2.17) with $g(r)$ and using the fact that $\overline{f(x)g(y)-xy}\in Z(R/P)$, we arrive at

$$-xy[r,g(r)]+x[y,g(r)](g(r)-r)+[x,g(r)]y(g(r)-r)\in P \quad \text{for all } r,x,y\in I. \tag{2.18}$$

Substituting rx for x in (2.18) and using it, we can see that

$$[r,g(r)]xy(g(r)-r)\in P \quad \text{for all } r,x,y\in I. \tag{2.19}$$

As a special case of (2.19), when we put $y=[r,g(r)]y$, we may write

$$[r,g(r)]x[r,g(r)]y(g(r)-r)\in P \quad \text{for all } r,x,y\in I. \tag{2.20}$$

Putting $y = yr$ in the last expression, we get $[r, g(r)]x[r, g(r)]y(rg(r) - r^2) \in P$. Right multiplying the relation (2.20) by r, we arrive at

$$[r, g(r)]x[r, g(r)]y(g(r)r - r^2) \in P.$$

Subtracting the two last expressions, we can easily find that

$$[r, g(r)]I[r, g(r)]I[r, g(r)] \subseteq P \quad \text{for all } \ r \in I. \tag{2.21}$$

In light of primeness of P, we conclude that $[r, g(r)] \in P$ for all $r \in I$. □

As an application of Theorem 2.7, we get the following result.

Corollary 2.8 ([4], Theorem 4)**.** *Let R be a semi-prime ring and I a nonzero ideal of R. If f and g are endomorphisms of R such that $f(x)g(y) - xy \in Z(R)$ for all $x, y \in I$, then g is commuting on I.*

Proof. Assume that $f(x)g(y) - xy \in Z(R)$ for all $x, y \in I$. So that $[f(x)g(y) - xy, r] = 0$ for all $r, x, y \in I$. According to semi-primeness, there exists a family Γ of prime ideals such that $\bigcap_{P\in\Gamma} P = (0)$ thereby obtaining $[f(x)g(y) - xy, r] \in P$ for all $P \in \Gamma$. Invoking Theorem 2.7, we conclude that $[g(x), x] \in P$ and because of $\bigcap_{P\in\Gamma} P = (0)$, assures that g is commuting on I. □

Corollary 2.9 ([4], Corollary 4)**.** *Let R be a semi-prime ring and I a nonzero ideal of R. If f is an endomorphism of R such that $f(xy) - xy \in Z(R)$ for all $x, y \in I$, then f is commuting on I.*

As a consequence of Theorem 2.7. The following proposition shows that under some additional assumptions, the ring R is commutative integral domain.

Proposition 2.10. *Let R be a prime ring and I a nonzero ideal of R. If f is an endomorphism of R and g is an epimorphism of R such that $f(x)g(y) - xy \in Z(R)$ for all $x, y \in I$, then $f = g = id_R$ or R is a commutative integral domain.*

Proof. Suppose that $f(x)g(y) - xy \in Z(R)$ for all $x, y \in I$. By view of Theorem 2.7, we arrive at $[g(x), x] = 0$ for all $x \in I$. Linearizing the above relation we get

$$[g(x), y] + [g(y), x] = 0 \quad \text{for all } \ x, y \in I. \tag{2.22}$$

Replacing x by yx in (2.22), we arrive at

$$[g(y), y]g(x) + g(y)[g(x), y] + [g(y), y]x + y[g(y), x] = 0 \quad \text{for all } \ x, y \in I. \tag{2.23}$$

Using the fact that $[g(x), x] = 0$, the relation (2.23) yields

$$g(y)[g(x), y] + y[g(y), x] = 0 \quad \text{for all } \ x, y \in I. \tag{2.24}$$

Left multiplying (2.22) by y and combining it with (2.24), we find that

$$(g(y) - y)[g(x), y] = 0 \quad \text{for all } \ x, y \in I. \tag{2.25}$$

Because of g is an epimorphism, we can see that

$$(g(y) - y)[z, y] = 0 \quad \text{for all } \ y, z \in I. \tag{2.26}$$

Accordingly

$$(g(y) - y)I[z, y] = 0 \quad \text{for all } \ y, z \in I. \tag{2.27}$$

By view of Fact 2.1, we get

$$g(y) = y \quad \text{or} \quad [z, y] \in P \quad \text{for all } \ y, z \in I.$$

Consequently, I is union of two additive subgroups I_1 and I_2, where

$$I_1 = \{y \in I \mid g(y) = y\} \ \text{ and } \ I_2 = \{y \in I \mid [I, y] = 0\}.$$

Since a group cannot be union of two of its proper subgroups, we are forced to conclude that $I = I_1$ or $I = I_2$. If $I = I_1$ then $g = id_R$ and the hypothesis becomes

$$[(f(x) - x)y, r] = 0 \quad \text{for all} \quad r, x, y \in I, \tag{2.28}$$

so that

$$(f(x) - x)[y, r] + [(f(x) - x), r]y = 0 \quad \text{for all} \quad r, x, y \in I. \tag{2.29}$$

Writing ys instead of y in (2.29) and using it, we find that

$$(f(x) - x)I[s, r] = 0 \quad \text{for all} \quad r, s, x \in I. \tag{2.30}$$

Using Fact 2.1, we get either $f(x) = x$ for all $x \in I$ or $[s, r]$ for all $r, s \in I$. By the first case we have $f = id_R$. In the second case the ring R is a commutative integral domain. Now assuming that $I = I_2$, then $[I, y] = 0$ for all $y \in I$. Therefore R is commutative. □

In 2004 M. Anwar et al [[1], Theorem 2.2] proved this result: Let R be a semi-prime ring and f an endomorphism of R, g an epimorphism of R such that $[f(x), g(x)] \in Z(R)$ for all $x \in R$, then $[f(x), g(x)] = 0$ for all $x \in R$.
Motivated by this result our aim in the following theorem is to investigate a more general context of differential identity involving a prime ideal without the semi-primeness assumption on the considered ring. This approach allows us to generalize the preceding results, indeed we will assume that the expression $\overline{[f(x), g(x)]} \in Z(R/P)$ for all x in a nonzero ideal I belong to the center of a quotient ring R/P where R is any ring and P is a prime ideal of R rather than $[f(x), g(x)] \in Z(R)$.

Theorem 2.11. *Let R be a ring, I a nonzero ideal of R and P be a prime ideal of R such that $P \subsetneq I$. If f is an endomorphism of R and g an epimorphism such that $\overline{[f(x), g(x)]} \in Z(R/P)$ for all $x \in I$, then $[f(x), g(x)] \in P$ for all $x \in I$.*

Proof. Suppose that

$$\overline{[f(x), g(x)]} \in Z(R/P) \quad \text{for all} \quad x \in I. \tag{2.31}$$

Linearizing this relation, we obviously obtain

$$\overline{[f(x), g(y)] + [f(y), g(x)]} \in Z(R/P) \quad \text{for all} \quad x, y \in I. \tag{2.32}$$

As a special case of (2.32) when we put $y = x^2$, we may write

$$\overline{[f(x), g(x)]g(x) + g(x)[f(x), g(x)]} + \overline{f(x)[f(x), g(x)] + [f(x), g(x)]f(x)} \in Z(R/P). \tag{2.33}$$

Using the fact that $\overline{[f(x), g(x)]} \in Z(R/P)$, we can see that

$$2(\overline{[f(x), g(x)]g(x) + [f(x), g(x)]f(x)}) \in Z(R/P) \quad \text{for all} \quad x \in I. \tag{2.34}$$

Commuting the last expression with $g(x)$ and using the hypothesis, we arrive at

$$2\overline{[f(x), g(x)]}^2 = \overline{0} \quad \text{for all} \quad x \in I, \tag{2.35}$$

which implies that

$$(2\overline{[f(x), g(x)]})^2 = \overline{0} \quad \text{for all} \quad x \in I. \tag{2.36}$$

Since the center of a semi-prime ring R/P contain no nonzero nilpotent elements, then the last relation reduces to

$$2\overline{[f(x), g(x)]} = \overline{0} \quad \text{for all} \quad x \in I. \tag{2.37}$$

Linearizing the above expression, one can see that

$$2(\overline{[f(x), g(y)] + [f(y), g(x)]}) = \overline{0} \quad \text{for all} \quad x, y \in I. \tag{2.38}$$

On the other hand, by using (2.31) and (2.37) we can see that

$$\overline{[f(x), g(x)^2]} = \overline{2[f(x), g(x)]g(x)} = \overline{0} \quad \text{for all} \quad x \in I. \tag{2.39}$$

Invoking relations (2.32), (2.37) and (2.38), we obtain

$$\overline{[f(x),g(xy)+g(yx)]+[f(y),g(x)^2]}=2\big(\overline{([f(x),g(y)]+[f(y),g(x)])g(x)+[f(x),g(x)]g(y)}\big)=\overline{0}. \tag{2.40}$$

Replacing y by yx in (2.40), we arrive at

$$\overline{(g([x,y])+2g(yx))[f(x),g(x)]+([f(x),g(xy)+g(yx)])g(x)} \\ +\overline{[f(y),g(x)^2]f(x)+f(y)[f(x),g(x)^2]}=\overline{0} \quad \text{for all} \quad x,y\in I.$$

According to (2.37) and (2.39), the above relation becomes

$$\overline{[f(x),g(xy)+g(yx)]g(x)+g([x,y])[f(x),g(x)]+[f(y),g(x)^2]f(x)}=\overline{0} \quad \text{for all} \quad x,y\in I. \tag{2.41}$$

Right multiplying (2.40) by $g(x)$ and then combining it with (2.41), we find that

$$\overline{g([x,y])[f(x),g(x)]+[f(y),g(x)^2](f(x)-g(x))}=\overline{0} \quad \text{for all} \quad x,y\in I. \tag{2.42}$$

On the other hand, by using (2.32) and (2.38), we obtain

$$\begin{aligned}\overline{[f(y),g(x)^2]} &= \overline{[f(y),g(x)]g(x)+g(x)[f(y),g(x)]} \\ &= \overline{-g(x)[f(x),g(y)]-[f(x),g(y)]g(x)}.\end{aligned}$$

Further the equation (2.42) reduces to

$$\overline{[g(x),g(y)][f(x),g(x)]-\Big(g(x)[f(x),g(y)]+[f(x),g(y)]g(x)\Big)(f(x)-g(x))}=\overline{0}.$$

Since g is an epimorphism of R, then the last expression yields

$$\overline{[g(x),z][f(x),g(x)]-\Big(g(x)[f(x),z]+[f(x),z]g(x)\Big)(f(x)-g(x))}=\overline{0}, \tag{2.43}$$

for all $x\in I$ and for all $z\in R$. As a special case of (2.43) when we put $z=f(x)$, we may write $\overline{[f(x),g(x)]}^2=\overline{0}$. Since the center of a semi-prime ring contain no nonzero nilpotent elements, we conclude that $[f(x),g(x)]\in P$ for all $x\in I$. This completes the proof of our theorem. □

As a consequence of Theorem 2.11, we get the following corollary which is an improved version of [[1], Theorem 2.2].

Corollary 2.12. *Let R be a semi-prime ring and I a nonzero ideal of R. If f is an endomorphism of R and g is an epimorphism of R such that $[f(x),g(x)]\in Z(R)$ for all $x\in I$, then $[f(x),g(x)]=0$ for all $x\in I$.*

Proof. Suppose that $[f(x),g(x)]\in Z(R)$ for all $x\in I$, then $[[f(x),g(x)],r]=0$ for all $x,r\in I$. According to semi-primeness, there exists a family $\mathcal{P}$ of prime ideals P such that $\cap_{P\in\mathcal{P}}P=(0)$. Accordingly $\overline{[f(x),g(x)]}\in Z(R/P)$ for all $x\in I$ and for all $P\in\mathcal{P}$. Using Theorem 2.11, we obtain $[f(x),g(x)]\in P$ for all $x\in I$ and for all $P\in\mathcal{P}$, thus we conclude that $[f(x),g(x)]\in\cap_{P\in\mathcal{P}}P=(0)$. □

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