Journal of Education and Future, year: 2014, issue:5, 39-56

A Study on the Forms of Perception of 7th Grade Students towards the Concept of Proof[*]

Ebru Aylar[**]

ABSTRACT

Mathematical proof is important in mathematics teaching in terms of the comprehension of mathematical knowledge. Thus, proof has critical value in the teaching process in terms of the prevention of memorization in mathematics, the construction of conceptual knowledge, and the realization of meaningful learning. This study aims to determine the perceptions of students towards proof after a teaching process with the objective of developing the perceptions and skills of 7th grade students towards proof. In line with this, an answer was sought for the question on the extent the concept of proof can be acquired by 7th grade students. Accordingly, the study was designed as action research, which is one of the qualitative research approaches, and descriptive analysis was employed in the study. Purposive sampling was preferred in the selection of the study group. The study group of the study consisted of a 7th grade from each of the two schools from the districts of Çankaya and Yenimahalle in the province of Ankara. First of all, a readiness test was applied to the classes in the application process of the study and then proof teaching for 1 hour a week was performed for 14 weeks. After this application, a questionnaire with the objective of determining the level of proof perception of students was utilized and semi-structured in-depth interviews were conducted with 16 students determined as a result of this test. As a result of the study, a development was observed in the perceptions of 7th grades students towards the concept of proof.

Keywords: Proof, proving, justification, perception of proving.

[*] This article includes a section of the doctorate dissertation titled " Examination of 7th Grade Students' Ability on Proving and Their Perception of Proving" and whose advisor is Assoc. Prof. Dr. Yeter Şahiner.

[**] Ress. Assist. Dr., Ankara University, Faculty of Educational Sciences, Department of Secondary School Science and Mathematics Education, Ankara, Turkey. E-mail: eaylar@ankara.edu.tr

7. Sınıf Öğrencilerinin İspat Kavramını Algılayış Biçimleri Üzerine Bir Çalışma*

ÖZ

Bu araştırmada 7. sınıf öğrencilerine ispat kavramının ne oranda kazandırılabileceği sorusuna yanıt üretebilmek amaçlanmıştır. Araştırmada ilk olarak 7. sınıf öğrencileri ile formel ispat yapabilecekleri uygulamalar üzerinde durulmuş, gerçekleştirilen bu uygulamaların ardından ispata yönelik algıları uygulanan sınav ve sonrasında gerçekleştirilen görüşme ile betimlenmeye çalışılmıştır. Bu araştırmada nitel ve nicel veriler birlikte kullanılmıştır. Uygulama sonrası toplanan veriler betimsel bir analize tabi tutulmuştur. Araştırma Ankara ilinde Yenimahalle ve Çankaya merkez ilçelerine bağlı iki ortaokulda, bu okulların birer 7. sınıf şubelerinde gerçekleştirilmiştir. Araştırma toplamda 54 öğrenci ile gerçekleştirilmiştir. Bu makale kapsamında veri toplama aracı olarak öğrencilerin ispata yönelik algılarını betimlemeyi amaçlayan bir soru formu, hazır bulunuşluk sınavının dördüncü sorusu ve bu sınavlarda verilen yanıtların gerekçelerini daha ayrıntılı alabilmek üzere hazırlanan görüşme formu kullanılmıştır. Buna ek olarak öğrencilerin ispat becerilerini geliştirmek üzere hazırlanan uygulama süreci için 14 haftalık ders planı hazırlanmış ve bu süreç pilot uygulama ile sınanarak plana son şekli verilmiştir. Bu araştırma kapsamında öğretim sürecinin etkililiği değerlendirilmeyeceği için ders sürecinin analizi yapılmamış, ders uygulaması sonrası öğrencilerin ispata yönelik kavrayışları betimlenmeye çalışılmıştır. 14 hafta süren uygulamanın ardından öğrencilere 4 sorudan oluşan bir sınav uygulanmıştır. Uygulanan soru formunun ardından, her iki sınıftan da öğrencilerin verdikleri yanıtların çeşitliliğini içerecek şekilde 16 öğrenci seçilmiş ve bu öğrencilerle yarı yapılandırılmış derinlemesine görüşme gerçekleştirilmiştir.

Gerçekleştirilen bu çalışmanın sonunda öğrencilerin örnek vererek doğrulama ile ispat arasındaki farka yönelik farkındalıklarında bir artış gözlenmiştir. Hazır bulunuşluk sınavı ile uygulama sonrası gerçekleştirilen sınavdan elde edilen bulgular karşılaştırıldığında öğrencilerin tümdengelimsel muhakemeyi içeren yanıtlara daha çok yöneldiği görülmüştür. Buna karşın öğrencilerin önemli bir bölümü birkaç durumun denenmesinin ispat için yeterli olduğu düşüncesini taşımaya devam etmişlerdir. Bu düşünceyi taşıyan öğrenciler cebirsel ifadeleri anlama ve uygulamada zorlanmaktadır, ispat için örnek vererek yapılan doğrulamalara yönelmişlerdir. Buna karşın sınav sonrası gerçekleştirilen görüşmelerde bu öğrencilere de yer verilmiştir. Onlarla yapılan görüşmelerde, öğrencilerin bazen zorlanarak ve uzunca tartışarak, bazen de kolaylıkla araştırmacının destek ve yönlendirmeleriyle ispat ile örnekle doğrulama arasındaki farkı algılayabildikleri gözlenmiştir. Sonuç olarak bu araştırmada 7. sınıf öğrencileri ile matematik dersi kapsamında yabancı oldukları ispat kavramına yönelik bir uygulama gerçekleştirilmiş ve bu uygulamanın ardından ispata yönelik algılarında bir gelişme gözlenmiştir.

Anahtar Sözcükler: İspat, ispat yapma, doğrulama, ispat algısı.

---

* Bu makale Doç. Dr. Yeter Şahiner danışmanlığında yürütülen "7. Sınıf Öğrencilerinin İspata Yönelik Algı ve İspat Yapabilme Becerilerinin İrdelenmesi" başlıklı doktora tezinin bir bölümünü içermektedir.

INTRODUCTION

Mathematical proof is an important part of mathematics. The importance of the concept of proof, which is one of the important concepts at the foundation of mathematics and mathematics teaching, (Lee, 2002) is referred to in the literature of both fields. Proof demonstrates the correctness or incorrectness of mathematical information (Tall & Mejia-Ramos, 2006) and it also is of importance in the construction of mathematical knowledge in terms of mathematics teaching.

Proof is an important instrument in the mathematics learning process (Knuth, 2002). According to Senk et al, proof, which is the core of mathematics (cited by: Bahtiyarı, 2010), is not only associated to what is correct, at the same time it is also associated to why it is correct. Proof is emphasized for the purpose of knowing and doing mathematics, constituting the foundation of the perception of mathematics, the comprehension, use, and development of mathematical knowledge (Hanna and Jahnke, 1996; Kitcher, 1984; Polya, 1981). All this emphasis demonstrates the importance of proof and establishes a strong relation between proof and mathematics teaching.

Mathematical proof is important in mathematics teaching in terms of the comprehension of mathematical knowledge. Thus, proof has critical value in the teaching process in terms of the prevention of memorization in mathematics, the construction of conceptual knowledge, and the realization of meaningful learning. Again, the tendency to consider proof as only a subject requiring advanced level mathematical knowledge is continuing. It would not be a mistake to say that report of “The Principles and Standards for School Mathematics” published by NCTM in 2000 created an important breaking point for this tendency. In this report, NCTM discussed proof as an important component of mathematics teaching for every age group and has led to interest and discussions being directed to this area. NCTM does not consider proof as a special activity of certain subjects of the curriculum conducted at certain times. Proof and reasoning must be a part of the process of teaching a lesson no matter what the subject is (NCTM, 2000).

“Reasoning and proof”, which NCTM dealt with as a process standard, is an important method of the comprehension of mathematical content and knowledge. NCTM mentions of the importance of comprehending proof in the understanding mathematics. Contrary to this, the most recent primary school and middle school curriculum attempting to largely include the process and content standards of NCTM in its content is observed to not emphasizing proof at the same degree.

With the transition to the practice of 12 years of compulsory education, curriculums were updated in 2013. Together with this correction, when curriculums are examined, it can be observed that proof is not included in the primary school and middle school curriculum as a concept. The skills that are required to be acquired by students have been listed as problem solving, association, communication, estimation and reasoning and proof has not been dealt with as a skill in the curriculum. In the curriculum, proof is dealt with in the curriculums of the 9^{th} and 11^{th} grades as mathematical skills and competencies that are aimed to be developed. The concept of proof is encountered by students for the first time in the 9^{th} grade in the subject area of "Equation and inequalities" in the proof of the number $\sqrt{2}$ not being a rational number. Afterwards, the proof of the Pythagoras theorem in the right triangle and the theories of sinus/cosines have been included in the curriculum.

Even though the concept of proof has not been included in the curriculum, the skill of reasoning has been defined as *"Reasoning is the process of obtaining new knowledge based on knowledge at hand and utilizing the unique instruments (symbols, definitions, relations etc.) and thinking techniques (induction, deduction, comparison, generalization etc.) of mathematics"* (MEB, 2013:5). Furthermore, it was emphasized that under the skill of reasoning, students were expected to advocate the correctness and validity of mathematical inferences and establish the relations between the relations underlying the rules, without having to memorize the rules. Within this context, it is possible to establish a relation between proof and reasoning, though it is indirect. Together with this, when other skill titles in the curriculum are examined, it is possible to associate ability to prove with some behaviors aimed to be acquired with these skills (Çalışkan, 2012). However, this indirect association is not adequate. In the first section of the master's dissertation completed by Çalışkan in 2012, the activities in the primary school 6^{th}, 7^{th}, and 8^{th} grade mathematics textbooks were analyzed according to the proof levels of Balacheff. The activities in the textbooks concentrated on lower groups according to the levels of Balacheff and for instance it was determined that there was no activities oriented at thought experiment, which is the highest level. All the activities in the textbooks are oriented at the levels of naive empricism and crucial experiment. All these levels are in the pragmatic proof level, which is the lowest level in terms of the development of the thought of proof. Rather than proof by reaching generalizations, they have the tendency of verification with examples. As a result, there is criticism on textbooks and mathematics curriculums not being adequate for developing the skill of proof of students.

In parallel with the increase of teaching of proof in high school and advanced levels of education, a large proportion of studies conducted regarding proof discuss the teaching of proof in primary and middle school and some studies can even state that proof in school mathematics is suitable for students in the advanced secondary education level and middle school students do not understand and do formal proof (Bell, 1976; Fischbein, 1982; Knuth, 2002). Contrary to these discussions, recently there is an increase in studies advocating that proof teaching can start in the early age group starting from preschool education (Ball et al., 2002; Cyr, 2011; Stylianides, 2007).

Aktaş (2002) states that the formation of the concept of proof starts in the preschool period in the cognitive development process. At the same time this process, called the intuitive stage by Piaget, is the transition process to logical thinking. It is aimed that concepts such as classification, pairing, ordering, and comparing constituting the basis of proof are acquired in this stage and these aims assume the role of being a bridge in the transition to logical thinking (Altıparmak, Öziş, 2005).

Children are in the concrete operation stage in primary school. Students should be enabled to reason and make assumptions on concrete objects and circumstances in this stage. Until the 3^{rd} grade students compare objects over physical materials, reason on their differences and similarities, and then reach generalizations over these and after the 3rd grade, they should be encouraged to test and advocate the generalizations they have reached and the assumptions. In order to test the assumptions of students in this level or demonstrate the correctness of their assumptions, they should be able to discuss that a few examples are not adequate, questions each other's reasoning, and use counter examples to refute their assumptions. The concept of mathematical argument forms during these ages (Altıparmak, Öziş, 2005).

In the middle age period, there is the development of abstract thought. Students should be able to should be able to test mathematical arguments with the induction and deduction methods and they should be able to present examples against incorrect expressions, and express mathematical expressions using symbolic language. Students should be encouraged to use the logic of deduction in this period.

All these transfers demonstrate that the idea of proof will gradually develop in the cognitive development process of individuals. This important part of development should be constructed in all stages of teaching starting from kindergarten. For the purpose of organizing the teaching process and presenting various examples in this context, it is necessary to describe the approach of each age group towards proof and describe the level they perceive and perform proof. Studies conducted in this area in our country are relatively limited. The title of "early age period and the relation of proof" for students in Turkey is an unknown title.

For the purpose of finding an answer to the problem above to some extent, the level of perception of 7th grade students towards proof was examined in this study. Primary school students are in the transition from concrete thinking to abstract thinking and start to make deductive inferences. Students enter the domain of algebra learning intensely after the 6th grade and start to use symbolic language more. Algebra is an important step in the development of the abstract thinking skill. The use of symbolic language is reinforced in the 7th grade. Thus, in this study it was assumed that 7th grade students could perform formal judgment by presenting generalisable judgments at a certain level of abstracting and use deductive reasoning.

By taking these factors into consideration, finding an answer to the question of the extent the concept of proof could be acquired by 7th grade students was aimed in this study. First of all in the study, applications that 7th grade students can perform proof were focused on and after these performed applications, their perceptions towards proof were attempted to be determined with the test and interview conducted subsequently.

RESEARCH METHOD

This study was designed as action research, which is one of the qualitative research approaches. Action research can be completed differently from each other. Each definition points out a different content of each action research. In action research, where the quality of the action in the social circumstance is aimed to be developed (Elliot, 1991), the improvement of the practices of educators and informing them can also be aimed (Calhoun, 2002). The changes in the perceptions of 7th grade students towards proof was attempted to be determined in this study at the end of a process attempting to develop the proof skills of 7th grade students. As specified by Yıldırım and Şimşek (2005), this study can be considered under the title of "the trial of a new approach". Students of the 7th grade were applied proof teaching, which is an extracurricular title, and a new approach was trialed and it was aimed that obtained findings were presented to educators as information for the development of teaching practices and curriculums.

In the data collection and analysis process of the action research, various techniques and methods can be used. In the literature, it can be observed that in actions researches, which are largely found in qualitative researches, quantitative methods and techniques have been also used. Kock (1997) evaluates action research being considered as a qualitative research approach to be a myth and also states that the method to be used in action research depends on the investigator and topic of the research and thus, both qualitative and quantitative methods could be used. With the purpose of achieving the objective in action research and support research results Kuzu (2009) states that while qualitative research methods are being used widely, quantitative research methods can also be used. Qualitative and quantitative data have been utilized together in this study.

The data collected after the application has been subject to a descriptive analysis. A validity committee consisting of the investigator and two field experts engaged in the first assessments in both the preparation of measurement instruments utilized in the study and also in the analysis of data. The measurement instruments prepared afterwards have been presented to the opinion of an expert and finalized.

Study Group

This study was conducted with a 7th grade in each of the middle schools located in the districts of Yenimahalle and Çankaya in the province of Ankara. As the investigator can enter the lesson and perform the application at an extent permitted by the school administration and mathematics teacher, the guidance of these two elements were taken into consideration in school selection. In this context, a purposive sample was formed rather than a random one. With the purpose of having the study present richer data, two schools and classes were selected from different socioeconomic sets. In class A located in Çankaya there was a total of 30 students consisting of 15 male and 15 female students. In class B located in the district of Yenimahalle there was a total of 24 students consisting of 11 females and 13 males. The study was conducted with a total of 54 students. Both classes shall be considered as a single study group in this paper.

Following the tests applied in the study, among the 54 students, 16 were selected in a manner covering the diversity of the answers provided by students in these tests. Semi-structured in-depth interviews were conducted with these students at the end of the application.

Data Collection Process and Instruments

A questionnaire aiming to determine the perceptions of students on proof, the fourth question of the readiness test, and interview forms prepared for the purpose of understanding the justifications of the answers in these tests were employed as data collection instruments under the scope of this study. These instruments were prepared by a validity committee consisting of 3 persons and finalized by being submitted to expert opinion. In addition to this, a 14 week lesson plan was prepared for the application process prepared for developing the proof skills of students and this period was tested with a pilot application and finalized. As the effectiveness of teaching process shall not be evaluated under the scope of this study, an analysis of the lesson process was not performed and attempts were made for the purpose of determining the comprehension of students towards proof after the lesson.

In the application process commencing in November, 2012, the readiness test consisting of four questions (the first three questions aims to measure their performance regarding proof, the final question attempts to determine their perception towards proof) was applied to both classes. Afterwards, a 14 week lesson application was performed in classes A and B with 1 hour a week. In this process, the subjects of numbers, sequences, unity, pairs, and divisibility were focused on and examples of proof regarding these subjects were provided in the classroom. A discussion was made on the difference between mathematical verification and proof over the examples discussed in the classroom for 14 weeks.

Following the application that lasted for 14 weeks, a test consisting of 4 questions was applied to the students. Together with mathematical propositions, answers presented as the proofs of these propositions were presented to the students in the questions and students were requested to evaluate and examine the quality of these provided answers of being proof. The data obtained following the application were encoded over the answers of students to each question and the percentages and frequencies of these codes were determined. The reliability of the encoding procedure was ensured through the Investigator Triangulation method.

Following the applied questionnaire, 16 students were selected in a manner covering the diversity of the answer provided by students of both classes and semi-structured in-depth interviews were conducted with these students. In these interviews, students were requested to explain and justify each answer and with the selected response, attempts were made to obtain more detailed information on their perception towards proof by questioning whether or not they reached a generalization. Audio recording of these interviews was performed with the consent of students and they were discussed together with findings obtained from the results of the applied test were reported.

RESULTS AND DISCUSSION

Data obtained under the scope of the study were analyzed in this paper on the basis of questions in the test. With the fourth question of the readiness test, the data of the four questions in the test applied for the purpose of determining the perceptions of students towards proof were examined and the data was encoded according to the responses of the students and presented in the form of a table. This data was detailed and supported with findings from the interviews.

First of all, the existing perceptions of students in conditions provided towards proof shall be attempted to be determined. Afterwards, findings towards the test following the lessons with the purpose of developing the skills and perceptions of students towards proof shall be titled and transferred in line with the purpose of each question.

The 4th question of the readiness test was arranged for the purpose of determining the perception of students towards proof within provided learning processes. In this question, together with a mathematical proposition (the total of 3 subsequent numbers, is three times the number in the middle) 4 options advocating that this proposition is evidence was provided to the students. Students were requested to select which of these options was the proof of the expression and explain the reasons.

In the first provided option, Ayşe verified the proposition by trying a single example and advocated that the verification was proof. In the second option, Belma tried two different number groups and verified the proposition this way and advocated that this was proof. In the option with Mert, algebraic expressions were used and the proposition was proved by such means. In the answer of Zeki, which was the final option, Zeki made a calculation error and reached an incorrect conclusion. By taking this incorrect conclusion into consideration, he advocated that the proposition was incorrect.

Students selecting Ayşe's response as proof were evaluated under Code 1 - "verification with an example – a single example", students selecting Berna's response as proof were evaluated under Code 2- " verification with an example – multiple examples", students selecting Mert's response as proof were evaluated under Code 3- "proof with algebraic expression", and students selecting Zeki's response as proof were evaluated under Code 4 - "providing a counter example".

Findings obtained regarding this question are as follows:

Table 1. Responses of students regarding the 4th question of the readiness test

Codes of answers	**Total (n=51)**	
	f	**%**
Code 1	17	33,3
Code 2	18	35,3
Code 3	6	11,8
Code 4	10	19,6

At the beginning of the application, an important proportion of the students (68.6%, Code 1 + Code 2) think that verification with an example is proof. Students advocating that verification with a single example is proof (Code 1) and verification with more than example (Code 2) is proof are proportionally close to each other. A proportion of 19.6% of students did not notice the error in Zeki's calculation and advocated that the proposition is incorrect. Only 6 students specified the correct answer, which is the proof that includes algebraic expression. The rate of students selecting the proof among provided options is only 11.8%.

If the findings associated to the test following the application of the lesson prepared for the purpose of developing the skills and perceptions of students towards proof,

Proof ? or Verification ?

The first question of the test after the application was designed for the purpose of determining the rate students differentiate proof and verification by providing an example. This question is the simplified form of the 4th question in the readiness test.

In this question, students were presented a mathematical proposition (The total of the 3 subsequent numbers is 3 times the number in the middle) and three options advocating that this proposition is proof. Students are asked which of the options is the proof of the proposition is. In one of these options the expression is verified with a single example (Ayşe's answer) and in the other options the expression was verified with three examples using one, two, and three digit numbers (Belma's answer). In the final answer, proof was performed using symbolic expressions (Mert's answer).

Students selecting Ayşe's answer as proof were assessed under Code 1 - " verification with an example - a single example ", students selecting Berna’s response as proof were evaluated under Code 2- " verification with an example – multiple examples", students selecting Mert’s response as proof were evaluated under Code 3- "proof with algebraic expression", and students selecting Zeki’s response as proof were evaluated under Code 4 - "providing a counter example”. The findings obtained regarding this question are as follows:

Table 2. Answers of students regarding the 1st question

Codes of answers	**Total (n=52)**	
	f	**%**
Code 1	9	16,7
Code 2	26	48,1
Code 3	17	31,5

An important proportion of the students 68.6% (Code 1 + Code 2) have considered verification by providing examples to be proof. Differently to the readiness test, this time students mostly tried many examples and they had the tendency of considering the verification to be proof. When the explanations of students for the answers they selected were examined, 7 of the students among the 9 students that accepted verification with a single example to be proof (Code 1) had stated their justification. Among these students, 4 stated that they considered Ayşe’s answer to be “comprehensible” and “logical”, 3 students sated that they selected answer as proof because it was “easy”.

The students that accepted verification by trying many examples to be (Code 2) proof approached the proof of the proposition with more suspicion and they were satisfied with a single example. As the number of examples tried increased, they advocated that the proof's "plausibility increased" and "correctness was reinforced". Among the 26 students accepting Berna's answer as proof, 21 stated their justification and 4 of these students considered Belma's answer to be "comprehensible" and 17 assessed the answer to be "more accurate" or "more correct".

> *"I believe Berna's answer is correct. Because she had more than one try. Furthermore, she reinforced her proof by providing single, double, and triple digit examples." (Nur)*

> *"I believe Belma's answer is proof, because; Berna proved it with tries and the number gradually increased. Ayşe proved with a single digit but I do not believe one proof is enough. I do not believe it when it is proved with a single proof and I do 2 more examples. Thus, I believe there is a need for 3 examples when proving something." (Beyza)*

The rate of students selecting the correct answer as proof in this question is 31.5%. The 10 students selecting algebraic expression as proof stated that providing examples is not an expression generalizable for all numbers and "definite" and "universal" judgments could be reached with algebraic expressions.

> *"It is Mert's answer because, algebraic expression is universal." (Berk)*

> *"I believe Mert's answer is correct because, even if he tries 10 numbers maybe the other number may not be obtained. We have to be sure. Mert's answer is proof based; this is why I selected Mert." (Sultan)*

In the interviews conducted with students, they were requested to justify their answer to this question in more detail. Students accepting providing examples to be proof were asked whether or not providing an example was sufficient to reach a generalization and attempts were made to question their responses. Even though 31.5% selected the answer with proof in the test, an important proportion of students changed their response and reached the correct answer during the interviews. Students insisting that verification with an example was proof, generally consider that algebra is not comprehensible. Students avoid algebraic expressions because they do not understand algebra or even if they do understand algebra, they think that their friends would not understand. Dicle is one of these students. She understands the algebraic notation provided in the question but, she does not select the answer with the algebraic notation. In the interview, Dicle explained her thoughts on the answer she selected and the answer containing algebraic expression as follows:

> ***Investigator:*** *ok, why did you select Berna and not Ayşe?*
> ***Dicle:*** *hmmm, [she reads Ayşe's answer] she tried 3, 4, 5. But, she only tried one example. I believe one example is not enough to prove this.*
> ***Investigator:*** *Then, should we try many examples for proof?*

Dicle: *Yes. This is why Berna's one is proof. Because, for instance, we cannot explain everything with a single example. But I believe we can convince the person across us with more than one example.*
Investigator: *OK, what do you think about Mert's answer?*
Dicle: *Mert provided a number; we do not know this number. The subsequent number is 1 greater than this. The other number is 2 greater than this. Thus, it is 3a+3. 3a+3 is three times a+1. This is also correct. But, I believe Berna is better. Because, in these symbols, everyone cannot understand the symbols provided by Mert. Everyone does not know. Thus, I selected Berna, it is more comprehensible.*

Can a proposition be both correct and incorrect?

In the second question of the applied test, students were provided an incorrect proposition (If you multiply any odd number with 3 and add 6 to the product, you will obtain a number that is a multiplier of 6). Refuting the proposition with counter examples (Ceyhun's answer) and also an answer verifying the proposition due to an incorrect calculation when using algebraic expressions (Canan's answer) have been presented as the proofs of this proposition. When students were asked which one of the provided answers was the proof of the proposition, they were also requested to perform separate evaluations for the presented answers.

While one of the presented answers was advocating that the expression was incorrect, the other one advocated that it was correct. With this question, the awareness of students on a mathematical expression not being correct and incorrect at the same time was attempted to be measured. The answers of the students were evaluated under Code 1 - "Ceyhun has proved", Code 2 - "Canan has proved ", and Code 3 - "they are both proof". The findings obtained regarding this question are as follow:

Table 3. Answers of students to the 2nd question

Codes of answers	**Total (n=53)**	
	f	**%**
Code 1	38	70,4
Code 2	9	16,7
Code 3	6	11,1

A majority of students (70.4%) stated that the expression provided was incorrect and expressed that the answer provided by Ceyhun was proof. The rate of students that think the answer containing the algebraic expression is proof (Code 2) and thus, advocated that the provided proposition is correct was only 16.7%. When the justifications written by students in the test was examined, a proportion of these students have made a preference over the idea that algebraic notations contain a generalizable judgment and another proportion (4 out of 9 students) advocated that a single example was inadequate in proving that proposition is incorrect.

Only 11.1% of students were in Code 3, in other words they stated that both answers were proof and advocated that it was correct and incorrect at the same time. The justification that verification performed with algebraic expression covers a generalizable judgment and thus, the answer of Ceyhun is also correct is a tendency in all students providing this incorrect response. These students, who accepted the incorrectness of Ceyhun's response after examining it, all thought this answer was proof when they saw the algebraic expression in Canan's answer. By advocating that the proposition in the question is both correct and incorrect at the same time, verifying proof, and providing counter examples they have been in a misconception on proof being together for the same proposition. In the interview conducted with Selda, who is one of these students, she was requested to detail her response.

> ***Selda:*** *They both seemed to be correct to me. I believe I could not exactly express it there but, both seem to be correct. Because, I don't how to say it, Canan's answer demonstrated it in general like a formula. However, Ceyhun's answer turns out to be incorrect even if we do it according to that formula. Actually, Ceyhun seems closer to me but, both have done it correctly, proved it.*

Even though Selda, who expressed her opinion as above, considers that the correct response is proof, the symbolic notation in the other answer caused her confusion. She was also asked to question whether or not a mathematical expression could be correct and incorrect at the same time and she explained that the expression given at the end of the discussion stated that the proof was Ceyhun's answer.

Orhan, another student, was in Code 2 with his response. By explaining his justification as;

> "*I believe Ceyhun answered incorrectly because Ceyhun answered by sampling and sampling cannot always be correct. Canan answered correctly. Because, Canan based these problems on formulas and formulas always provide correct answers."*

He demonstrated his view on an example indicating the contrary to a proposition not being adequate in proving that a proposition is incorrect. In the conducted discussion, Orhan noticed his mistake and explained that he answered as such because he did not look over the answers in detail because he saw the algebraic expression and thought that the algebraic notation was proof. Among the 9 students, the 4 students that advocated that Canan's answer was proof and not Ceyhun's answer stated in their test paper that a single expression was inadequate in advocating that an expression is incorrect, in a manner similar to Orhan's statement. Based on the idea that the examples used are inadequate in proving the correctness of a proposition, these students evaluated the example used by Ceyhun and made a mistake by evaluating the example presented for the purpose of proving that the proposition is incorrect in this axis.

Can a proposition have more than one proof?

In the third question, students were presented three answers regarding the proposition (**The total of two odd numbers is always an even number**). One of these is proof using algebra (Cem's proof), the other is proof through explanation (Buse's proof), and the final one is verification by providing an example (Mehmet's answer). In this question, students were asked which of the provided answer/answers was proof. With this question the aim was to observe whether or not students were aware that a proposition could be proved by more than one method. Furthermore, an attempt was made to determine whether or not they were aware of the relation between proof and presenting a generalizable proof.

In this question, students could select more than one answer as proof. Those selecting Mehmet's answer were encoded under Code 1, those selecting Buse's answer under Code 2, those selecting Cem's answer under Code 3. The answer desired to achieve in this question was Code 2 and Code 3.

In the analysis of the answers of students, these codes were evaluated under three main titles: students accepting only verification with an example as proof, students including verification with an example in their answer, and students that include proofs in their answers. The findings obtained from students responding to the question are as follows:

Table 4. Answers of students to the 3rd question

Codes of Answers		**Total (n=49)**	
		f	**%**
Students selecting verification with an example	Code 1	12	22,2
Students selecting verification with an example in addition to answers including proof	Code 1 & Code 2	4	7,4
	Code 1 & Code 3	7	13
	Code 1 & Code 2 & Code 3	6	11,1
	Total	17	31,5
Students only selecting he provided proofs	Code 2	4	7,4
	Code 3	11	20,4
	Code 2 & Code 3	5	9,3
	Total	20	37

The rate of students selecting verification with only an example as proof was lower in this question compared to other questions. This rate was only 22.2%. Contrary to this, the rate of students that include verification by providing an example together with proofs in the answers they have selected as proofs is 315% when all students are taken into consideration. When this is considered together with students that have selected verification with an example as proof, it is revealed that half of the students are clearly not aware of the difference between proof and verification.

Contrary to this, while the rate of students that selected proof through both algebraic and expressional means as an answer was 37%, the rate of students that answered correctly by specifying both proofs was only 9.3%. Among these students Orhan explained his justification with the expression of;

> "*I believe Cem and Buse answered correctly, because Cem used formulas in the problems and Buse based it on the characteristics of odd numbers*".

Considering the 1st and 3rd questions in the exam together, it is observed that most of the students accepting the verification as a proof by giving examples in the 1st question selected answers including generalizations as proof in this question. Yeliz, who chosed Ayşe's answers (verification with just an example) as proof, and Sude, who chosed Belma's answer (verification with multiple examples) as proof in the first question, chose algebraic notation which was Cem's answer as proof in this question. During interviews, these students were asked about the reason for this difference in their answers; and both of them answered this question in a similar way and said that they didn't pay much attention to what was demanded in the first question with the following statements:

> ***Investigator:*** *When we reach the third question, there were three answers. When I asked "Which one is the proof?", you selected the answer including an algebraic expression. Why did you choose the answer with the algebraic expression in this question while you didn't choose the algebraic expression in the first question?*
> ***Yeliz:*** *I don't know. As it was the first question, maybe, I couldn't get used to the questions yet. Probably, I examined the other questions in a more detailed way.*
> ....
> ***Investigator:*** *When we look at the third question, it is interesting for me: although you accepted verification with example as a proof in the first question, you selected Cem who used algebraic expression in this question. I found it interesting; why do you think there is such a difference between these two questions?*
> ***Sude:*** *I don't know, as the other one was the first question, I think I immediately chose the option that was the most understandable one for me.*

As an overall evaluation of the question, when students are provided with opportunities, in other words when students were provided with an opportunity to make multiple selections among some given options, they enhanced variations in their answers for proofs. In addition to that, the tendency of students to think that a proof should provide a generalisable judgment within the data set where the proposition is defined increased in this question; 37 % of the students stated that they evaluated Buse or Cem's answers as proof for this reason in their statements both during the exam and also in the interviews. Tuna, who selected Buse's answers as proof, stated his reasons as follows:

> ***Tuna:*** *In fact, numbers are infinite; maybe such a number would be found that the result would be wrong. But Buse didn't give an example; she also drew a figure and explained with it. I think this became more understandable. She described the odd number by stating when we group them in twos. In other words, it was defined for all numbers. What she says will be correct for all the numbers.*

Which algebraic expressions?

In the fourth question, three answers related to the provided proposition were given to students (All odd numbers can be written as sum of two consecutive numbers). One of the answers was a proof of the proposition through an algebraic expression (Sedat's answer), another one was a proof of converse of the provided proposition (Sum of two consecutive numbers are always an odd number.) through algebraic expression (Deniz's answer), and the last one was the verification made by giving an example (Berk's answer). In this question, students were asked which one of the provided answers is the proof; and it was assessed whether they could put forward the distinction between two algebraic proofs and also indicate their awareness on the difference between verification and proof by giving an example.

Answers given by the students were coded as follows: those choosing verification with an example Code 1; those choosing proof of the provided proposition Code 2, those choosing proof of converse of proposition Code 3. Findings obtained from the students answered this question are as follows:

Table 5. Answers provided by the students for the question 4

Codes of answers	**Total (n=50)**	
	f	**%**
Code 1	15	27,8
Code 2	21	38,9
Code 3	14	25,9

When the answers provided by the students for this question are examined, a decrease was observed in the rate of students assessing the verification by giving example as a proof compared to the other questions. Students choosing Berk's answer as the proof found Berk's answer simple and understandable and the common tendency observed in the reasons they provided by all the students was the failure in understanding the algebraic expressions. They described their reasons in the test paper as follows:

> *"Berk's answer is both explained and proven, and more detailed. I didn't understand the other answers." (Deniz)*

> *"Berk's answer is correct because it is simple and descriptive. Sedat's answer is too complex; Deniz's answer is wrong because the proof with x numbers is not clear enough. " (Ünal)*

Also during the interviews, students accepting verification with an example as the proof stated that they had difficulty in understanding and applying algebraic expressions.

The option including the proof for the proposition provided in the question, i.e. Sedat's answer, was selected at the highest rate by the students answering this question. 38.9% of the students selected the correct answer. When the reasons given by the students in the test are examined, it is observed that students associated the proof with the proposition and understood the algebraic notation involved in the proof. One of these students, Berk explained that the proof should start with notation of the odd numbers included in the proposition with the following words:

"[*correct answer*] *I think Sedat's answer because it started with the formula of odd number and then reach the proof, it is correct under any conditions."*

On the other hand, some of the students mentioned that Sedat's answer presented a generalisable judgment and it is valid for all the numbers as Recep did with the following statement:

"*Sedat's answer is correct because such a thing happens in the 30th number, and it is the same in the 4th number.*"

On the other hand, students accepting the other answer including the other algebraic answer (Deniz's answer – Code 3) stated that they found Sedat's answer too long and difficult to understand. However, they chose Deniz's answer as the proof of the proposition on the basis of the idea that the algebraic notation is the proof.

CONCLUSIONS

At the end of this study conducted, an increase was observed in the awareness of students on the difference between verification with an example and proof. When the findings from the readiness test and the test after the application are compared, it was observed that students tended to prefer answers containing deductive reasoning. Contrary to this, an important proportion of the students continued to have the idea that the trial of a few conditions would be adequate for proof. Students with this idea have difficulties in understanding and applying algebraic expressions. As a result, they prefer verifications with examples for proof. Contrary to this, these students were included in the interviews conducted after the test. In the interviews conducted with them, it was observed that students could perceive the difference between verification with examples and proof sometimes with difficulty and long discussions and sometimes with ease with support and guidance of the investigator.

In a question in the test at the end of the lesson process for developing the skill and perception towards proof, an incorrect proposition and 2 options as the proof of this proposition were provided to students. In one of these options, the proposition was refuted with a counter example. In another one, there was an error in the calculation in the proof initiated with algebraic proof and as a result it was advocated that the result was correct. A proportion of 16.7% of the students preferred this option when answering because symbolic representation is more mathematical and appears to be like proof. In the studies conducted by Martin and Harel (1989) and

Healy and Hoyles (2000) there has also been reference to this tendency. However, the tendency was determined to be lower in this study compared to the said studies.

In the same question, a proportion of 11.1% of students demonstrated a similar approach and advocated that both options were proof as they considered proof with an additional counter example to be logical. With this answer, students advocated that the proposition was correct and incorrect at the same time and they also advocated that direct proof and proof by providing a counter example for the same proposition can be valid together. In the study conducted by Stylianides and Al-Murani (2010), they referred to the perception of students towards the relation between proving and refuting correctness. The rate of students making this mistake in this study was determined to be lower than the finding of Stylianides and Al-Murani. Again, it is possible for students to make this mistake in the proof teaching process and it should be taken into consideration. This incorrect approach developing during the test was also dealt with in the study and was corrected with face-to-face dialogue with students.

Another finding that is striking in the study is that some students advocated that a single example would not be adequate in the proof of an incorrect proposition. In the study conducted by Galbraith (1981) with students in the 12-17 age group, it was concluded that 18% of students thought that a single counter example was not sufficient in refuting the proposition. In this study, the tendency of students in line with this was lower and this tendency was able to cause students to prefer algebraic expressions in the proof of an incorrect proposition.

In conclusion, an application was performed with 7th grade students in this study under the scope of the mathematics course on the concept of proof, which they were not familiar with, and following this application, a development was observed in their perception towards proof.

REFERENCES

Aktaş, Y. (2002). *Okul öncesi dönemde matematik eğitimi*. Adana: Nobel Tıp kitap evi.

Albayrak Bahtiyari, Ö. (2010). 8. Sınıf Matematik Öğretiminde İspat ve Muhakeme Kavramlarının Ve Önemlerinin Farkındalığı, Yayınlanmamış Yüksek Lisans Tezi, Atatürk Üniversitesi Fen Bilimleri Enstitüsü, Erzurum.

Altıparmak, K.; Öziş, T. (20005). Matematiksel İspat ve Matematiksel Muhakemenin Gelişimi Üzerine Bir İnceleme, *Ege Eğitim Dergisi*, 6 (1), 25-37.

Ball, D.L.; Hoyles, C.; Jahnke, H.N.; Movshovitz-Hadar, N. (2002). The teaching of proof, Proceedings of the International Congress of Mathematicians (Ed. L.I.Tatsien), Vol. III, Higher Education Press, Beijing, 907–920.

Bell, A. W. (1976). A Study of Pupils' Proof-Explanations in Mathematıcal Situations, Educational Studies in Mathematics, 7, 23-40.

Calhoun, E. F. (2002). Actian Research for School Improvement. *Educational Leadership*, 59 (6), March, 18-24.

Cyr, S. (2011). Development of beginning skills in proving and proof writing by elementary school students, Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education, University of Rzeszów, Poland, http://iep.univalle.edu.co/archivos/CENDOPU/CERME%207%202011%20TABLA%20DE%20CONTENIDO.pdf adresinden 13 Şubat 2013 tarihinde indirilmiştir.

Çalışkan, Ç. (2012). 8. Sınıf Öğrencilerinin Matematik Başarılarıyla İspat Yapabilme Seviyelerinin İlişkilendirilmesi, Yayınlanmamış Yükseklisans Tezi, Uludağ Üniversitesi Eğitim Bilimleri Enstitüsü, Bursa.

Elliot, J. (1991). Action Research for Educational Change. Buckingham : Open University Press.

Fischbein, E. (1982). Intuition and Proof. For The Learning of Mathematics, 3 (2), 9-18.

Galbraith, P. L. (1981). Aspects of Proving: A Clinical Investigation of Process. *Educational studies of Mathematics*, 12(1), 1-28.

Hanna, G.; Jahnke, H. N. (1996), Proof and proving. International Handbook of Mathematics Education (Ed. A. J. Bishop, K. Clements, C. Keitel, J. Kilpatric, & C. Laborde), Dordrecht, Netherlands: Kluwer Academic Publishers, p. 877 – 908.

Healy, L., & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education,* 31 (4), 396-428.

Kitcher, P. (1984), The nature of mathematical knowledge. New York: Oxford University Press.

Knuth, E. J. (2002). Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teachers Education,* 5, 61 – 88.

Kock, N. F., Jr. (1997). Myhts in Organizational Action Research: Reflections on a Study of Computer-Supperted Process Redesign Groups. Organizations & Society, 4 (9), 65-91.

Kuzu, A. (2009). Öğretmen Yetiştirme ve Mesleki Gelişimde Eylem Araştırması. *The Journal of International Social Research,* 2(6), 425-433.

Lee, J. K. (2002). Philosophical perspectives on proof in mathematics education, *Philosophy of Mathematics Education,* 16.

M.E.B. (2013b). Orta Öğretim Matematik (9.10.11 ve 12. Sınıflar) Dersi Öğretim Programı, Ankara: Milli Eğitim Basımevi.

Martin, G. & Harel, G. (1989). Proof Frames of Preservice Elementary Teachers, Journal for Research in Mathematics Education, 20 (1), 41-51.

NCTM (National Council of Teachers of Mathematics), (2000). Principles and standards for school mathematics, www.nctm.org

Polya, G. (1981), Mathematical discovery: on understanding, learning and teaching problem solving . New York: Wiley.

Stylianides, A. J. (2007), Proof and Proving in School Mathematics, *Journal for Research in Mathematics Education,* 38 (3), 289-321.

Stylianides, A. J. & Al-Murani, T. (2010). Can a proof and a counterexample coexist? Students' conceptions about the relationship between proof and refutation, *Research in Mathematics Education*, 12(1), 21-36.

Tall, D. & Mejia-Ramos, J. P. (2006). The Long-Term Cognitive Development of Different Types of Reasoning and Proof, presented at the Conference on Explanation and Proof in Mathematics: Philosophical and Educational Perspectives, Essen, Germany.

Yıldırım, A.; Şimşek, H. (2005). Sosyal Bilimlerde Nitel Araştırma Yöntemleri (5. Baskı), Ankara: Seçkin Yayınevi.